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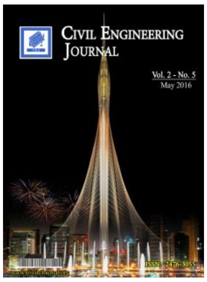

Modeling of Crack Propagation in Layered Structures Using Extended Finite Element Method

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Abstract

Crack propagation in structures is an important issue which is engineers and designers should consider. Modeling crack propagation in structures and study the behavior of this phenomenon can give a better insight to engineers and designers for selecting the construction's materials. Extended finite element method (XFEM) was used successfully in the past few years for simulating crack initiation and propagation in sophisticated and complex geometries in elastic fracture mechanics. In this paper, crack propagation in three-point bending beam including initial crack was modeled based on ABAQUS software. The following consequences were attained through the study of simulation data. First, the effects of young's modulus and fracture energy on force-displacement curve at three-point bending beam were investigated. It was observed that, by increasing the value of young's modulus and fracture energy, three-point bending beam was showed more load carrying against initiation. Second, in multi-layer beam, the effect of young's modulus on force-displacement curve was investigated. In case I (the thin upper layer is harder than the substrate) the value of young's modulus in substrate was kept constant and the amount of young's modulus in thin layer was risen in each step rather than the substrate, the peak in force-displacement curve was ascended and three-point bending beam resisted better against crack initiation. Next, similar conditions was considered in case II (the thin upper layer is softer than the substrate), by decreasing the value of young' modulus in top layer, peak in force-displacement curve was declined and crack initiation was happened in lower loading in each step. Finally, sensitivity analysis for thickness of top layer was conducted and the impact of this parameter was studied.

Keywords: Extended Finite Element Method (XFEM); Fracture; Three-Point Bending Beam; Crack Propagation.

1. Introduction

One of the most significant aspects of structures is their ability to resist the service loads that are subjected to them. The most prevalent reasons that cause early failure in structures are: environmental conditions, construction's error, voids, microscopic flaws, and cracks. Modelling crack propagation is a practical solution to predict failure in structures. Finite element method (FEM) implements different cracks that are occurred in various shapes, sizes, and locations. The requirement of re-meshing the discontinuous of crack's domain is the notable restriction of FEM, which leads to lots of problems for modelling the crack propagation in complex geometry. In order to mitigate the difficulties of computational crack propagation in FEM, Belytschko and Black [1] suggested the extended finite element method (XFEM), as a powerful method to resolve the problem of FEM by enriching in the proximity of the crack and simulate the domain without requiring re-meshing which is based on the partition of unity. Later, XFEM was boosted by Moes et al. [2] and Sukumar et al. [3]. Computer implementation of XFEM was defined by Sukumar et al. [4] then Areias et al. [5] Developed the XFEM to 3D. Dolbow et al. [6] modeled fracture with frictional contact on the crack face and modeling dynamic crack propagation was done by Belytschko et al. [7, 8], Grégoire et al. [9], and Prabel et al. [10]. XFEM is powerful and more effective than boundary element method [11], Re-meshing method [12, 13] and element deletion methods [14], these advantages convinced the researchers to select this method. This paper was presented an XFEM

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procedure to investigated 2D crack propagation in elastic, homogenous and isotropic three-point bending beam with initial crack. The modelling was done by the finite element software ABAQUS version 6.10.1.

2. Extended Finite Element Method

1.1. Basic Formulation

High accuracy and independence to mesh refinement in crack's domain caused this method has been preferred to the other methods. The enriched displacement approximation in 2D crack modeling is written as following:

$$u^h = \sum_{i\in I} u_i\phi_i + \sum_{j\in J} b_j\phi_j H(x) + \sum_{k\in K} \phi_k \tag{1}$$

Where ϕ_i, ϕ_j and ϕ_k are the shape function associated with node i, j and k. H is the modified Heaviside function and is used to introduce discontinuity in crack faces and shows by the following formulation:

$$\mathrm{H(x)} = \begin{cases} -1 & \text{above the carck} \\ +1 & \text{blow the crack} \end{cases} \tag{2}$$

F_l is enrichment function and describes by following:

$$\{F_l(r,\theta)\} \equiv \sqrt{r}\sin\left(\frac{\theta}{2}\right), \sqrt{r}\,cos\left(\frac{\theta}{2}\right), \sqrt{r}\,sin\left(\frac{\theta}{2}\right)sin(\theta), \sqrt{r}\,cos\left(\frac{\theta}{2}\right)sin(\theta) \tag{3}$$

Where (r, θ) are the local polar coordinates in proximity of crack tip. In (1) u_i, b_j and c_l^k are the degree of freedom to common DOFs, I is the set of all nodes in the domain, J and K are the set of nodes enriched by discontinuous enrichment function, J is cut completely by the crack and K exist in two side of crack tip which are shown in Figure 1.

