



Effect of Semi-Rigid Connection on Post-Buckling Behaviour of Frames Using Finite Element Method

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Abstract

It is very important task to estimate the post buckling for structures that have slender elements, since post-buckling state means loss the structures stability related with large displacement and that lead to demolition the structures. On the other hand, in the design and analysis of steel frame, the beam-columns connection is assumed perfect pin or fully rigid, this assumption leads to incorrect estimation of the structural behaviour. Practically, beam-column connection is between these two assumptions and this type of connection is called semi-rigid. This study presents a numerical analysis using finite element method to investigate the effect of semi-rigid connections on post-buckling behaviour of two-dimensional frames with different supporting types and different lateral loading cases. The semi-rigid connections are modelled as rotational spring in linear elastic stage, using COMBIN14 element which has rotational stiffness value. The numerical results showed that; the effect of changing the beam-column connections from rigid to semi rigid for toggle frame with rotational joint stiffness $25EI/L$ to $15EI/L$ and $10EI/L$ led to decrease the initial peak load of the frames of fixed-fixed supports with percentages 3.36 %, 5.6% and 8.95% respectively as compared with that of the rigid connection frame, While, the frames with fixed-pin and pin-pin supports cases did not affected by this changing. The fixed-fixed support case is more affected by changing the joint stiffness from other cases and the effect of changing the joint stiffness in pin-pin support model is less significant from others. This can be attributed to that, the fixed-fixed supports is restrained in all degree of freedom and will be affected by any rotation and presence the pin in other cases makes the frame less affected by the rotation of semi-rigid connection. The effect of changing the beam-column connection from rigid to semi rigid decreases with presence the lateral load. Thus, the semi-rigid connection should be considered in analysis and design of steel frames to obtain more realistic results.

Keywords: Post-Buckling; Semi-Rigid Connections; Finite Element Method; Steel Frame.

1. Introduction

Structures which has slender elements, after reaches the applied loads to buckling loads value, the loads remain unchanged or it decrease, but the deformations continue to increase. For some cases and after a certain value of deformation, the structure begins to carry more loading to retain the continuity of deformations, and the second buckling occurs. This cycle recur several times, this phenomenon is called post buckling behaviour [1]. It very important to estimate the post-buckling behaviour for the structures having slender elements, since post buckling state means loss the structures stability related with large displacement and that led to demolition the structures. On the other hand, in the design and analysis of steel frames, the beam-column connection is assumed either perfectly pinned or fully rigid;

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this assumption leads to incorrect estimation of the structural behaviour, as practically, beam-column connections are between these two extreme assumptions and this type of connection is called semi-rigid [2].

For a long time, engineers have been interested in the buckling of structures, but they did not observed the behaviour of structures after the onset of buckling. The agreeable idea was that the buckling load was the ultimate performance of the structure, that for the reasons of safety, the actual load had to be kept far below the critical load, and the research on post-buckling behaviour had no practical significance [3].

The problem of buckling in spatial structures has grown in the last two decades of several interrelated developments, the need to provide large span of structure without bracing or intermediate support and also to provide very small ratio of weight to the unit area of structure due to economic considerations, made the buckling capacity of these structures a determining for their design.

Many researches were presented to study the post-buckling behaviour and semi-rigid connections of steel frames. In 2001, Al-Mahdawi [4], presented a theoretical analysis to predict the pre- and post-buckling behaviour of plane steel frames with prismatic and non-prismatic members. In 2006, Al-Sarraf et al. [5] presented a non-linear post-buckling analysis of steel frame having prismatic and non-prismatic members with end gusseted plates using beam column theory. In 2012, Novoselac et al. [6], presented a linear and non-linear buckling and post-buckling analysis of a bar with influence of imperfections, the non-linear buckling analysis is achieved by using Riks method. In 2015, Ismael and Salman [7], presented a numerical analysis to investigate the post-buckling behaviour of plane structure steel member using finite element method. In 2018 Szymczak et al. [8], presented a study concerns torsional buckling and initial post-buckling of thin-walled aluminium alloy column with bisymmetrical cross section. In 2019 Szymczak et al. [9], presented a study concerns flexural buckling and initial post-buckling of a column made of aluminium alloy and there are many investigations and theoretical predications about studying the semi-rigid connections, these studies mainly focused on the moment-rotation relationship. In 1980, Jones et al. [10], studied the influence of connection stiffness on the strength of steel column. In 2004, Degertekin and Hayalioglu [11], presented analysis and design method for steel frames with semi-rigid connection and semi-rigid column bases. The semi-rigid connections are modelled by using Frye and Morris polynomial model. In 2005, Ozturk and Secer [12], developed a model to examine the dynamic response of two-dimensional frames with semi-rigid connection. The connection flexibility is modelled by linear elastic rotational spring. In 2007, Yan [13], presented a non-linear behaviour of plane steel frames with semi-rigid connection using ANSYS software package. In 2008, Xinwu [14], presented a non-linear analysis of two-dimensional steel frame with semi-rigid connection by using FORTRAN program. Consider connective, geometrical, and material non-linear. In 2010, Dave and Savaliga [15], used Frye and Morris polynomial model for semi-rigid connection to analysis and design of 3-storey building frame with one, two and three bays respectively. In 2016 Ibrahim and Ismael [16], presented a numerical analysis using finite element method to study the effect of semi-rigid connection on post-buckling behaviour of steel frames with prismatic members. In 2018, Bahaz et al. [17], presented a three-dimensional finite element model of steel beam-column connection using ABAQUS computer program to identify the parameters that effect on the behaviour of semi-rigid end-plate connections. In 2018, Elvin et al. [18], generalised a virtual work optimization method (VWOM) to consider the structures with semi-rigid connections. The VWOM is an automated that minimizes the mass of structure. All the previous studies did not investigate the effect of semi-rigid connections on post-buckling behaviour of steel frames. The objective of the current work is to present a numerical analysis using finite element method to investigate the effect of semi-rigid connection on post-buckling behaviour of frames with different support type (fix-fix, fix-pin, and pin-pin) and different lateral load cases (with and without lateral loading).

