



Investigation of Nonlinear Behavior of Concrete on Seismic Performance of an Arch Dam Using Finite Element Method

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Abstract

In this paper the effect of nonlinear behaviour of concrete is investigated on seismic performance of a double curvature concrete dam. The Morrow Point concrete dam has been selected as the case study and dam-reservoir-foundation interaction considered in the model. Finite element method has been used for modelling and analysis of case study by applying the El Centro earthquake components considering nonlinear behaviour of concrete. The obtained results of nonlinear dynamic analysis illustrate the increasing of displacement of dam crest along the river and decreasing of maximum principle stresses in critical points. The results demonstrate the importance of consideration of nonlinear behaviour of material in seismic performance of arch dams to achieve the optimal design of models.

Keywords: Arch Dam; Time History Analysis; Nonlinear Behaviour; Finite Element; Interaction.

1. Introduction

Arch dams are found widely around the world and some of them have been built in high seismic activity areas. Besides, the request for the construction of arch dams in high seismic regions still exists even today due to increasing demand for both water supply and flood protection. The effects of dam-water-foundation interaction on linear response of arch dams subjected to earthquake have been studied by researchers where a combination of finite and boundary element methods has been utilized to simulate the system. Some researchers developed the interaction studies like as added-mass concept of dam-water interaction during earthquake wider form of the Westergaard added-mass [1, 2]. Researchers have investigated the seismic performance of arch dam. They investigated seismic performance of dams according to different parameters such as the effects of near and far fault earthquakes on seismic behavior of the dam-reservoir-foundation system [3] and the effects of different criteria on seismic performance of concrete arch dams [4]. Seismic response of arch dams is affected by interaction of dam-reservoir-foundation rock; the interaction of system is studied by many researchers [5]. The correct performance of a structure is related to the strength and deformation capacity of elements that they are bigger than the requirement imposed on structures due to earthquake. According to the seismic performance of structure, the system must be analyzed by static and dynamic time history methods. The concrete have a complex relationship between stress and strain that it's related to time and loading history. The effects of the materials nonlinearity on the performance of arch dams are investigated by many researchers [6, 7], But in these researches there is low attention to the material nonlinearity of the dam bodies; the researchers often have investigated the geometry nonlinearity and nonlinearity joints behavior of dams [8, 9]. When a serve earthquake is happened the materials are in the nonlinear phase and this is so important. Nonlinear behavior causes increased damping and it absorbed the high energy of earthquake. The nonlinearity behavior is prevented rigid behavior of structure and the maximum capacity is used [10]. Therefore in these studies the material nonlinearity is utilized. The concrete dams are

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designed to resist initial loads such as gravity and hydrostatics. In the design criteria for static loads, the calculated compressive stress is so lower than the compressive strength of the concrete. However the earthquake can produce the great compressive and tensile stresses according to the combination of static and dynamic stresses and exceed of the linear range of concrete [11, 12]. The effect of nonlinearity behavior is less studied on the arch dams [13].

In this study the arch dams with the effects of nonlinear behaviors of dam body concrete is investigated. All the effective domains are considered on the seismic behavior on the model. The mass less model for foundation is selected and the water of reservoir is assumed compressible [14]. The Marrow point double curvature arch dam is selected as case study and the finite element method is used for simulation and analysis using ANSYS software.

2. Governing Equations

The problem is formulated according to the structural and hydrodynamic aspects. The material of the foundation is assumed linearity and the linearity and nonlinearity material are assumed for the dam body.

2.1. Modeling of Dam

The motion equation of dam-water-foundation system by fluid hydrodynamic pressure of the dam-water and the interaction effects of the system is:

$$M\ddot{U} + C\dot{U} + KU = M\ddot{U}_g + F^{Pr} \quad (1)$$

In which M , C and K are mass, damping and stiffness matrices respectively. U and \ddot{U}_g are the vectors of the displacement and the gravity acceleration and F^{Pr} is the fluid pressure load.

