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Simplified Irregular Beam Analysis and Design

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Abstract

This paper presents simple method to estimate the strength design of reinforced concrete beam sections based on structural safety and reliability. Irregular beam shaped sections are commonly used nowadays in the construction industry. This study reveals the simplified method to analyze and design the different irregular shaped beam sections. In this study, the selected irregular beam shaped sections are divided mainly into three groups, beams with straight edges, beams with sloped edges and circular beams. Each group contains the most commonly used beam shaped sections in that category. Six beams sections (B-1 to B-6) are selected for group-1 whereas five beam sections (B-7 to B-11) and a circular beam section (B-12) are chosen for group 2 and 3 respectively. Flexural beam formulas for three groups of reinforced concrete beams are derived based on section geometry and ACI building code of design. This study also analyzed numerical examples for some of the sections in each group category using the proposed simplified method to determine the strength design of the irregular beams. The results obtained using simplified method for all of the three groups are compared with the finite element software (SAP v2000). The percentage difference of simplified method with the finite element software ranges within 5% to 10%. This makes the simplified method for irregular shaped beam sections quite promising.

Keywords: Reinforced Concrete Beams; Irregular Shaped Beam Cross Section; Circular Beams; Sloped Edged Beams; Internal Compressive Force.

1. Introduction

Beams are very important structure members and the most common shape of reinforced concrete beams is rectangular cross section. Safety and reliability are used in the flexural design of reinforced concrete beams of different sections using ultimate-strength design method USD under the provisions of ACI building code of design [1]. Lu et al. (1994) worked on the evaluation of time-invariant reliability for designing of reinforced concrete under ACI building code [2]. Their study concluded that the reliability indices are most critical to live load, uncertainties of models and the strength of materials. Investigation of the reliability of reinforced concrete beams for high rise buildings based on the New ACI 318-05/ASCE 7-05 are done by Baji et.al and their study indicates that the different limit states at the controlling stations are not consistent for low values of wind to dead load ratios [3].

Beams with single reinforcement are the preliminary types of beams and the reinforcement is provided near the tension face of the beam [4]. Beam sizes are mostly governed by the external bending moment Mc. The flexural beam formula for the rectangular shaped beam sections are derived in several books [5-6]. These also includes the detailed design of singly and doubly reinforced rectangular and T-shaped section beam sections. The analysis and design of irregular shaped sections are not illustrated in detail in these books. Several studies were also conducted on the design and analysis of irregular shaped sections subjected to flexure but are limited to certain shaped beam sections.

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Mahzuz, H.M.A Mahzuz used the working stress design method (WSD) to evaluate the performance of singly reinforced triangular shaped section only [7]. Mansur et.al focused mainly on the analysis and design of beams with openings of irregular shaped sections to allow the essential services like water supply, sewerage, air-conditioning, telephone, computer network etc. to pass through them. In their study, they analyzed and designed the circular, trapezoidal and triangular shaped openings in the rectangular beam section [8]. Al-Ansari worked on the reliability and flexural behavior of triangular and T-shaped beam sections. His research work indicated the triangular shaped beam sections as more reliable than the T-shaped section beams with an equal area of concrete and steel reinforcement [9]. Solmon Teminsui used circular, rectangular, circular with openings as well as rectangular with openings and triangular shaped beam section subjected to flexure to develop their universal design model [10]. Further, in another study conducted by Cosenza et al. [11], the bending moment capacity of reinforced concrete members of circular cross-section has assessed only.

The previous research studies are limited to certain irregular shaped beam cross sections. This study presents the simple method to estimate the flexural capacity of all possible irregular shaped beam sections used commonly in the construction practices. In this study, the flexural beam formulas for the different irregular beam sections are derived based on section geometry and ACI building code of design.

The beam sections are divided mainly into three groups; beams with straight edges, beams with sloped edges and circular beams. The formulation of the flexural formulas for each group is discussed separately in this study. Furthermore, this study also analyzed numerical examples using the proposed simplified method to determine the strength design of these irregular beam sections and the obtained results are later compared with the finite element software (SAP v2000). All of the calculations for the proposed simplified method are done on the Mathcad Software [12]. The flexural beam formula for the rectangular beam cross-section is shown in Figure 1.