2. Materials and Method

The post-buckling is a special case of large displacement. So, large displacement option is activated in ANSYS to use arc-length method as a technique for the analysis the post-buckling of the frames. The maximum and minimum multipliers of arc-length method are set to be 10 and 0.0001 respectively. In the present study, ANSYS release 15 was used for analysis the steel frames with rigid and semi-rigid connection and tracking their post-buckling behaviour. In modelling the frames elements of this study in ANSYS, the linear elastic isotropic model was used for simulating the stress-strain curve of steel for all members. The linear model required the value of modulus of elasticity and Poisson's ratio of steel.

2.1. Modelling of Beams and Columns of Frame

The members of frame (beam and column) were modelled using BEAM3 elements which is a uniaxial element with tension, compression, and bending capabilities. The element has three degree of freedom at each node: translations in the nodal x and y directions and rotation about the nodal z axis. Figure 1 shows BEAM3 element [19].

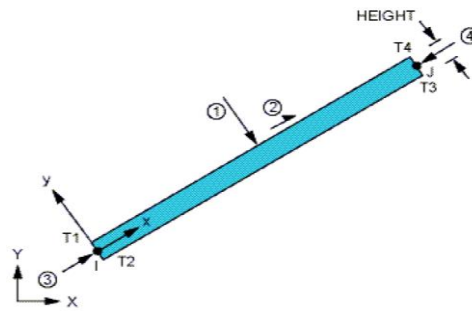


Figure 1. BEAM3 element [19]

2.2. Modelling of Beam-Column Connection

The beam-column connection of frame are represented as rotational spring and is modelled in ANSYS using COMBIN14 element which has longitudinal or torsional capability in 1-D, 2-D, or 3-D applications. The longitudinal spring-Damper option is a uniaxial tension-compression element with up to three degree of freedom at each node: translations in the nodal x, y and z directions as shown in Figure 2. The rotational spring-damper option is a purely rotational element with three degrees of freedom at each node: rotations about the nodal x, y, and z axes [19]. COMBIN14 element is defined with two coincident nodes and modelled into the frame with one rotational degree of freedom in the z-axis. Since each two nodes of combin14 element shall have translation in the x and y direction, each pair of the nodes that belong to each of the combin14 element should be couple in the x and y directions.

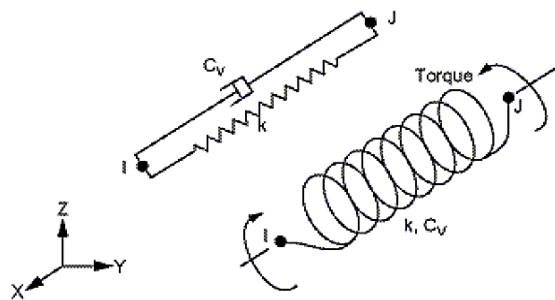


Figure 2. COMBIN14 element [19]

The behaviour of semi-rigid connection is represented by linear spring has rotational stiffness value. The connections stiffness (K) assumed by Lightfoot and Lemessurier [20] as:

$$K = \lambda \frac{EI}{L} \quad (1)$$

In which E modulus of elasticity of the beam, I moment of inertia of the beam, L length of the beam, and λ is the rigidity index that indicate the degree of connection flexibility. The value of λ ranges from zero for perfectly pinned connection and is infinite for perfectly rigid connection [21].

4. Result and Discussion

In order to check the validity and accuracy of the finite element models which were constructed to simulate the post-buckling behaviour, analyzed two frame and comparisons the results with previous study results.

4.1. Williams' Toggle Frame

The geometry, material properties and loading condition of this frame shown in Figure 3 [4]. There are many researchers has been analyzed Williams' toggle frame to verify the numerical accuracy of their studies. In 1964 Williams studied and presented a theoretical study using a beam-column model to describe the behaviour of individual member of this frame [22], Hsiao and Hou analyzed this frame by taking only half of the frame in the analysis [23] and Al-Mahdawi [4] also analyzed this frame using arc-length strategy by beam-column theory. All previous studies modelled the connections between the beam and the column as rigid connections. In this study, Williams' toggle frame was analyzed using FEM. Two types of connection (rigid and semi-rigid) were adopted for representation joint B of this frame. In the semi-rigid connection, three values of joint stiffness ($10EI/L$, $15EI/L$, and $25EI/L$) were used (where E is modulus of elasticity, I is moment of inertia and L is length of the beam). The cross-sectional area of frame parts is 117.9998 mm^2 , the modulus of elasticity is 70906.4 MPa . Figure 4 shows finite element modelling of the frame in ANSYS.