2.2. Modeling of the Water Domain

The dynamic equation according to the fluid-structure interaction effects must be considered with Navier-Stokes, momentum and continuity equations of fluid. With the assumption that the water within the reservoir of the studied dam is non-viscous, compressible and with small displacements, continuity equation and momentum will be summarized into wave equation:

$$\frac{1}{c^2} \frac{\partial^2 P}{\partial t^2} - \nabla^2 P = 0 \quad (2)$$

In which P , t and C are the hydrodynamic pressure, time and velocity of the acoustic waves respectively.

$$C = \sqrt{\frac{K}{\rho_0}} \quad (3)$$

In which K and ρ_0 are the stiffness modulus and mass density of the fluid.

2.3. Boundary Conditions

The effect of surface waves is inconsiderable for the free surface of water, and then P is equal to zero at free surface ($P = 0$).

Applied Boundary condition at interface of dam-reservoir and reservoir-foundation is:

$$\rho \vec{a}_{ns} = - \frac{\partial P}{\partial n} \quad (4)$$

a_{ns} is normal component acceleration of the boundary. The Sommerfeld boundary condition is selected for the reservoir domain:

$$\frac{\partial P}{\partial x} = - \frac{1}{C} \frac{\partial P}{\partial t} \quad (5)$$

In which C is the velocity of the acoustic waves.

2.4. Finite Element Formulation

The governing equations of dam-water-foundation system utilizing FEM are extended in matrix form. The system discretization dynamic equation is achieved by finite element formula. Also Reservoir elements matrices are achieved by wave equation discretization. The applied fluid pressure load on the structure is added to formula for applied the interaction effects. The extension of first and second order derivations of displacement are used to achieve the matrices.

2.5. Finite Element Model of Fluid

The Eq. 2 includes the fluid pressure and displacement parameters of structure which includes the unknown parameters to analyze the system. The approximate shape functions for pressure variables and displacement are as follows:

$$P = \{N\}^T \{P_e\} \quad (6)$$

$$u = \{N'\}^T \{u_e\} \quad (7)$$

In which $\{N\}$ and $\{N'\}$ are the element shape function, and $\{P_e\}$ and $\{u_e\}$ are the vectors of node pressure and node displacements respectively.

The second time derivative of variables and pressure variation from above equations is written as follows:

$$\frac{\partial^2 P}{\partial t^2} = \{N\}^T \{\ddot{P}_e\} \quad (8)$$

$$\frac{\partial^2 \{u\}}{\partial t^2} = \{N'\}^T \{\ddot{u}_e\} \quad (9)$$

$$\delta P = \{N\}^T \{\delta P_e\} \quad (10)$$

By applying $\{L\}$ operator on element shape function:

$$[B] = \{L\} \{N\}^T \quad (11)$$

The finite element wave equation is achieved by substitution the Eqs.7-12 in the Eq.1 and applying the boundary conditions to the model:

$$\int_v \frac{1}{C^2} \{\delta P_e\}^T \{N\} \{N\}^T dV \{\ddot{P}_e\} + \int_v \{\delta P_e\}^T [B] [T] dV \{P_e\} + \int \rho_0 \{\delta P_e\}^T \{N\} \{N\}^T [N'] dS \{\ddot{u}_g\} = 0 \quad (12)$$

In which $\{N\}$ is in the normal direction to fluid boundaries. $\{\delta P_e\}$ is a optional parameter that it shows the changing of the node pressure; and it can be neglected.