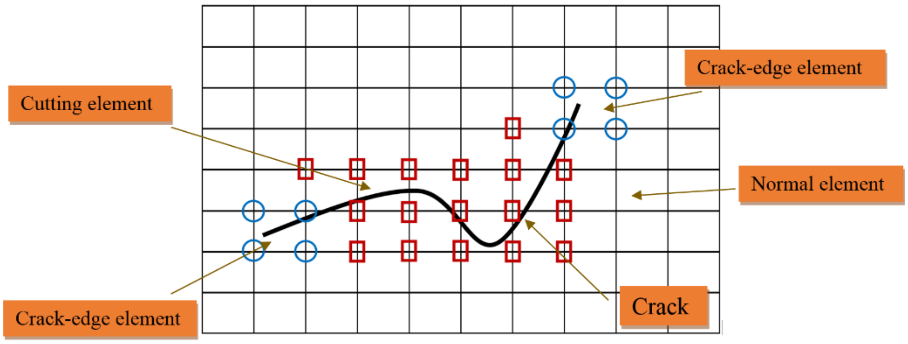

Figure 1. Enriched nodes in the XFEM

Starting crack initiation is a result of beginning the degeneration of enriched elements. In the following, some criteria in ABAQUS implementation which are related to stress and strain are shown below [15].

The maximum nominal stress criterion:

$$f = \left\{\frac{\langle\sigma_{max}\rangle}{\sigma_{max}^0}\right\} \tag{4}$$

The maximum nominal stress criterion:

$$f = \max\left\{\frac{\langle t_\mathrm{n}\rangle}{t_\mathrm{n}^0}, \frac{t_s}{t_s^0}, \frac{t_\mathrm{t}}{t_\mathrm{t}^0}\right\} \tag{5}$$

The quadratic traction- interaction criterion:

$$f = \left\{\frac{\langle t_n\rangle}{t_n^0}\right\}^2 + \left\{\frac{t_s}{t_s^0}\right\}^2 + \left\{\frac{t_t}{t_t^0}\right\}^2 \tag{6}$$

The maximum principal strain criterion:

$$\mathrm{f} = \left\{\frac{\langle\varepsilon_{\max}\rangle}{\varepsilon_{\max}^0}\right\} \tag{7}$$

The maximum nominal strain criterion:

$$f = max\left\{\frac{\langle \varepsilon_n \rangle}{\varepsilon_n^0}, \frac{\varepsilon_s}{\varepsilon_s^0}, \frac{\varepsilon_t}{\varepsilon_t^0}\right\} \quad (8)$$

The quadratic separation- interaction criterion:

$$f = \left\{\frac{\langle \varepsilon_n \rangle}{\varepsilon_n^0}\right\}^2 + \left\{\frac{\varepsilon_s}{\varepsilon_s^0}\right\}^2 + \left\{\frac{\varepsilon_t}{\varepsilon_t^0}\right\}^2 \quad (9)$$

In this paper, crack propagation was occurred when material was reached to max principal stress. In (4), σ_{max}^0 expresses maximum principal stress and the symbol $\langle\ \rangle$ represents Macaulay bracket that disallows the compressive stress leads to damage initiation:

$$\langle \sigma \rangle = \begin{cases} 0 & \sigma < 0 \\ \sigma & \sigma \geq 0 \end{cases} \quad (10)$$

Damage was occurred in the material when f in expression (6) reaches to a value of one. Scalar damage parameter, D, represents damage evolution which was equal to zero at first. In this paper, damage evolution was modeled and its effect on normal and shear stress components is defined as below:

$$t_n = \begin{cases} (1-D)T_n & T_n \geq 0 \\ T_n & T_n < 0 \end{cases} \quad (11)$$

$$t_s = (1-D)T_s \quad (12)$$

$$t_t = (1-D)T_t \quad (13)$$

Where T_n, T_s and T_t represents the normal and shear stress components.