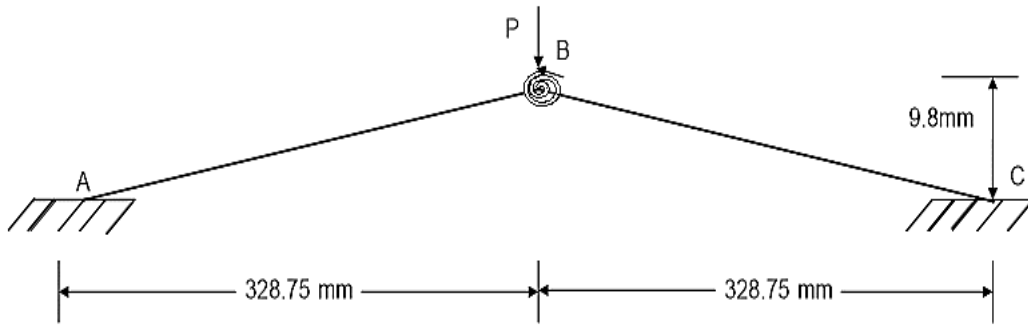


Figure 3. Geometry and loading Williams' toggle frame [4]

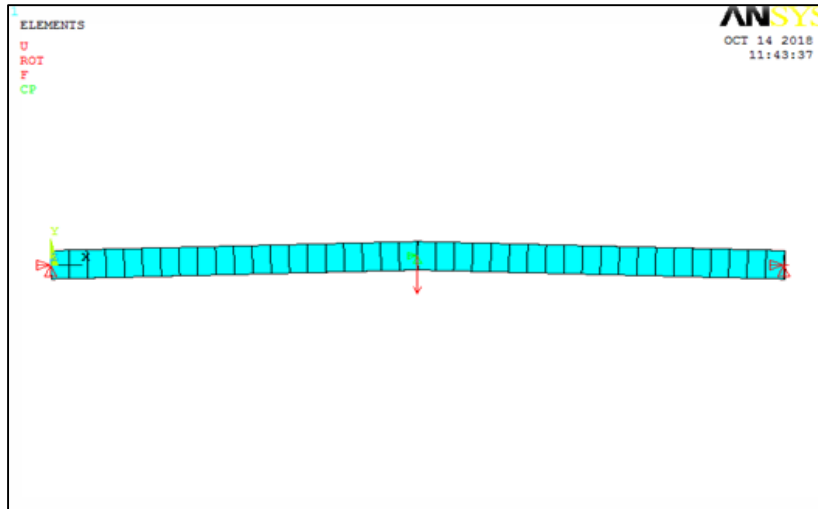


Figure 4. Modelling of toggle frame in ANSYS

The post-buckling in terms of load-displacement curve for vertical displacement of joint B shown in Figures 5 and 6. Shows the variation of vertical displacement along the frame in post-buckling stage at ultimate load using ANSYS at $10EI/L$.

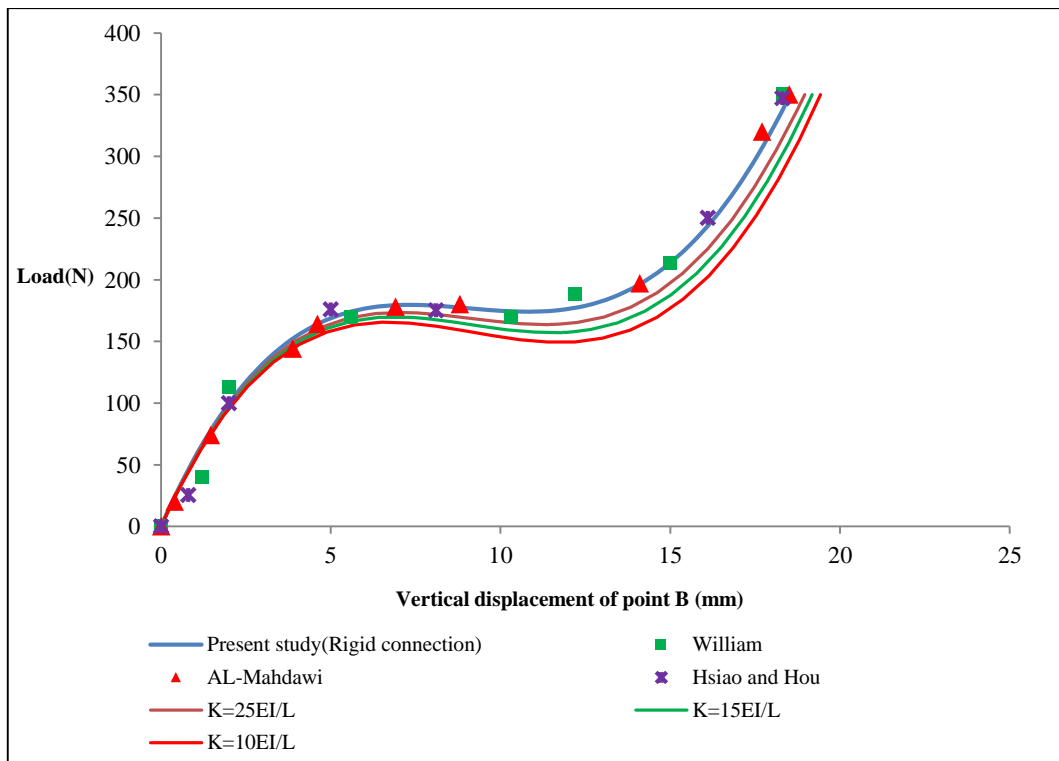


Figure 5. Load-vertical displacement of point B of toggle frame

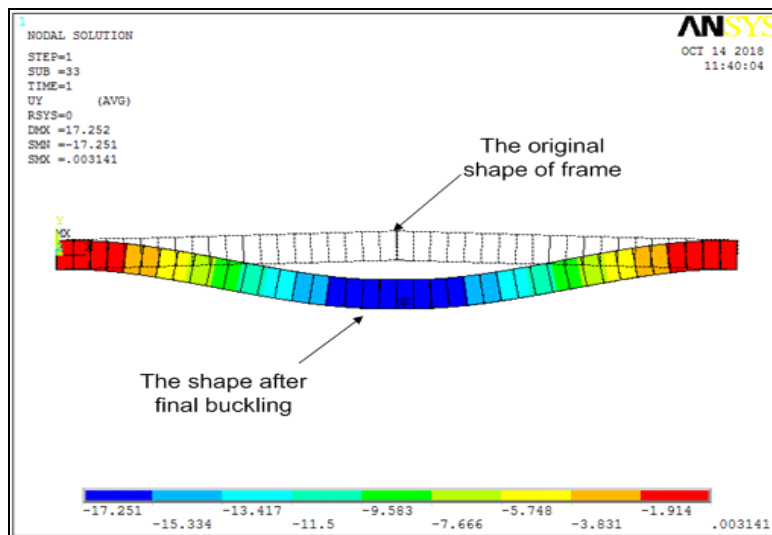


Figure 6. Variations in vertical displacement for the Williams toggle frame at ultimate load using ANSYS at 10EI/L