Also sentences that aren't changing on the element can be placed outside the integral, so:

$$\frac{1}{C^2} \int_v \{N\} \{N\}^T dV \{\ddot{P}_e\} + \int_v [B] [T] dV \{P_e\} + \rho_0 \int \{N\} \{N\}^T [N'] dS \{\ddot{u}_g\} = 0 \quad (13)$$

The wave equation of discrete model by rewriting the above equation in matrix form is:

$$[M_e^P] \{\ddot{P}_e\} + [K_e^P] \{P_e\} + \rho_0 [R_e]^T \{\ddot{u}_g\} = 0 \quad (14)$$

In which $[M_e^P]$, $[K_e^P]$ and $\rho_0 [R_e]^T$ are the matrices are defined as bellow equations:

$$[M_e^P] = \frac{1}{C^2} \int_v \{N\} \{N\}^T dV \quad (15)$$

$$[K_e^P] = \int_v [B]^T [B] dV \quad (16)$$

$$\rho_0 [R_e]^T = \rho_0 \int_s \{N\} \{N\}^T [N'] dS \quad (17)$$

2.6. Finite Element Model of Dam

The finite element equation of the dam is:

$$[M_e] \{\ddot{u}_e\} + [C_e] \{\dot{u}_e\} + [K_e] \{u_e\} = \{F_e\} + \{F_e^{Pr}\} \quad (18)$$

In which $\{F_e^{Pr}\}$ is the vector of fluid applied pressure load; and it achieved with the integration on the contact location:

$$\{F_e^{Pr}\} = \int_s \{N'\} P \{N\} dS \quad (19)$$

In which $\{N'\}$ and $\{N\}$ are the interpolation functions for discretization displacement parameters and normal vector on the contact location:

$$\{F_e^{Pr}\} = \int_s \{N'\} \{N\}^T \{n\} dS \{P_e\} \quad (20)$$

Or

$$\{F_e^{Pr}\} = [Re]\{P_e\} \tag{21}$$

In which:

$$[R_e]^T = \int_s \{N'\} \{N\}^T \{n\} dS \tag{22}$$

The finite element equation of the motion is achieved by substituting the Eq.20 in the Eq.19:

$$[M_e]\{\ddot{u}_e\} + [C_e]\{\dot{u}_e\} + [K_e]\{u_e\} - [Re]\{P_e\} = \{F_e\} \tag{23}$$

It can be extract desired results for system by either simultaneous solving or repeated trial-error method.

2.7. Nonlinear Behavior of Material

Rate independent plasticity analyze refers to a condition that to achieve a certain level of stress, strain is irreversible (Figure 1). The plastic strain will become apparent if the calculated equivalent stress from elastic resolving exceed than the yield stress. In this case plastic strain reduces the stress how the yield criterion is satisfied. Then the plastic strain is changing time and it depends on amount of stress and stress history. Plastic analysis have three pillars: Stress-strain curve and yield criterion, flow rule and stiffness rule [13]. In the multi-axial loading the yield criterion show the function of principle stresses that reagent the yield level.

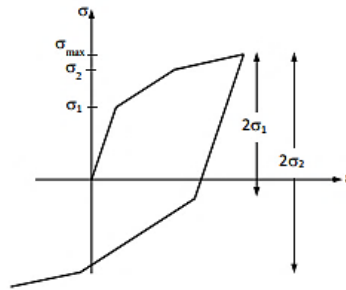


Figure 1. Stress-Strain curve

$$\sigma_e = f[\{\sigma\}] \tag{23}$$

That means the material will be flow when the equivalent stress reach to yield stress. The flow rule submitted the amount and direction of variation of plastic strain and it presented as followed:

$$\{d\varepsilon^{pl}\} = \lambda \left\{ \frac{\partial Q}{\partial \sigma} \right\} \tag{24}$$

In which λ and Q are the plastic ratio and plastic potential function respectively and them show the direction of plastic strain.

The flue rule type is associated flue rule when the plastic potential function is selected as the yield level; in this case the plastic strain direction is in the normal direction of yield level.

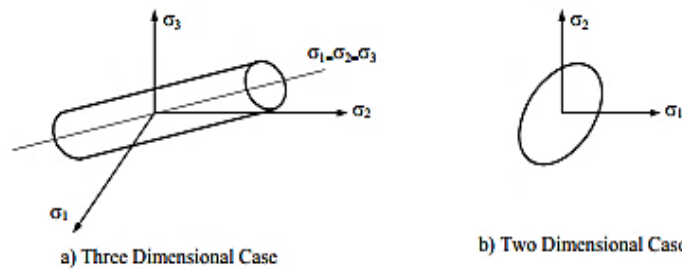


Figure 2. Yield surface of kinematic hardening: a. 2D case, b. 3D case

The hardening rule submitted the transmigration of the yield surface with increasing the plastic strain. In this case the yield equation must be satisfied by increasing the plastic strain. There are two rules for hardening: isotropic and kinematic hardening.