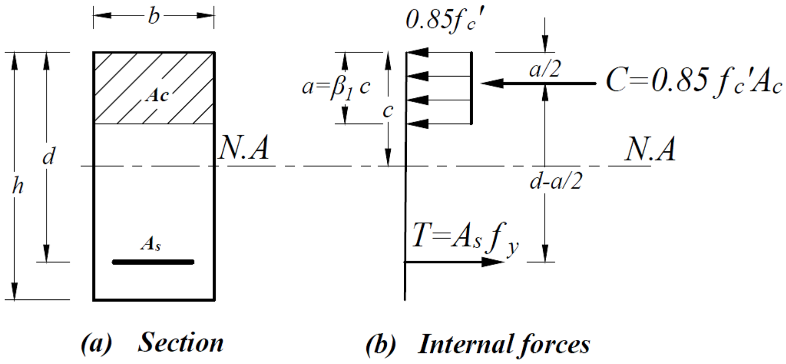


Figure 1. Rectangular cross section with single reinforcement [6]

With assumed balance failure condition, the tensile force T is equal to the concrete compressive force C.

$T = C$

$$A_s f_y = 0.85 f_c' A_c \quad (1)$$

The compression area *(Ac)* will be equal to

$$A_c = \frac{A_s f_y}{0.85 f_c'} \quad (2)$$

The depth of the compression block can be computed as;

$$a = \frac{A_s f_y}{0.85 f_c' b} \quad (3)$$

Thus, the design moment flexural strength is formulated as;

$$M_c = \emptyset_b A_s f_y \left(d - \frac{a}{2}\right) \quad (4)$$

Where:

φ_b= Bending reduction factor;

f_y = Specified yield strength of non-prestressed reinforcing;

f_c' = Specified compression strength of concrete;

A_s = Area of tension steel;

A_c = Compression area;

d = Effective depth;

a = Depth of the compression block;

b = Width of the beam cross section;

h = Total depth of the beam cross section;

A_g = Gross cross-sectional area of a concrete member;

In this present study, the flexural capacities for different beam sections of each group are discussed separately. Moreover, the complete analysis for some of these sections from each group are also performed in this study.

2. Beams with Straight Edges (Group -1)

Straight edges beams are the most common type of the beam section used in the construction Industry. Some commonly used beam sections with the straight edges are T-beams, Inverted T-beams, Rectangular beams with duct opening, I shaped beams, tube section beams and several other sections. Some of these sections are displayed in Figure 2.

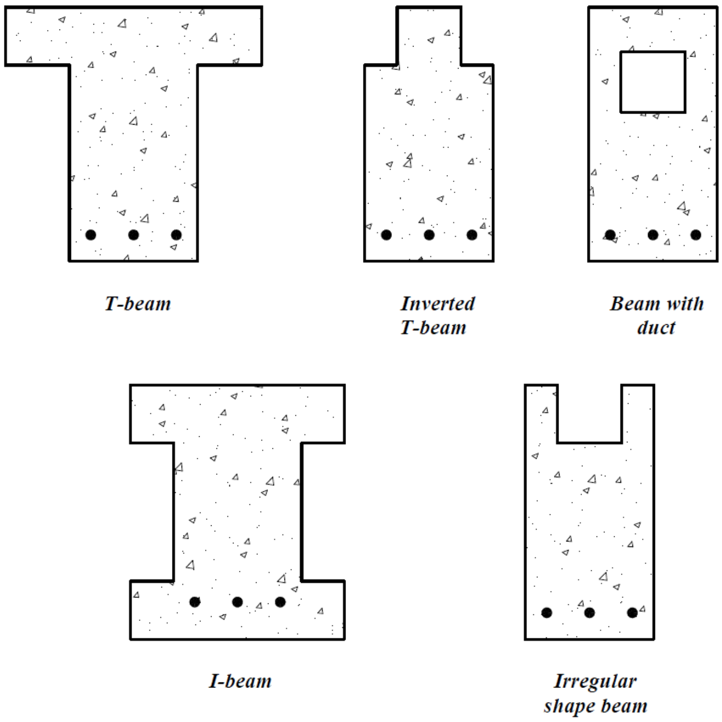
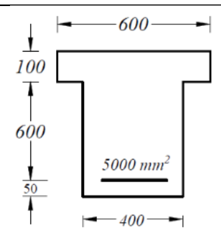
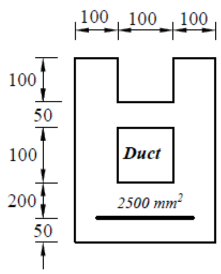