2. Finite Element Modeling

This section divides into three parts; in the first part, a procedure for simulating the growth and propagation of localized tensile cracks [16] was verified. In the second part, the influences of young's modulus and fracture energy on force-displacement curve in one layer three-point bending beam were investigated. In the third part of this section the effect of young's modulus on force-displacement curve in a thin layer beam was studied. Ultimately, sensitivity analysis for thickness of thin layer was discussed.

2.1. Model Verification

This example is a three-point test on a notched beam under the vertical displacement $U = 1\ mm$ that was applied to in the midpoint. Geometry and boundary conditions are given in Figure 2.

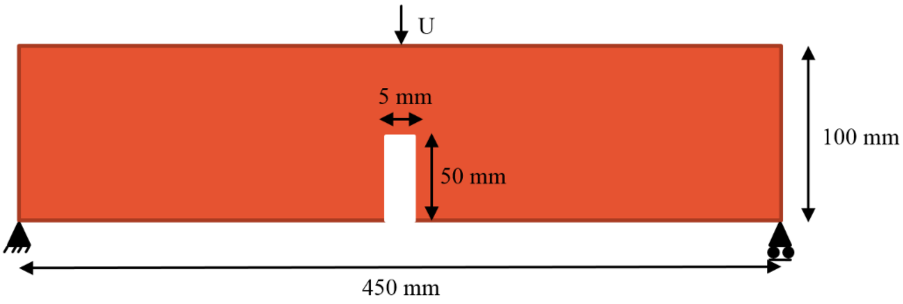

Figure 2. Geometry and boundary conditions of notched beam

Table 1. Mechanical properties [16]

Young's modulus (GPa)	Poisson ratio -	Tensile strength (MPa)	Mode I fracture energy (N/m)
20	0.2	2.4	113

Figure 3 illustrates force-displacement curve for different steps and compares the results between experimental [17] and numerical analysis [16]. The force-displacement curve that was modeled with XFEM is represented in Figure 4. Both curves in Figures (3 and 4) are to support each other.

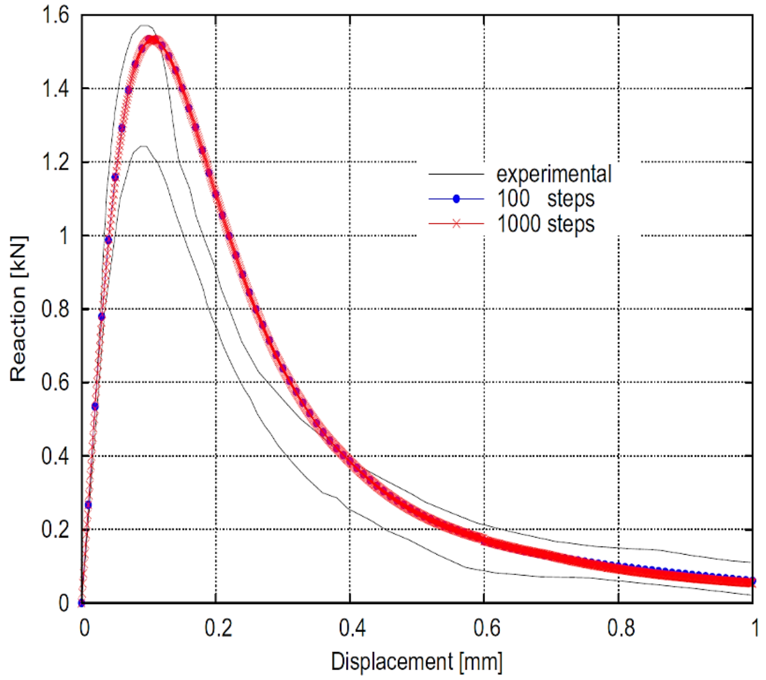

Figure 3. Force-displacement curve for different magnitude of step [16]