In the kinematic hardening the yield level is transported with no volume changing in the principle stresses space when the plastic strain is increased. But the in the case of isotropic hardening the yield level is changed the volume without transporting (Figure 2).

Figure 3. shows the changing of yield surface according to these two rules:

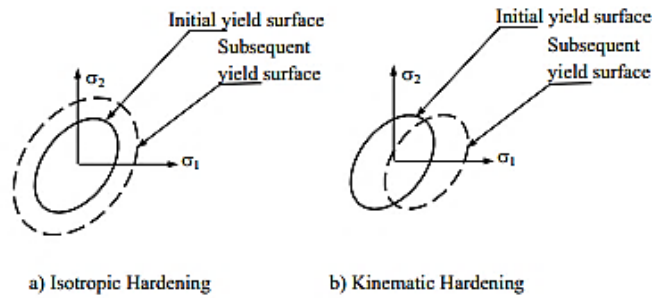


Figure 3. Evolution of yield surface: a. isotropic, b. kinematic hardening

The hardening rule determines that the yield level changes by one of isotropic hardening or kinematic hardening. That means:

$$F\{\sigma\}, k, \{\alpha\} = 0 \tag{25}$$

In which K and $\{\alpha\}$ are elastic work and yield surface transfer respectively. The plastic work is the sum total done plastic work in the whole of loading process that C is the material parameter.

Using the above equations (ANSYS 13 help):

$$k = \int \{\sigma\}^T [M] \{d\varepsilon^{pl}\} \tag{26}$$

$$\{\alpha\} = \int C \{d\varepsilon^{pl}\} \tag{27}$$

$$dF = \left\{ \frac{\partial F}{\partial \sigma} \right\}^T [M] \{d\sigma\} + \frac{\partial F}{\partial \alpha} dk + \left\{ \frac{\partial F}{\partial \alpha} \right\}^T [M] \{d\alpha\} = 0 \tag{28}$$

By substituting in the above equations:

$$\left\{ \frac{\partial F}{\partial \sigma} \right\}^T [M] \{d\sigma\} + \frac{\partial F}{\partial \alpha} \{\sigma\}^T [M] \{d\varepsilon^{pl}\} + C \left\{ \frac{\partial F}{\partial \alpha} \right\}^T [M] \{d\varepsilon^{pl}\} = 0 \tag{29}$$

$$\{d\sigma\} = [D] \{d\varepsilon^{pl}\} \tag{30}$$

$$dk = \{\sigma\}^T [M] \{d\varepsilon^{pl}\} \tag{31}$$

$$\{d\varepsilon^{el}\} = \{d\varepsilon\} - \{d\varepsilon^{pl}\} \tag{32}$$

$$\{d\alpha\} = C \{d\varepsilon^{pl}\} \tag{33}$$

$$M_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \tag{34}$$

$$\lambda = \frac{\left\{ \frac{\partial F}{\partial \sigma} \right\}^T [M] [D] \{d\varepsilon\}}{-\left\{ \frac{\partial F}{\partial k} \right\} \{\sigma\}^T [M] \left\{ \frac{\partial Q}{\partial \sigma} \right\} - C \left\{ \frac{\partial F}{\partial \alpha} \right\} [M] \left\{ \frac{\partial Q}{\partial \sigma} \right\} + \left\{ \frac{\partial F}{\partial \sigma} \right\}^T [M] [D] \left\{ \frac{\partial Q}{\partial \sigma} \right\}} \tag{35}$$