Figure 2. Irregular Beam Cross sections with straight edges

2.1. Numerical Examples for Group-1

Six different beam sections (B-1 to B-6) are selected to find the flexural capacities for this case. Two out of six beam sections (B-1 and B-2) are solved numerically with complete analysis steps. The concrete compressive strength (f_c') and the steel yield strength (f_y) for this group are 30 MPa and 400 MPa respectively. The results for these beam sections are shown in Table 1.

Table 1. Design Strength of Beam with Straight Edges

Beam ID	Irregular beam shapes (Straight Edges)	Depth of compression area *(a) mm*	Design Strength *Mc (kN-m)*	Finite Element Software *Mc (kN-m)*
B1		146	1139	1136
B2		171.1	323.5	338.9

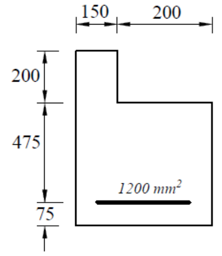
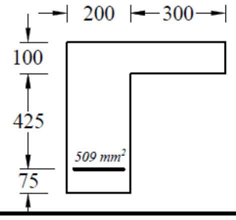
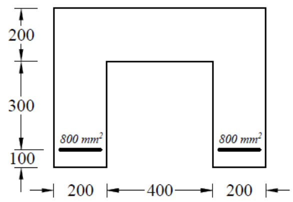
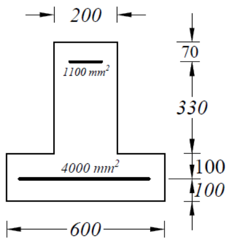
B3		125.5	264.5	291.1
B4		15.97	94	103.4

Table 2. Design Strength of Beam with Straight Edges (Continued)

Beam ID	Irregular beam section (Straight Edges)	Depth of Compression Area *(a)* mm	Design Strength *Mc* *(kN-m)*	Finite Element Software *Mc (kN-m)*
B5		31.37	279	308
B6		219.4	573.5	573.15

2.1.1. Beam B1 (Analysis)

These examples are solved in such a way to follow simple steps from 1 to 6 as mentioned in the following examples. The results are depicted in Table 1 and 2 respectively as the main concern of these solved problems is to find the section capacity ***(Mc)*** of irregular shaped beam sections and to validate them with the finite element software. Therefore, the steps mentioned here describe precisely to find the required moment capacities.

Input Data (Figure 2a):

$A_s = 5000\ mm^2$ $\quad f_y = 400\ MPa$

$f_c' = 30\ MPa$ $\quad E = 200{,}000\ MPa$

(All dimensions are in mm)

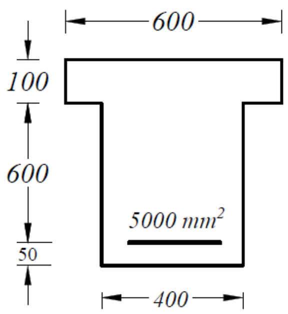


Figure 2 (a) Beam section B-1

Solution:

1- Tensile force in Steel $T = A_s f_y$

$T = 5000 \times 400 = 2000{,}000\ N$

2- Balanced condition, to find the area of compression (A_c);

$$T = C$$

$$2000{,}000 = 0.85 f_c' A_c$$

$$A_c = \frac{2000000}{0.85 \times 30} = 78431\ mm^2$$

3- To check the location for the area of compression (Figure 2b);

$$A_c = 78431 > 600 \times 100 = 60000 \ \ (The\ compression\ area\ also\ includes\ part\ of\ the\ web\ portion)$$