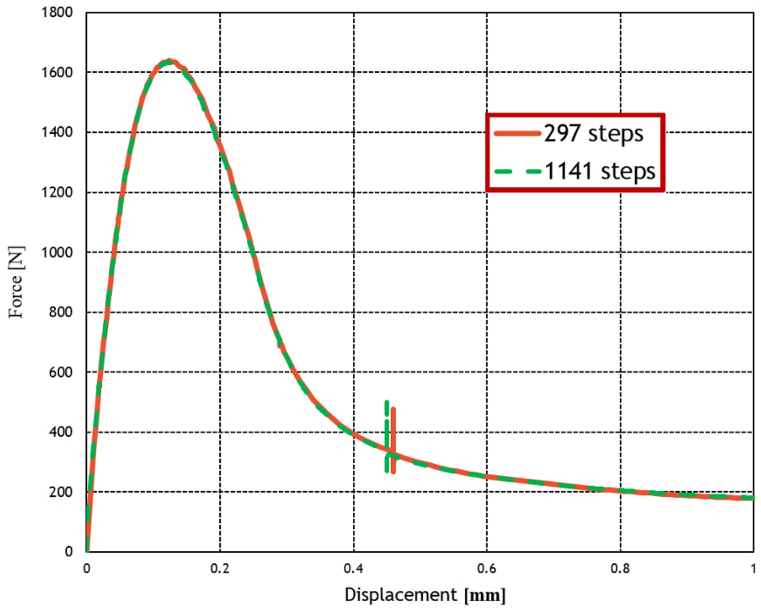

Figure 4. Force-displacement curves obtained from XFEM simulation

2.2. One Layer Three-Point Bending Beam Specimen

In this part, the influence of young's modulus and fracture energy on force-displacement curve were investigated. Geometry and boundary conditions of one layer beam including initial crack with values $L = 55\ mm$, $B = 10\ mm$ and crack length $a = 2\ mm$ are schematically shown in Figure 5. Material was considered as elastic, homogenous and isotropic solids in 2D and plain strain condition was assumed. Vertical displacement was imposed at the upper mid-point of specimen for 1 mm in 60 second and with a constant speed; morever, pure mode I was considered for this study. The material parameters are taken to be as follows in table 2. The finite element mesh density zones and CPE4R plane strain meshes with 7354 elements which was used to attain the results is illustrated in Figure 6.

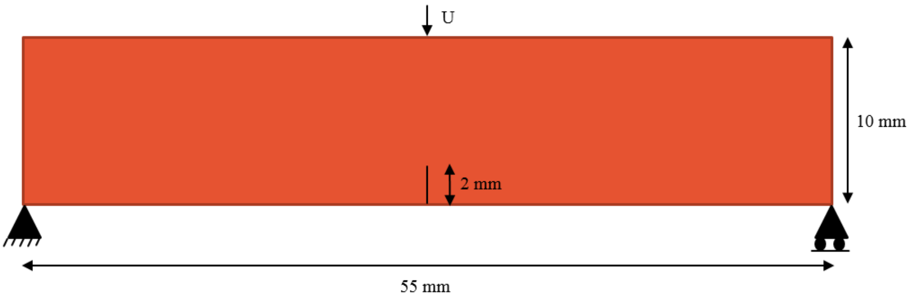

Figure 5. Geometry and boundary conditions for one layer beam

Table 2. Mechanical properties for three-point bending beam

Young's modulus (GPa)	Poisson ratio -	Tensile strength (MPa)	Mode I fracture energy (N/m)
80	0.3	500	15

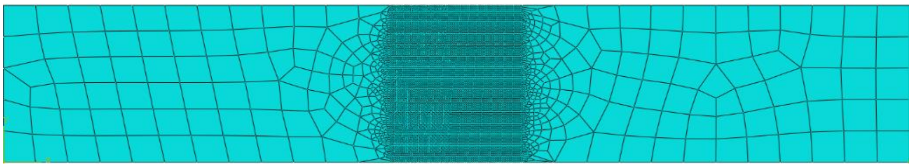

Figure 6. CPE4R plane strain meshes with 7354 elements of one layer three-point bending beam

Hardness is an intrinsic property of materials that affects load carrying; this parameter is dependent on young's modulus. Figure 7 displays force-displacement curve for different values of young's modulus in one layer three-point bending beam taken as: 80, 100, 120, 140, 160, 180, 200, and 210 GPa. The other parameters were kept constant according to table 2. Due to Figure 7, by increasing the value of young's modulus, element's hardness was grown and three point-bending beams was simultaneously shown better resistance against crack initiation. On the other hand, by ascending the value of peak in force-displacement curve, the displacement corresponding to the peak, changes backward and leads to brittle failure rather than before.