In which λ is the plastic ratio and $[D]$ is the stress-strain matrix. One of the most common models for stress-strain curves is the kinematic multilinear hardening. This model is considered the Buschinger effect. The material behavior

is assumed by considered different parts (interconnected segments but with different slopes) that all of them defined under a total strain (Figure 4). But each part has a yield strength and slope of own. In this case the following happens:

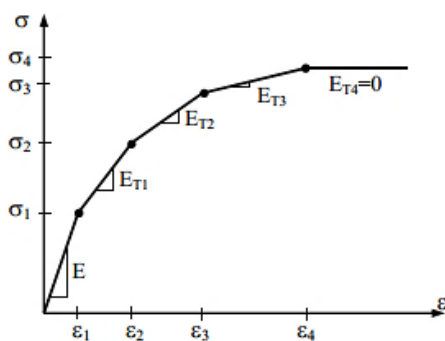


Figure 4. The uniaxial stress-strain curve of multilinear kinematic hardening

Yield strength is calculated for each part. The development of plastic strain is calculated for each part. The total development of plastic strain is calculated. The recovered plastic strain and elastic strain are calculated.

$$W_k = \frac{E - E_{TK}}{E - \frac{1 - 2\vartheta}{3} E_{TK}} - \sum_{i=1}^{k-1} W_i \tag{36}$$

W_k : Weight factor of the part “k”

E_{TK} : The slope of part “k” of stress-strain curve

$\sum_{i=1}^{k-1} W_i$: The calculated total weight factors

$$\sigma_{yk} = \frac{1}{2(1 + \vartheta)} (3E \varepsilon_k - (1 - 2\vartheta)\sigma_k) \tag{37}$$

In which σ_k and ε_k are the break points of stress-strain curve.

The calculation method is based on the following formula:

$$\{\Delta\varepsilon^{pl}\} = \sum_{i=1}^{N_{sv}} W_i \{\Delta\varepsilon_i^{pl}\} \tag{38}$$

In this method the hardening rule is the kinematic hardening. In this study for the nonlinear analyze and the stress-strain model the multilinear kinematic hardening in three dimensional is selected.

3. Model Analysis

The system is analyzed by ANSYS software based on finite element method. This software has the capability to seismic analysis by considering effect of interaction between reservoir, foundation and the irregular geometry domains. $\beta = 0.25$ and $\gamma = 0.5$ are the selected parameters for Newmark method and the $\Delta t = 0.02 \text{ Sec}$ is the selected time step.

The Morrow Point dam is selected as the model for simulation and analyze. Morrow Point dam is a 143m height concrete double curvature dam on the Gunnison River located in Colorado, the first dam of its type built by the U.S. Bureau of Reclamation. The geometry data of dam is followed as table 1:

Table 1. Geometry data

Total dam height (m)	143
Thickness of dam at top (m)	3.66
The maximum thickness at the base (m)	15.73
Length of dam crest (m)	184
Internal curve radius at the crest level (m)	114.3
Internal curve radius at the zero level (m)	71.57

For discretization of system the 868 eight node solid (Solid45) elements are chosen for dam body and foundation and the 875 eight node fluid elements (Fluid30) for simulation of reservoir. The Solid45 and Fluid30 are demonstrated 3D

solid curvilinear geometry of dam and compressibility property of fluid in the reservoir model respectively. Table 2 shows the material properties.

Table 2. Material properties of system

	Dam	Foundation
Weight (N/m ³)	24800	26430
Modulus of elasticity (GPa)	27.5	22
Poison ratio	0.2	0.2

Figures 5 to 8 demonstrate the finite element discretization of system. The length of dam at crest and zero level are divided into 13 equal parts. The height of the dam is divided into 8 equal parts and also the reservoir and the foundation models are meshed comply with dam meshing. The El Centro earthquake ground motion that happened in 1940 is selected for seismic analysis.