4- Finding the centroid ȳ ;

$$\bar{y} = \frac{(A_f = t_f \times b_f)\left(\frac{t_f}{2}\right) + (A_c - A_f)\left(\frac{A_c - A_f}{b_w} \times \frac{1}{2} + t_f\right)}{A_c}$$

$$\bar{y} = \frac{(60000)\left(\frac{100}{2}\right) + (18431)\left(\frac{18431}{400} \times \frac{1}{2} + 100\right)}{78431}$$

$$\bar{y} = 67.16\ mm$$

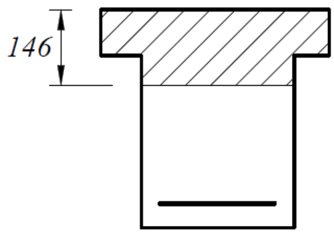


Figure 2. (b) Finding location of "a"

5- Verifying that the steel is yielding. $(f_s = f_y)$

$$a = 100 + 46 = 146\ mm$$

$$c = \frac{a}{\beta} = \left(\frac{146}{0.85}\right) = 171.76\ mm$$

$$\epsilon_s = \left(\frac{d-c}{c}\right) 0.003 = 0.009226$$

$$\epsilon_y = \frac{F_y}{E_s} = 0.002$$

$\epsilon_s > \epsilon_y$, the assumption is OK. ∴ $(f_s = f_y)$

6- Flexural capacity $M_c = \emptyset_b A_s f_y (d - \bar{y}) \ = 0.9 \times 5000 \times 400 \times (700 - 67.16) \times 10^{-6}$

$$M_c = 1139\ kN.m$$

2.1.2. Beam B2 (Analysis)

Input Data (Figure 2c):

$A_s = 2500\ mm^2$ $\quad$ $f_y = 400\ MPa$

$f_c' = 30\ MPa$ $\quad$ $E = 200{,}000\ MPa$

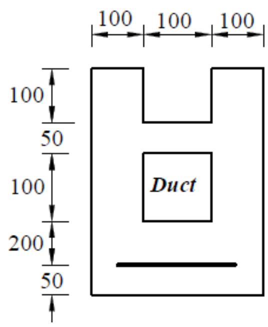


Figure 2. (c) Beam section B-2

<u>Solution:</u>

1- Tensile force in Steel $T = A_s f_y$

$$T \ = 2500 \times 400 = 1000{,}000\ N$$

2- Balanced condition, to find the area of compression (A_c) (Figure 2d);

$$T = C$$

$$1000{,}000 = 0.85 f_c' A_c$$

$$A_c = \frac{1000000}{0.85 \times 30} = 39215.69\ mm^2$$

3- Finding the centroid ȳ;

$$\bar{y} = \frac{(20{,}000)(50) + (15{,}000)(125) + 4215.686\left(150 + \frac{21.078}{2}\right)}{39215.69}$$

$$\bar{y} = 90.57\ mm$$

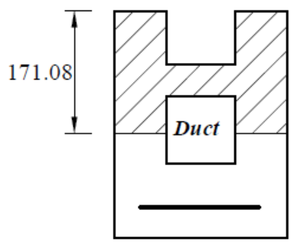


Figure 2. (d) Finding location of "a"

4- Verifying that the steel is yielding. $(f_s = f_y)$

$$a = 100 + 50 + 21.078 = 171.078$$

$$c = \frac{a}{\beta} = \left(\frac{171.078}{0.85}\right) = 201.27\ mm$$

$$\epsilon_s = \left(\frac{d-c}{c}\right) 0.003 = 0.00371$$

$$\epsilon_y = \frac{F_y}{E_s} = 0.002$$

$\epsilon_s > \epsilon_y$, the assumption is OK. $\therefore (f_s = f_y)$

5- Flexural capacity $M_c = \emptyset_b A_s f_y (d - \bar{y}) = 0.9 \times 2500 \times 400 \times (450 - 90.57) \times 10^{-6}$

$$M_c = 323.49\ kN.m$$

3. Beams with Sloped Edges (Group -2)

This group contains the study of the sloped edges beams used quite often in the construction industry. Some commonly used beam sections with sloped edges are Triangular beams, Trapezoidal beams, Inverted triangular beams, Inverted trapezoidal beams, hexagonal shaped beams and many more sections. Some of these sections are shown in Figure 2.