Figure 5 shows that the load-vertical displacement curve of the rigid connection finite element model shows very good agreement with the curves of the previous studies, also the figure reveals that increasing the rotational stiffness value of the semi-rigid finite element model from (10EI/L to 15 EI/L and 25EI/L) makes the curves close to that of the rigid finite element model and that of the previous studies, this agreement prove the validation of semi-rigid connection model that modelled by finite element method.

4.2. Effect of Semi-Rigid Beam-Column Connection on Different Support Type for Williams' Toggle Frame

To study the effect of semi-rigid beam-column connection in terms of different beam-column rotational stiffness values (10EI/L, 15EI/L, 25EI/L as well as fully rigid case) on the post-buckling behaviour of frame with different support type (fixed-fixed, fixed-pin, pin-pin), Williams toggle frame was used:

4.2.1. Frame with Fixed-Fixed Support

In this type of frame supporting, the joint A and C of Williams toggle frame are assumed to be fixed. Figure 7 shows the effect of rotational joint stiffness on the post-buckling behaviour in terms of load-vertical displacement curve at the point B extracted from the finite element analysis. It can be noted from this figure that, the effect of changing the rotational joint stiffness is negligible before the load 125N, and beyond this load the effect starts to increase and become significant. Also this Figure and Table 1 reveals that changing the beam-column connection from the fully rigid case to the semi-rigid case of the rotational joint stiffness 25EI/L, 15EI/L and 10EI/L leads to decrease the initial peak load with percentages 3.36%, 5.6% and 8.95% and increase the ultimate vertical displacement with percentages 2.32%, 3.45% and 4.75%. These decreases in initial peak load and increase in ultimate vertical displacement can be attributed to the fact that, changing the joint connection from rigid to semi rigid connection with low rotational stiffness, decrease the stability of the frame and increase the flexibility, thus the frame exhibit less carrying capacity against the applied loads and increase the displacement.

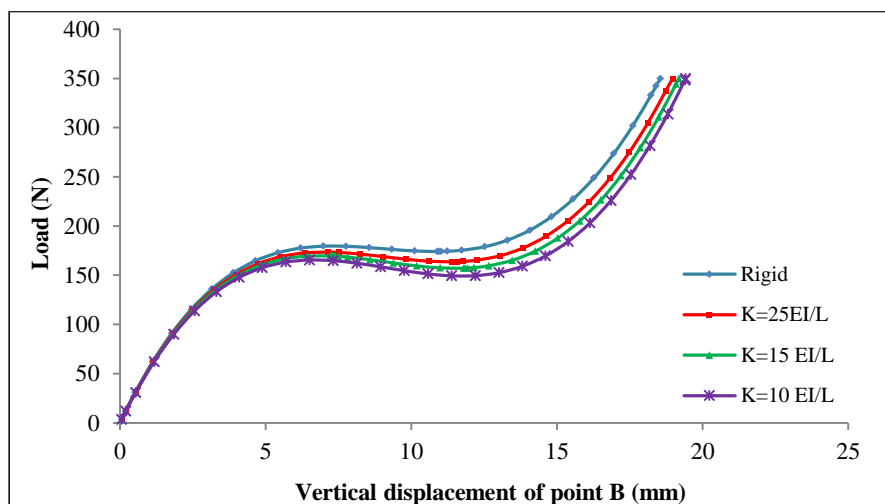


Figure 7. Effect of rotational joint stiffness value on load-vertical displacement at point B of Williams toggle frame with fixed-fixed support

Table 1. Effect of rotational joint stiffness on peak load and ultimate displacement of Williams toggle frame with fixed-fixed support case

Stiffness	Rigid	25EI/L	15EI/L	10EI/L
Initial peak load (N)	179.58	173.54	169.5	163.51
Percentage decrease in initial peak load (%)	-	3.36	5.6	8.95
Ultimate vertical displacement (mm)	18.54	18.97	19.18	19.42
Percentage increase in vertical displacement (%)	-	2.32	3.45	4.75

4.2.2. Frame with fixed-pin support

In this type of frame supporting, the joint A is assumed to be fixed, while joint C is assumed to be pin. Figure 8 shows the effect of rotational joint stiffness on the post-buckling load-vertical displacement curve which was obtained from the finite element analysis. It can be noted that the effect of changing the rotational joint stiffness is started after the load of 75.8N beyond the peak load, and before that the effect is negligible, this behaviour can be attributed to that presence of the pin in point C reduced the effect of rotational motion of the beam BC of the toggle frame therefore, the effect of the rotation in joint C become lesser

Also, this figure shows that the initial peak load decreases with percentage 46.67% as compared to fixed-fixed support case. Figure 8 and Table 2 reveals that changing the beam-column connection from the fully rigid case to the semi-rigid case of the rotational joint stiffness 25EI/L, 15EI/L and 10EI/L leads to increase the ultimate vertical displacement with percentages 1.75 %, 2.75% and 3.8% as compared with fully rigid case.