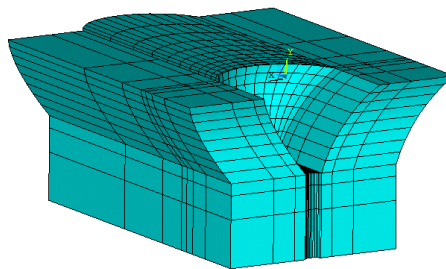


Figure 5. Finite element model of dam-reservoir-foundation system

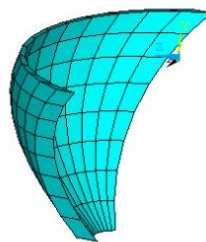


Figure 6. Finite element model of dam body

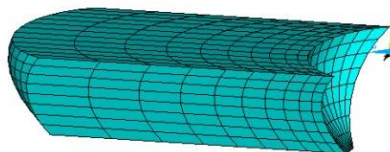


Figure 7. Finite element model of reservoir

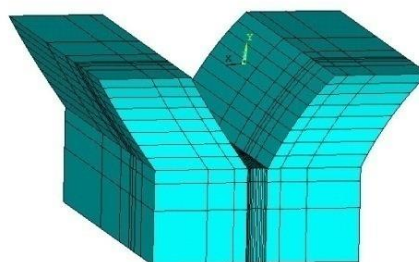


Figure 8. Finite element model of foundation

Figures 9 to 11 demonstrate the North-South, East-West and the Vertical components of El Centro earthquake. The model is analyzed assuming linear and nonlinear behavior of concrete.

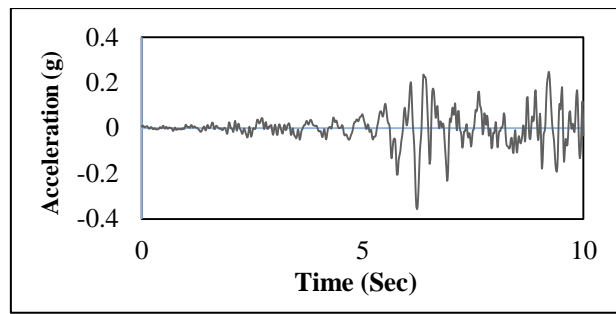


Figure 9. East-West component of El-Centro earthquake

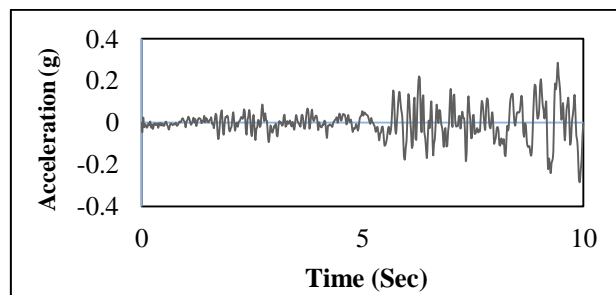


Figure 10. North-South component of El-Centro earthquake

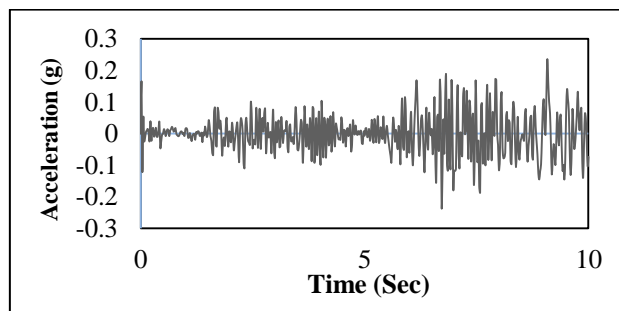


Figure 11. Vertical component of El-Centro earthquake

The achieved results are provided in time history cases. The results of the study are provided for critical point of displacement and principle stresses [14].

Figures 12-14 show the results of displacement of the midpoint of dam crest along the river and principle stresses in dam body for two cases.