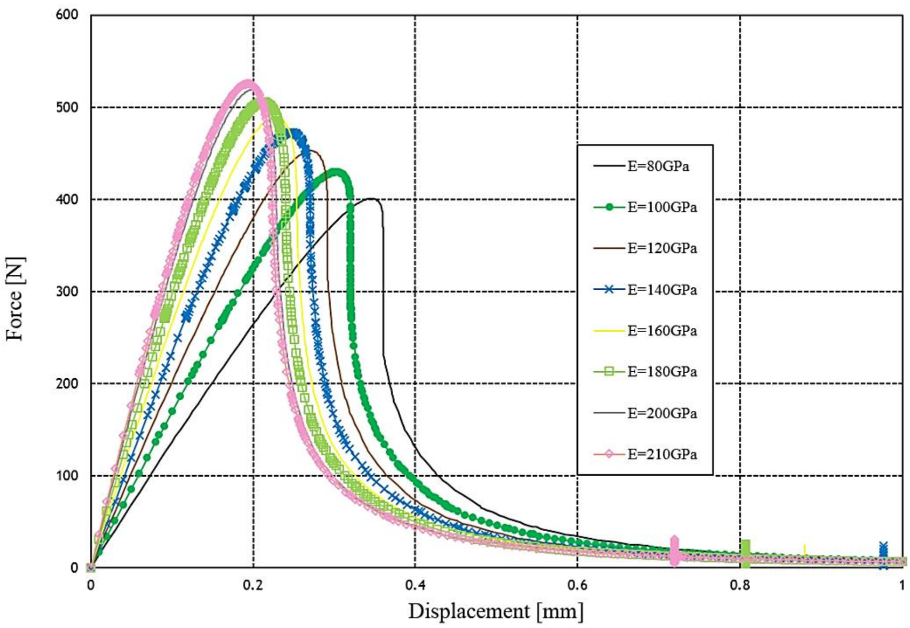

Figure 7. Force-displacement curve for different values of young's modulus

In the second section of this part the influence of fracture energy on force-displacement curve was studied, fracture energy is an intrinsic parameter that independence to the geometry and loading. The values for this parameter were chosen as: 7.2, 11, 15, 19, 23 and 24 N/mm, and the other parameters were kept fixed as stated in Table 2.

According to Figure 8, by growing the amount of fracture energy in each step, peak in force-displacement curve was gone upward and crack initiation was occurred in superior loading.