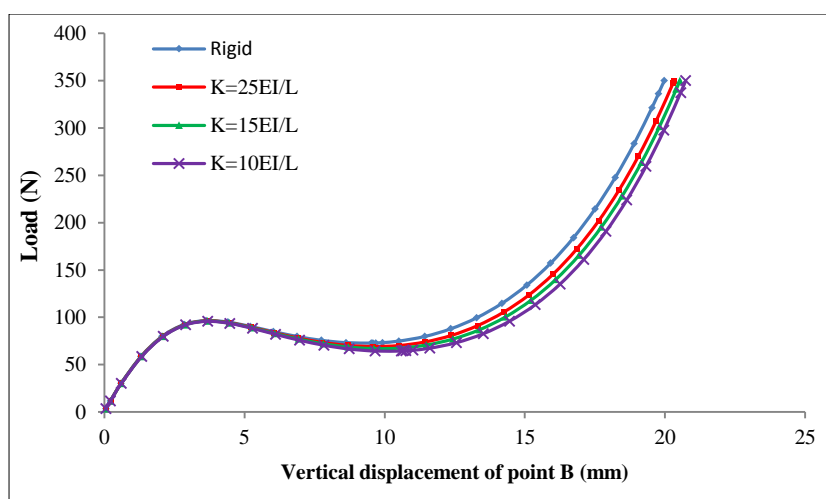


Figure 8. Effect of rotational joint stiffness value on load-vertical displacement at point B of Williams toggle frame with fixed-pin support

Table 2. Effect of rotational joint stiffness on ultimate displacement of Williams toggle frame with fixed-pin support case

Stiffness	Rigid	25EI/L	15EI/L	10EI/L
Ultimate vertical displacement (mm)	19.98	20.33	20.53	20.74
Percentage increase in vertical displacement (%)	-	1.75	2.75	3.8

4.2.3. Frame with Pin-pin support

In this type of frame supporting, the joint A and C are assumed to be pin. Figure 9 shows the effect of rotational joint stiffness on the load-vertical displacement of point B. This figure shows that the initial peak load decreases with percentage 53.1% as compared to fixed-fixed support case. Also, it can be noted that the effect of changing the rotational joint stiffness is started after the load of 23.56N beyond the peak load, and before that there is no effect on peak load and vertical displacement. Figure 9 and Table 3 reveals that changing the beam-column connection from the fully rigid case to the semi-rigid case of the rotational joint stiffness 25EI/L, 15EI/L and 10EI/L leads to increase the ultimate vertical displacement with percentages 1.39%, 2.18% and 3.2.

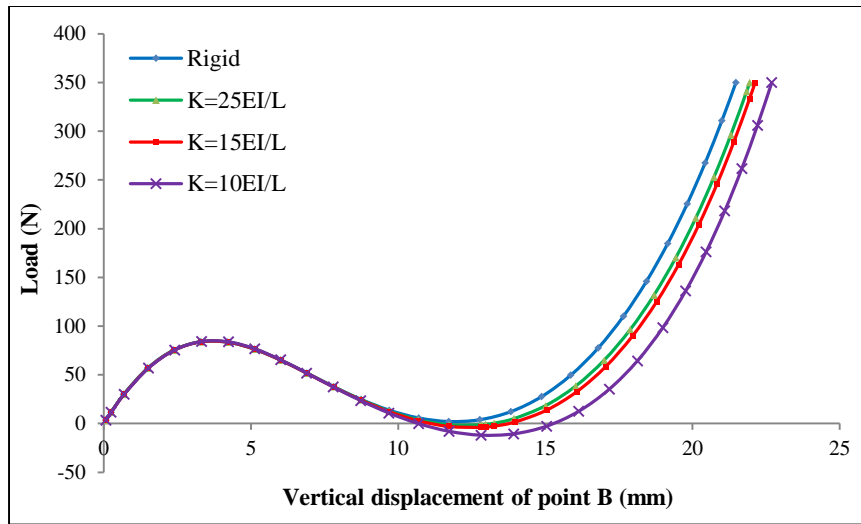


Figure 9. Effect of rotational joint stiffness value on load-vertical displacement at point B of Williams toggle frame with pin-pin support

Table 3. Effect of rotational joint stiffness on ultimate displacement of Williams toggle frame with pin-pin support case

Stiffness	Rigid	25EI/L	15EI/L	10EI/L
Ultimate vertical displacement (mm)	21.48	21.78	21.95	22.17
Percentage increase in vertical displacement (%)	—	1.39	2.18	3.2

When compared the results of three type models under the same load, it can be noted that, the fixed-fixed support case is more affected by changing the joint stiffness from other cases and the effect of joint stiffness in pin-pin support model is less significant from others. This can be attributed to that, the fixed-fixed supports is restrained in all degree of freedom and will be affected by any rotation, while presence the pin in other cases makes the frame less affected by the rotation of semi-rigid connection.

4.3. Portal Frame with Inclined Columns

This frame was analyzed by Al-Mahdawi [4] with rigid beam-column connections using beam column theory. In this study, this frame was analyzed using FEM. Two types of connection (rigid and semi-rigid) were adopted for representation joint B and C of this frame. Figure 10 shows the geometry, section properties and springs position of portal frame, the modulus of elasticity is 216977 N/mm², the cross-sectional area is 20.16125 mm² and the moment of inertia is 4.234125 mm⁴. The FE modelling of frame in ANSYS shown in Figure 11. In the semi-rigid connection, three values of joint stiffness (2EI/L, 5EI/L, and 20EI/L) were used.