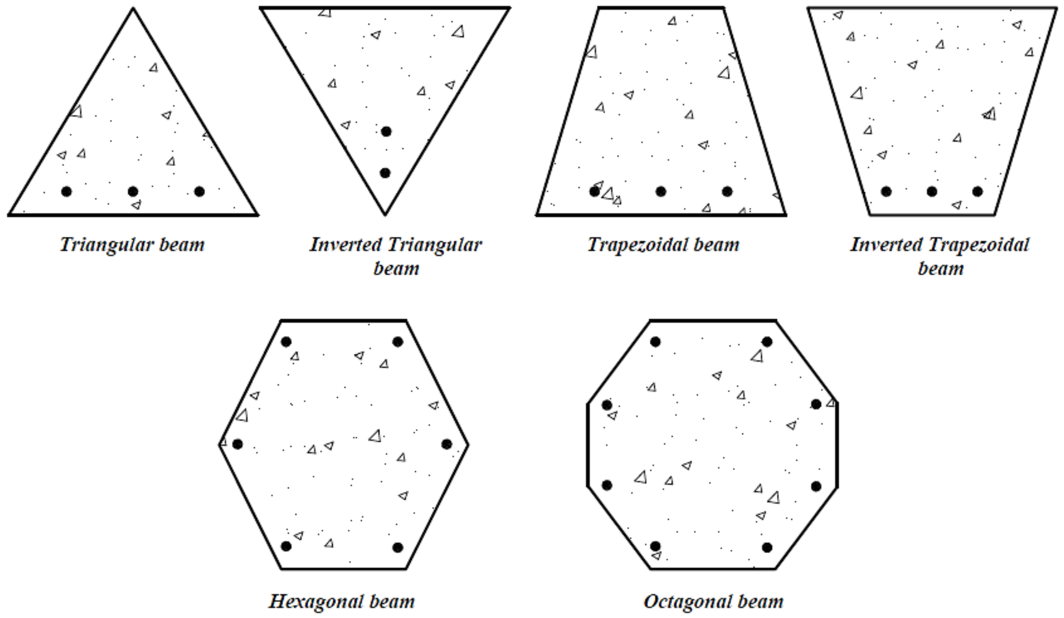


Figure 3. Irregular Beam Cross sections with sloped edges

3.1. Flexural Formula for Sloped Edges Beams

The flexural capacity for the sloped edged beams can be obtained by following the same procedure of analysis for the rectangular beam with single reinforcement and making use of its geometry. The geometry shapes for the trapezoidal and inverted trapezoidal sections are shown in Figure 4.

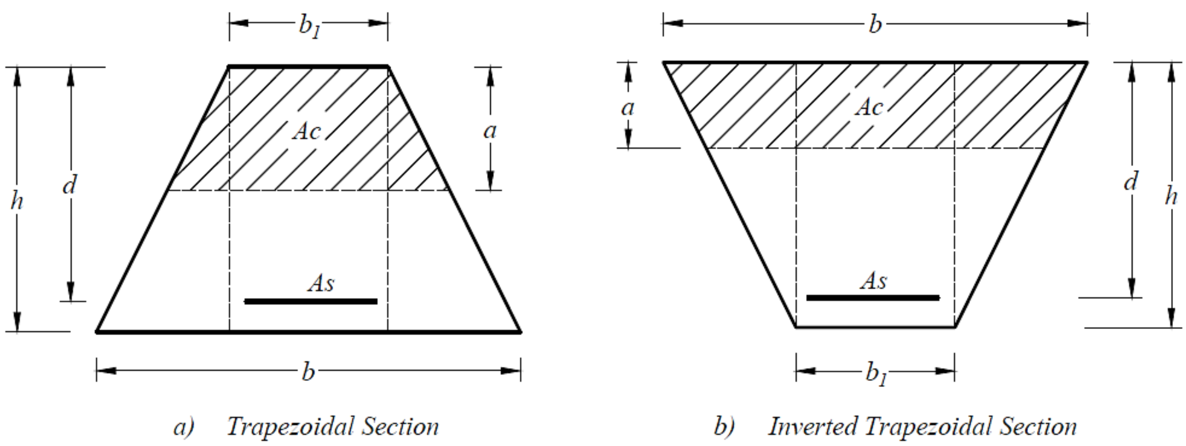


Figure 4. Geometrical shapes for some of sloped edged beams

Where:

φ_b= Bending reduction factor

f_y = Specified yield strength of non-prestressed reinforcing

A_s = Area of tension steel

A_c = Compression area

d = Effective depth

a = Depth of the compression block

b = Width of the beam cross section

b_1 = Smaller width of the trapezoidal beam cross section

h = Total depth of the beam cross section

$\bar{y}$ = Center of gravity of the compression area

The triangular and inverted triangular beam sections are special cases of the trapezoidal and inverted trapezoidal sections section and it could be easily obtained by setting the least width dimension (b_1) equal zero.

The moment capacity for these sloped edged beams can be found by using the Equation 5, the similar equation in case of rectangular beam with single reinforcement.

$$M_c = \emptyset_b A_s f_y (d - \bar{y}) \quad (5)$$

3.2. Numerical Examples for Group -2

Four different shaped sections (B-7 to B-11) are selected to find the flexural capacities for this case. Each section is solved with different leg dimensions. The analysis of trapezoidal and inverted trapezoidal section (B-8 and B-9) is also solved numerically. The concrete compressive strength (f_c') and the steel yield strength (f_y) for this group are 30 *MPa* and 420 *MPa* respectively. These beam section results are compared with the finite element software and are displayed in Table 2.

3.2.1. Beam B-8 (Analysis)

Input Data (Figure 3a):

$A_s = 2500\ mm^2$ $\quad E = 200{,}000\ MPa$

$f_y = 400\ MPa$ $\quad f_c' = 30\ MPa$

b= 400 *mm and* b_1=200 *mm* $\quad$ *d*= 450 *mm*

h= 500 *mm*

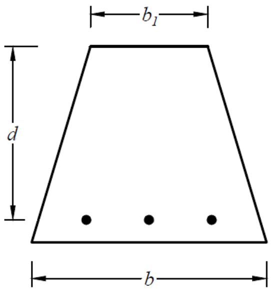


Figure 3. (a) Beam section B-8

Solution (Slope Method):

1- *Slope* $= \frac{500}{100} = \frac{a}{X}$, $X = \frac{a}{5}$

2- Tensile force in Steel $T = A_s f_y = 2500 \times 400 = 1{,}000{,}000\ N$

3- Balanced condition, to find the area of compression (A_c);

$$T = C$$

$$1{,}000{,}000 = 0.85 f_c' A_c$$

$$A_c = \frac{1{,}000{,}000}{0.85 \times 30} = 39{,}215.69\ mm^2$$

4- To find the location for the area of compression (Figure 3b);

$$A_c = A_1 + 2A_2 \Longrightarrow 39215.69 = 200a + 2\left(\frac{1}{2} \times a \times \frac{a}{5}\right)$$

$$\frac{1}{5} \times a^2 + 200a - 39215.69 = 0$$

a = 167.891 mm

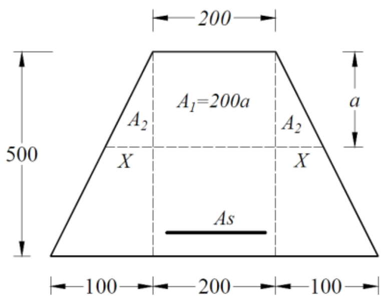


Figure 3. (b) Finding location of "*a*"