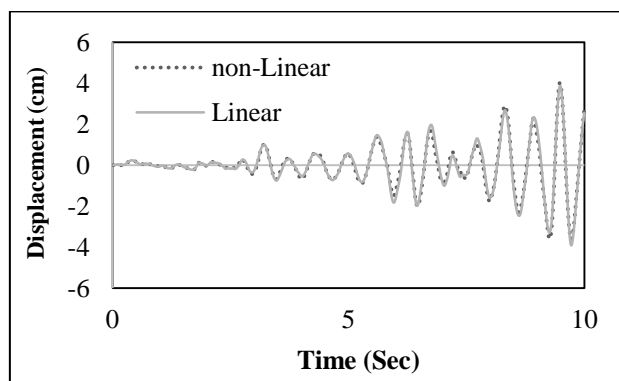


Figure 12. Time history response of maximum dam crest displacement

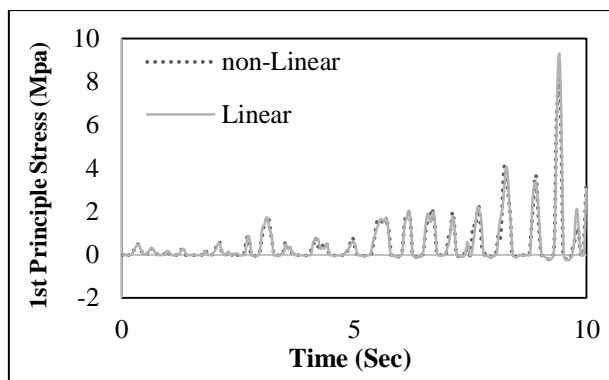


Figure 13. Time history response of maximum 1st principle stress

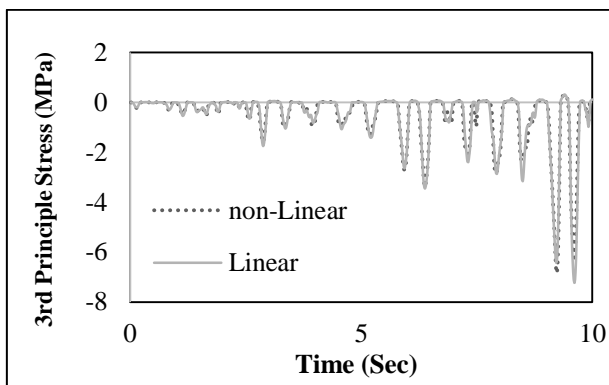


Figure 14. Time history response of maximum 3rd principle stress

The numerical values of results for more accurate review of linear and nonlinear behavior of concrete are provided in Table 3.

Table 3. The numerical values and comparison of the results

The behavior of concrete	Maximum displacement of dam crest (cm)	Principle tensile stress (MPa)	Principle compressive stress (MPa)
linear	3.85	9.31	-7.23
nonlinear	4.01	8.15	-6.83
The variation of responses percent	4.16	12.45	5.53

According to obtained results it is obvious the seismic responses are decrease by applying the nonlinear behavior to the concrete by the applied earthquake. According to the occurring stresses in the body of arch dams is one of most important design criteria it is necessary attention to the nonlinear behavior of materials for safe and economical design.

According to the time history of Figure12 and Table 3 it is obvious the displacement of dam crest along the river for nonlinear behavior case is bigger than the linear behavior case. In dynamic loading of the earthquake acceleration in the initial seconds is low; in this case the structure is not in the nonlinear phase and has the elastic behavior. The structure is yielded and it is in the nonlinear phase after peak acceleration, it means the plastic length is increasing and it is flowed by increasing the stress and decreasing the stiffness. On the other hand the earthquake loading isn't push loading and it is cycle loading, Then this factors are reduced the seismic forces and this results show the importance of considering the nonlinear behavior of materials in design and assessments.

According to Figures 13 to 14 it is obvious that the principle stresses for linear case are bigger than nonlinear case and according to stress-strain curve of two cases it is obvious for linear case the curve is linear, and in this part the strain is increased by stress increasing, but in the nonlinear phase by force increasing the material is flowed. Distribution of displacement and principle stresses is shown in the Figures 15 to 17 for better understanding of the effect of nonlinear behavior of material for critical cases of study.