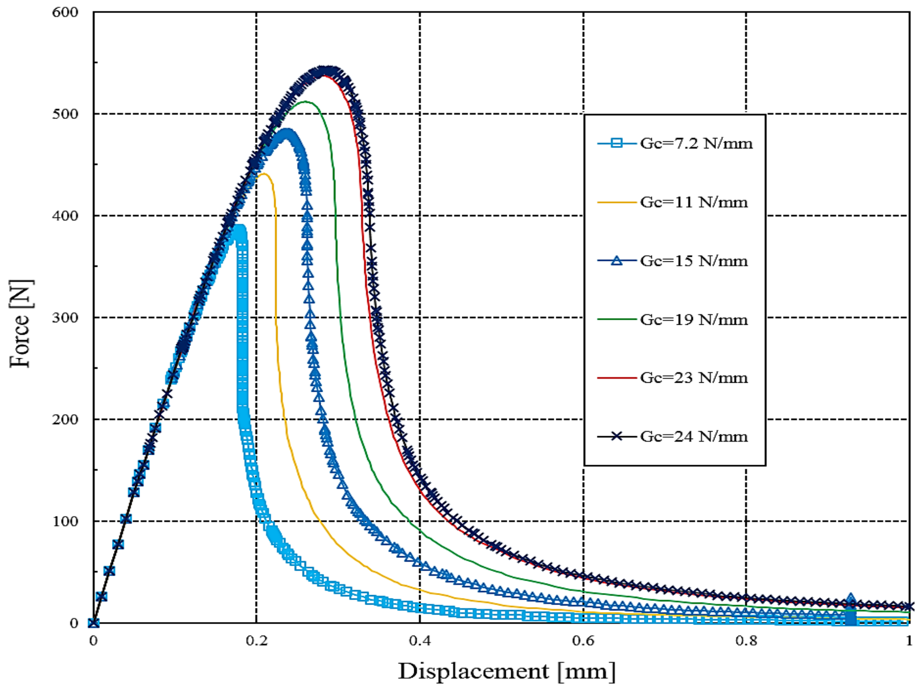

Figure 8. Force-displacement curve for different values of fracture energy

2.3. Two Layer Three-Point Bending Beam Specimen

In this part the effect of young's modulus on force-displacement curve in a multilayer beam was presented. Loading conditions were considered similar to the pervious part. Geometry and boundary conditions are shown in Figure 9, thickness of top layer was taken as $t = 1\ mm$. The interface between two layers was modeled as perfectly-bonding. The beam was constructed by CPE4R plane strain meshes with 7385 element (See Figure 10). In case I the values of young's modulus for top layer were considered as: 100, 120, 140, 160, 180, 200, and 210 GPa, and for substrate was chosen as 80 GPa, the other parameters were considered as the same as Table 2 for both layer. Figure 11 represents force-displacement curve for different values of young's modulus assigned to the top layer. Increasing the value of young's modulus at top layer led to rising the element's participation in absorbing the load rather than the elements in substrate. As a result, superior load for crack initiation has been achieved.

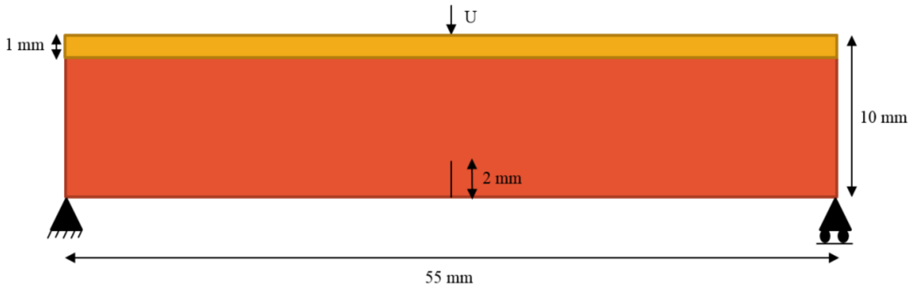

Figure 9. Geometry and boundary conditions for multilayer beam

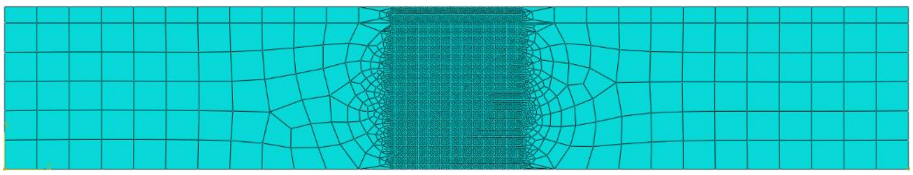

Figure 10. CPE4R plane strain meshes with 7385 element of multilayer beam

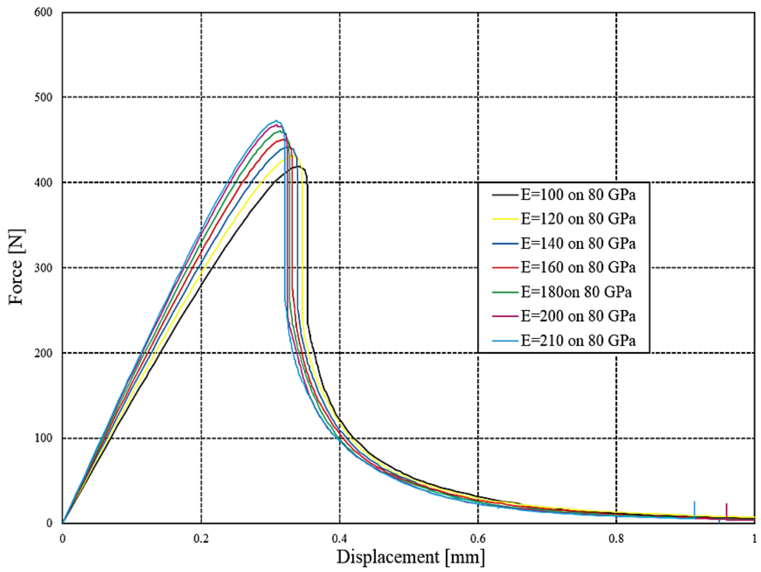

Figure 11. Force-displacement curve for case I (thin upper layer is harder than the substrate)

In the following, the effect of young's modulus on force-displacement curve in case II is depicted in Figure 12. The values of young's modulus for top layer were chosen: 80, 100, 120, 140, 160,180, and 200 GPa, and for substrate was considered as: 200GPa, the other parameters were kept constant similar to pervious section. In contrast with case I, participation of elements in top layer compared with substrate for absorbing load was decreased. As a result, by decreasing the amount of young's modulus in the top layer, peak in load-displacement curve decreased and crack initiation occurred in lower loading.