Table 2. Design Strength of Beam with Sloped Edges

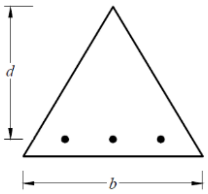
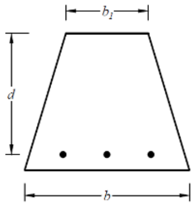
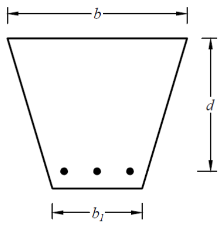
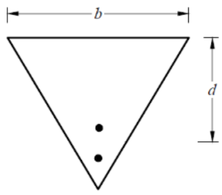
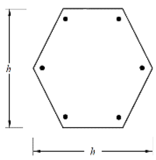
Beam ID	Irregular beam section (Sloped Edges)	Optimized Section Dimensions				Mc (kN-m)	
		b_1 *(mm)*	b *(mm)*	d *(mm)*	As *(mm²)*	Flexural Equations	Finite Element Software
B-7		NA	500	300	1200	139.1	130.5
		NA	300	600	628	109	107.7
		NA	300	600	660	113	112.3
		NA	350	760	920	203	201
B-8		200	400	450	2500	325.8	345.6
		200	600	430	880	132	132
		200	750	415	1000	142	143.2
		250	700	470	1100	180	181.4
B-9		230	600	450	900	148.8	150.5
		200	600	400	900	132	132.5
		250	550	470	1000	172	170
		230	600	450	900	149	151
B-10		NA	300	650	981.8	220.5	239
		NA	450	485	1100	193	193.1
		NA	500	400	900	131	130.9
		NA	500	450	730	121	120.8

Table 2. Design Strength of Beam with Sloped Edges (Continued)

Beam ID	Irregular beam section (Sloped Edges)	Section Dimensions		Mc (kN-m)	
		h *(mm)*	As *(mm²)*	Flexural Equations	Finite Element Software
B-11		380	1889.95	88.9	80.6
		450	2945.24	169.8	157.4

5- Finding the value of $\bar{y}$

$$\bar{y} = \frac{\left(200 \times (167.891)^2 \times \frac{1}{2}\right) + \left(\frac{1}{5}(167.891)^2 \times \frac{1}{2} \times 2 \times \frac{2}{3} \times 167.891\right)}{39215.69}$$

$$\bar{y} = 87.968\ mm$$

6- Verifying that the steel is yielding. $(f_s = f_y)$

$$a = 167.891\ mm$$

$$c = \frac{a}{\beta} = \left(\frac{167.891}{0.85}\right) = 197.51\ mm$$

$$\epsilon_s = \left(\frac{d-c}{c}\right) 0.003 = 0.00383$$

$$\epsilon_y = \frac{F_y}{E_s} = 0.002$$

$\epsilon_s > \epsilon_y$, the assumption is OK. $\therefore (f_s = f_y)$

7- Flexural capacity $M_c = \emptyset_b A_s f_y (d - \bar{y}) = 0.9 \times 2500 \times 400 \times (450 - 87.968) \times 10^{-6}$

$$M_c = 325.83\ kN.m$$

The triangle beam (B-7) with single reinforcement is a special case of trapezoidal section and it could be easily obtained by setting the least width dimension b_1 equal zero.

3.2.2. Beam B-9 (Analysis)

Input Data (Figure 3c):

$A_s = 900\ mm^2$ $\quad E = 200{,}000\ MPa$

$f_y = 420\ MPa$ $\quad f_c' = 30\ MPa$

$h = 510\ mm$ $\quad d = 450\ mm$

$b = 600\ mm$ and $b_1 = 230\ mm$

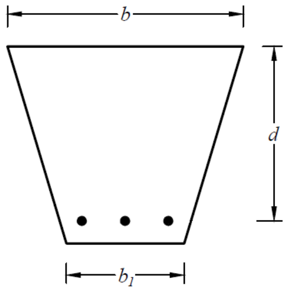


Figure 3. (c) Beam section B-9

<u>Solution (Slope Method):</u>

1- *Slope* $= \frac{510}{185} = \frac{a'}{X}$,

$X = \frac{a'}{2.757}$

2- Tensile force in Steel $T = A_s f_y = 900 \times 420 = 378{,}000\ N$

3- Balanced condition, to find the area of compression (A_c);