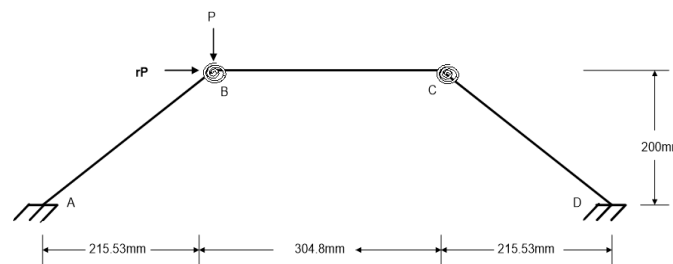


Figure 10. Geometry and loading condition of portal frame [4]

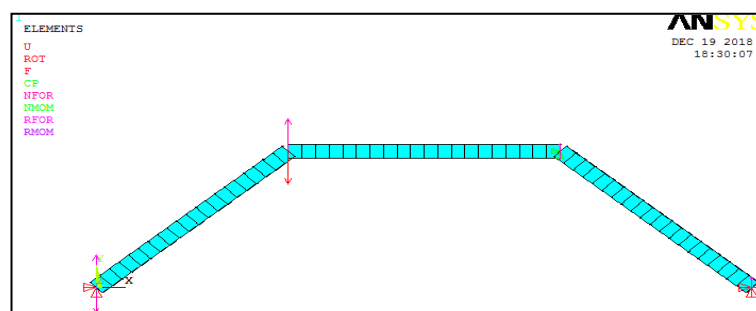


Figure 11. Modelling of portal frame in ANSYS

Post-buckling behaviour in terms of load- vertical displacement of point B shown in Figure 12. Before the curves met at point 1, when increase the stiffness, noted increase initial peak load, while after point 1 noted the joint with low stiffness have high load value, while there is no noticeable difference in final displacement. And this figure reveals that, there is a good agreement of curves for rigid connection FE model of this study and curve of Al-Mahdawi study (rigid connection), that gives a proof of the validity of these models that achieved by FE. And also, when increase the joint stiffness that makes the curves closer with rigid case.

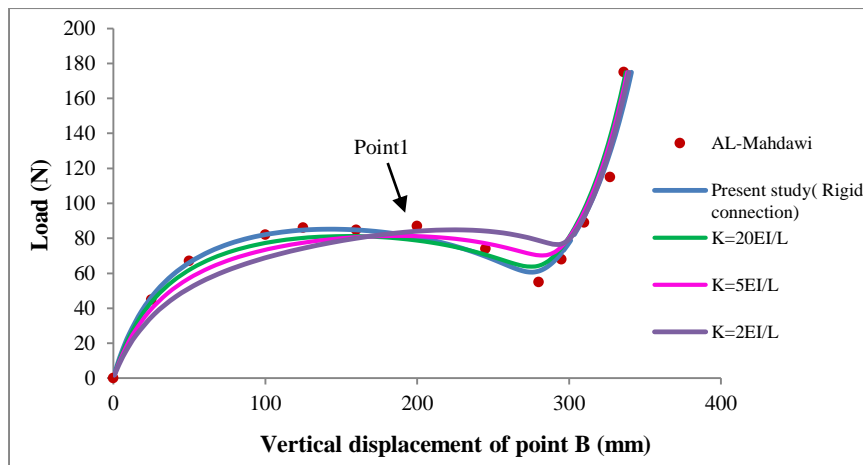


Figure 12. Load-vertical displacement of point B for portal frame with inclined columns

Figure 13 show the load-horizontal displacement of point B. From this figure it can noted that, these curves consist of two parts, in the first part increase the applied load lead to increase the displacement and in the second part increase applied load lead to decrease the displacement. The curves make closer with rigid when increase the joint stiffness and gives a good agreement with rigid case of Al-Mahdawi result. Figure 14 shows the variation in vertical displacement of portal frame with inclined column at ultimate load at 2EI/L.

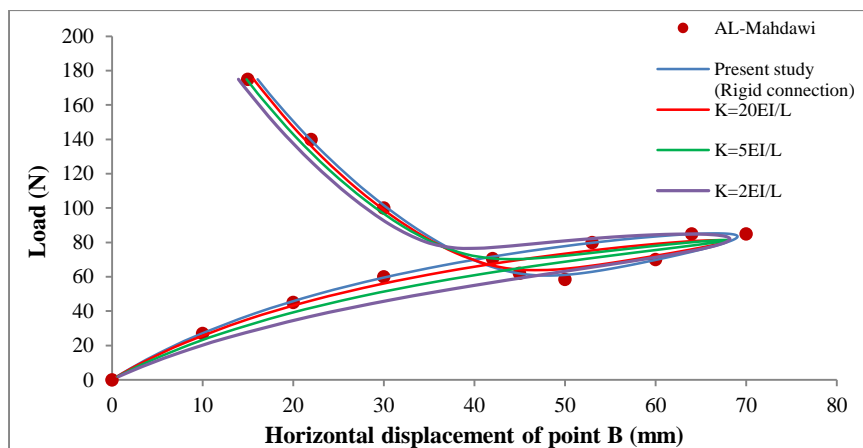


Figure 13. Load-horizontal displacement of point B for portal frame with inclined columns

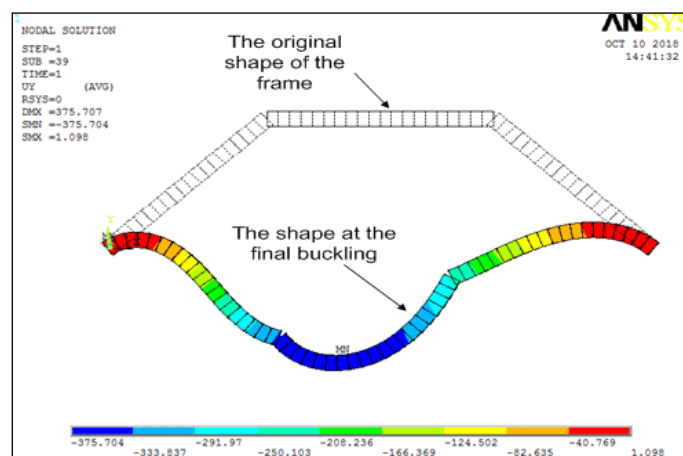


Figure 14. Variation in vertical displacement of portal frame with inclined column at ultimate load at 2EI/L

4.4. Effect of Semi-Rigid Beam-Column Connection on Different Lateral Loading Cases on Portal Frame with Inclined Columns

To study the effect of semi-rigid beam-column connection in terms of different beam-column rotational stiffness values ($2EI/L$, $5EI/L$, $20EI/L$ as well as fully rigid case) on the post-buckling behaviour take two cases of lateral load (without and with lateral load).