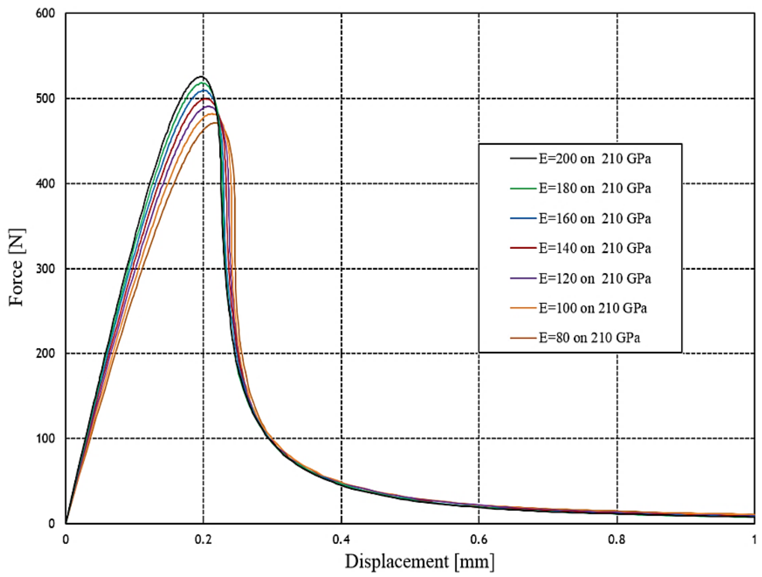

Figure 12. Force-displacement curve for case II (thin upper layer is softer than the substrate)

In this study, a sensitivity analysis for thickness of top layer in case I and case II with perfectly-bonded was conducted for five different simulation thicknesses namely 1, 1.5, 2, 2.5, and 3 mm. Mechanical material properties for case I are given in Table (3).

Table 3. Mechanical properties for case I (the thin upper layer is harder than the substrate)

Location	Young's modulus (GPa)	Poisson ratio	Tensile strength (MPa)	Mode I fracture energy (N/m)
Top layer	210	0.3	500	15
Sub layer	140	0.3	500	15

Figure 13 depicts force-displacement curve for different thickness layers in case I. By increasing the thickness of top layer, the number of elements in top layer made a larger contribution of the beam in absorbing the load rather than sub's elements and led to the higher peak in force-displacement curve which due to higher crack resistance against propagation.

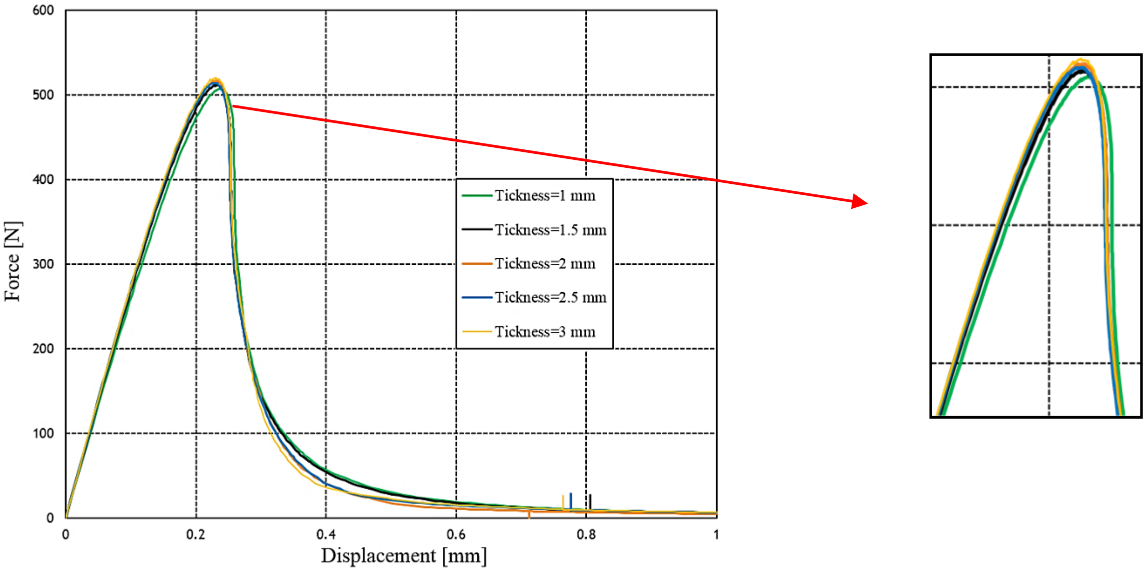

Figure 13. Force-displacement curve at different thickness for case I (thin upper layer is harder than the substrate)