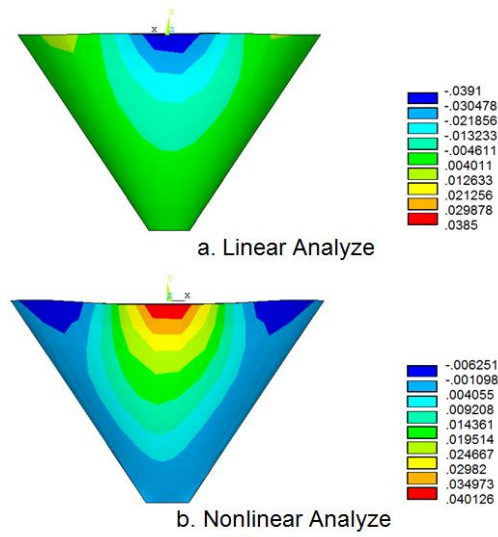


Figure 15. Contour of distribution of maximum dam crest displacement along the river (cm)

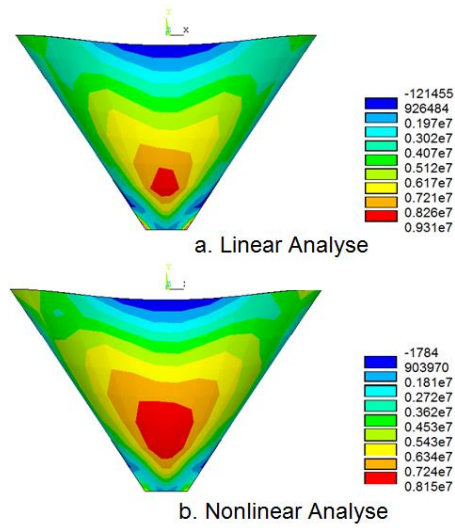


Figure 16. Contour of distribution of maximum principle tensile stress in dam body (MPa)

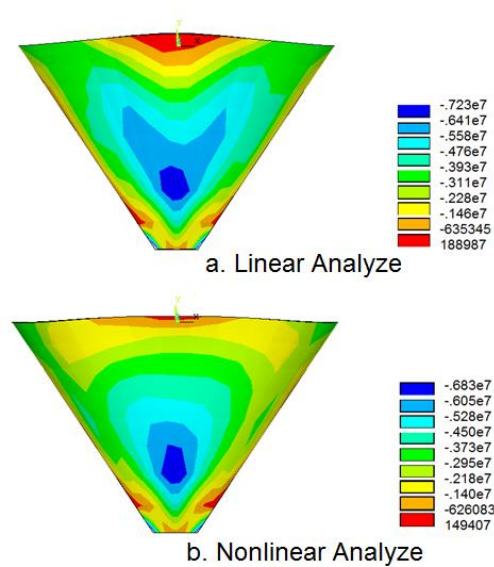


Figure 17. Contour of distribution of maximum principle compressive stress in dam body (MPa)

By comparing the distributions of principle stresses for both cases it is obvious the principle stresses is decreased. This counters show the decreasing of the stress concentration area of nonlinear case, and it is note that the site of stress concentration is on the dam body.

4. Conclusion

In this paper the effect of nonlinear behavior of material is investigated on seismic performance of Marrow Point double curvature dam considering the interaction effects. Finite element method was used for modeling and analysis by applying the El Centro earthquake ground motion components. The model was analyzed considering linear and nonlinear behavior of concrete of dam body to show the nonlinearity effect on the seismic performance of arch dams. Comparison of obtained results for linear and nonlinear cases indicates that the displacement of dam crest along the river is increased and the principle stresses of dam body are decreased for the case of nonlinear behavior. Based on the results obtained from nonlinearity dynamic analysis, it must be concluded that the maximum capacity of the structure can be used by applying nonlinear behavior of material and it can consider the safe design issues such as damping of seismic energy.

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