4.4.1. Without Lateral Load

Figures 15 and 16 shows the effect of changing rotational joint stiffness on the post buckling in term load-vertical displacement and load-horizontal displacement curve of point B of portal frame without lateral loading respectively. It can be noted from Figure 15 that the load decrease with decrease joint stiffness until the point 1, beyond this point, the begun increase with decreasing joint stiffness until the point 2, after that point the effect of changing the joint stiffness become negligible. From Figure 16 it can noted that, these curves consist of two parts, in the first part increase the applied load lead to increase the displacement and in the second part increase applied load lead to decrease the displacement.

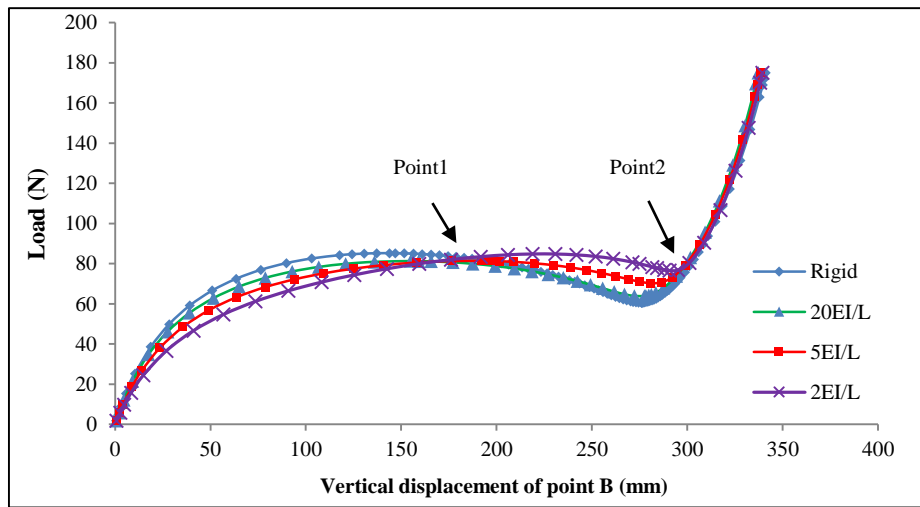


Figure 15. Effect of rotational stiffness on Load-vertical displacement for portal frame without lateral loading ($r=0$)

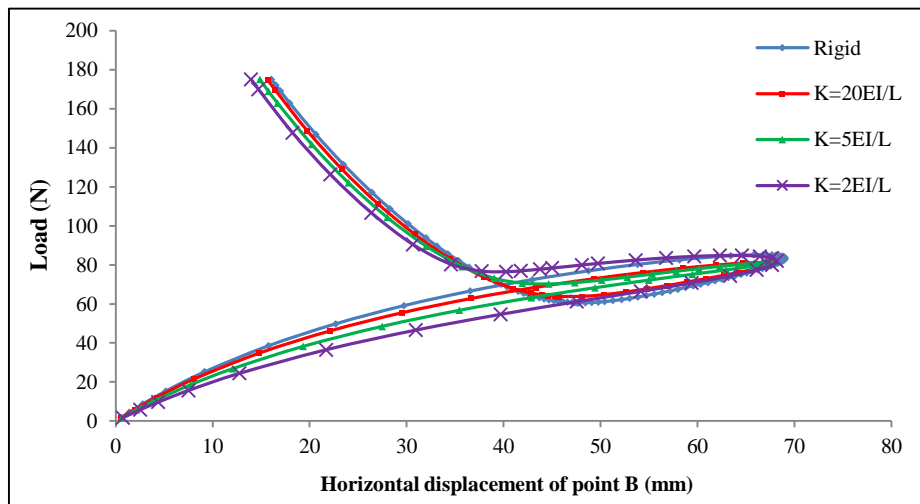


Figure 16. Effect of rotational stiffness on Load-horizontal displacement at point B for portal frame without lateral loading ($r=0$)

4.4.2. With Lateral Load ($r=0.5$)

Figures 17 and 18 shows the effect of changing rotational joint stiffness on the post buckling in term load-vertical displacement and load-horizontal displacement curve of point B of portal frame with lateral loading ($r=0.5$) respectively. It can be noted from Figures 17 and 15 that previously discussed that, presence lateral load decrease the ultimate vertical displacement with percentage (3.22%) and the curves become stiffer comparing with that of absence lateral loading. The figures also reveal that, there is no different in ultimate vertical displacement when changing the joint rotational stiffness.