At the end of this part, the sensitivity analyze for thickness of top layer in case II will be discussed. The thicknesses for top layer were considered similar to the pervious step. Figure 14 illustrates force-displacement curve for different thickness in case II. With increasing the thickness of top layer, the number of elements that resist against the load was weaker than before this procedure caused crack resistance was decreased and crack initiation was occurred in lower loading than before.

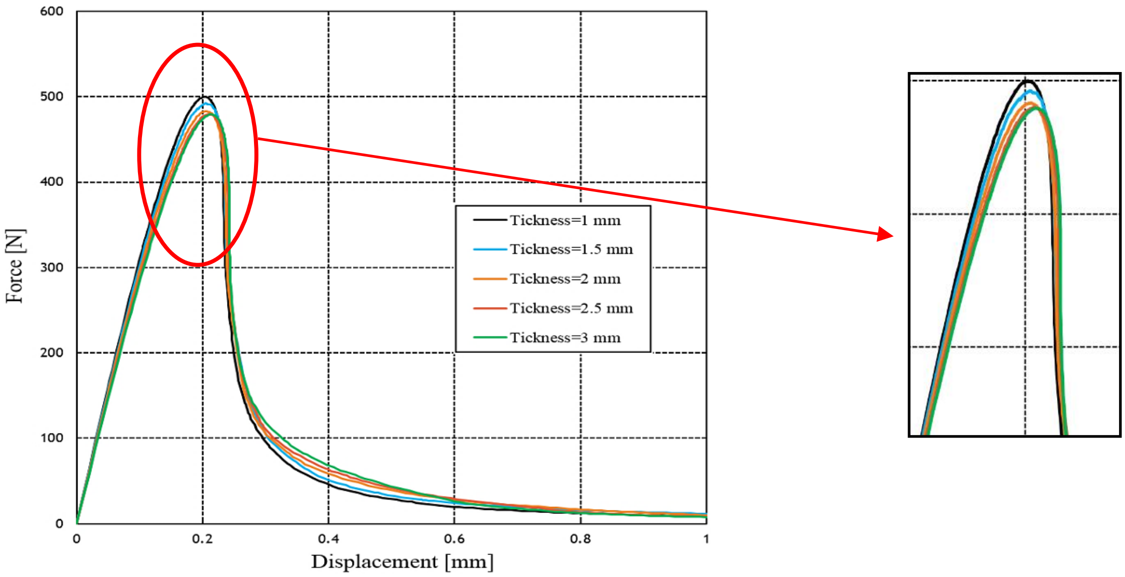

Figure 14. Force-displacement curve at different thickness for case II (thin upper layer is softer than the substrate)

3. Conclusion

In this paper, simulation of crack propagation was presented in three-point bending beam with XFEM procedure. The effect of young's modulus and fracture energy on force-displacement curve in one layer three point bending beam were investigated and the results are as follows. By increasing the value of young's modulus, the peak in force-displacement curve was risen and the beam resisted better against crack propagation; meanwhile, the displacement which is related to the peak in force-displacement curve decreased and failure was gone to the brittle manner. With increasing the value of fracture energy, both the peak and displacement which is related to the peak in force-displacement curve had higher value than before and the behavior of the beam in failure was gone to the ductile manner. In the next part of this study, sensitivity analysis for young's modulus and thickness of top layer in case I and case II has been conducted. In case I, by increasing the amount of young's modulus in top layer, element's participation for absorbing the load were increased and the peak in force-displacement curve was increased, this procedure led to better resistance against crack initiation. On the other hand, in case II, by decreasing the value of young's modulus in top layer, beam was failed in lower loading than before. At the end of this paper, a sensitivity analysis for the thickness of top layer has been conducted. Increasing the thickness of top layer in case I, results in increased the percentage of hard material and the number of stronger elements that was participated in load carrying in each step, as a result, three-point bending beam had better resistance against crack propagation. On the other hand by increasing the thickness of top layer in case II, the percentage of soft layer in the beam was increased and terminated to diminish the capability of beam against crack propagation.

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