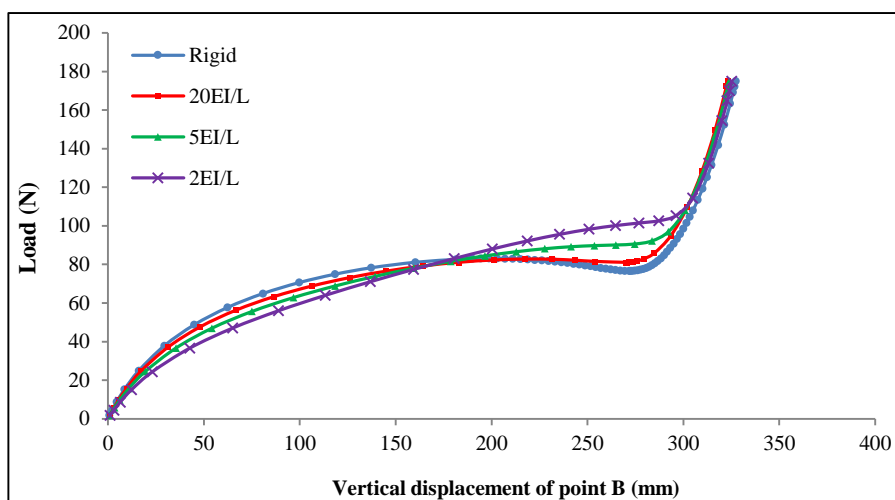


Figure 17. Effect of rotational stiffness on Load-vertical displacement at point B for portal frame with lateral loading of (r=0.5)

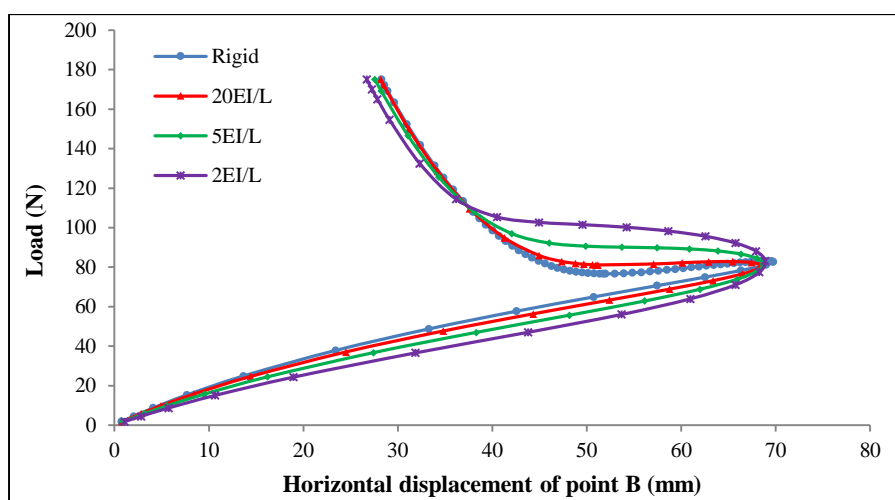


Figure 18. Effect of rotational stiffness on Load-horizontal displacement at point B for portal frame with lateral loading (r=0.5)

When make a comparison between the Figures 18 and 16 shows that presence the lateral load (r=0.5) makes the final horizontal displacement increase and the curves become less stiff. While, the ultimate horizontal displacement not effected by increasing lateral load ratio and changing the joint rotational stiffness. Also, it can be noted from Table 4. that, changing the beam-column connection from the fully rigid case to the semi-rigid case of the rotational joint stiffness 20EI/L, 5EI/L and 2EI/L leads to decrease the final horizontal displacement with percentages 2.6%, 7.5% and 13.1% for frame without lateral loading and 0.35%, 2.42% and 5.528% for frame with lateral loading.

Table 4. Effect of rotational stiffness on final horizontal displacement for portal frame with and without lateral loading

Frame type	Stiffness	Final horizontal displacement (mm)	Percentage decrease in final horizontal displacement (%)
Portal frame with load ratio (r=0)	Rigid	16.066	-
	20EI/L	15.648	2.6
	5EI/L	14.86	7.5
	2EI/L	13.959	13.1
Portal frame with load ratio (r=0.5)	Rigid	28.27	-
	20EI/L	28.17	0.35
	5EI/L	27.585	2.42
	2EI/L	26.707	5.528

A comparison between the results, it can be observed that, the effected of semi-rigid connection is less significant when increasing lateral load ratio.

5. Conclusions

In this research, post-buckling behaviour of two-dimensional frames with semi-rigid beam-column connection has been achieved and compared with that of the frames with rigid beam-column connection to study the effect of semi-rigid beam column on different cases of frame. Based on the numerical results that conducted in this study, the following conclusions can be drawn:

- The post-buckling behaviour of the frames obtained from the finite element analysis using ANSYS computer program showed very good agreement with that of the previous studies.
- For toggle frame changing the beam-column connection from rigid to semi rigid with different rotational stiffness value ($25EI/L$ to $15EI/L$ and $10EI/L$) led to increase the ultimate vertical displacement with percentages 2.32%, 3.45% and 4.75% for the frame with fixed-fixed supports, 1.75 %, 2.75% and 3.8% for the frame with fixed-pin supports and 1.39%, 2.18% and 3.2% for the frame with pin-pin supports respectively as compared with that of the rigid connection frame. This can be attributed to that, the fixed-fixed supports is restrained in all degree of freedom and will be affected by any rotation and presence the pin in other cases makes the frame less affected by the rotation of semi-rigid connection.
- The effect of changing the beam-column connection from rigid to semi-rigid on the vertical displacement at the connection joint in William toggle frame with fixed-fixed support case is more significant than that of fixed-pin and pin-pin cases.
- For the portal frame with presence lateral load, changing the beam-column connection from the rigid to the semi-rigid with different rotational stiffness ($20EI/L$, $5EI/L$ and $2EI/L$) makes the load-displacement curve more stiffer as compared with that of the frame without lateral load and led to decrease the final horizontal displacement with percentages 0.35%, 2.42% and 5.528% for the frame with lateral load and 2.6%, 7.52% and 13.1% for the frame without lateral loading, respectively as compared with that of the rigid frame.

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7. Conflicts of Interest

The authors declare no conflict of interest.

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