$T = C$

$378{,}000 = 0.85 f_c' A_c$

$A_c = \frac{378,,000}{0.85 \times 30} = 14{,}823.53\ mm^2$

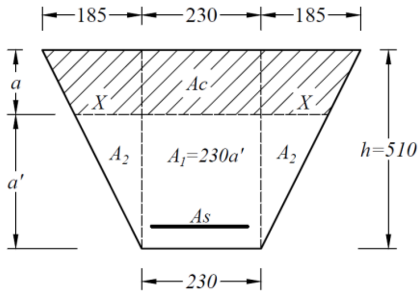


Figure 3 (d) Finding location of *"a"*

4- To find the location for the area of compression (Figure 3d)

Gross Area of Inverted Trapezoidal = $A_g = 211{,}650\ mm^2$

$A_c' = A_g - A_c$

$A_c' = 211{,}650 - 14{,}823.53 = 196{,}826.47\ mm^2$

$A_c' = A_1 + 2A_2$

$196{,}826.47 = 230a' + 2\left(\frac{1}{2} \times a' \times \frac{a'}{2.757}\right)$

$\frac{1}{2.757} \times a'^2 + 230a' - 196{,}826.47 = 0$

$a' = 484.914\ mm$

Therefore, depth of compression block $\Longrightarrow a = h - a'$

$a = (510 - 484.914)\ mm$

$a = 25.086\ mm$

5- Finding the value of $\bar{y}$ (Figure 3e);

$$\bar{y}_1 = \frac{a(2b + b_1 + 2X)}{3(b + b_1 + 2X)}$$

$$\bar{y}_1 = \frac{25.086\ (2(600) + 230 + 2(175.9))}{3\ (600 + 230 + 2(175.9))}$$

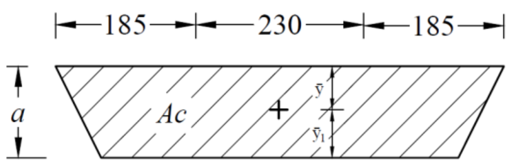


Figure 3. (e) Finding location of "$\bar{y}$"

$\bar{y}_1 = 12.607\ mm$

$\bar{y} = a - \bar{y}_1$

$\bar{y} = 12.479\ mm$

6- Verifying that the steel is yielding. $(f_s = f_y)$

$$a = 25.086\ mm$$
$$c = \frac{a}{\beta} = \left(\frac{25.086}{0.85}\right) = 29.52\ mm$$
$$\epsilon_s = \left(\frac{d-c}{c}\right) 0.003 = 0.043$$
$$\epsilon_y = \frac{F_y}{E_s} = 0.002$$
$\epsilon_s > \epsilon_y$, the assumption is OK. $\therefore (f_s = f_y)$

7- Flexural capacity $M_c = \emptyset_b A_s f_y (d - \bar{y}) = 0.9 \times 900 \times 420 \times (450 - 12.479) \times 10^{-6}$

$$M_c = 148.845\ kN.m$$

The Inverted triangle beam (B-10) with single reinforcement is a special case of Inverted trapezoidal section and it could be easily obtained by setting the least width dimension b_1 equal zero.

3.2.3. Beam B-11 (Hexagonal Beam Analysis)

Input Data (Figure 3f):

$A_{sT} = 6\ \emptyset 20 = 1884.954\ mm^2$
$f_y = 400\ MPa$
$f_c' = 30\ MPa$
$E = 200{,}000\ MPa$
h= 380 mm

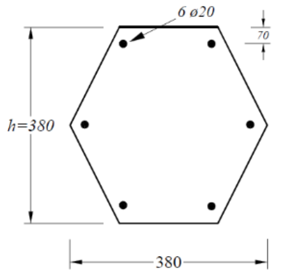


Figure 3. (f) Beam section B-11

Solution:

To solve the Hexagonal shaped beam, it should be converted to equivalent square shape to find the required moment capacity (Figure 3g).

1- The height for the equivalent square shape can be found as; $H_{square} = h \times 0.93$

$H_{square} = 380 \times 0.93 = 353\ mm$
$d' = 70\ mm$
$$A_s = \frac{A_{sT}}{2} = 3\ \emptyset 20 = 942.48\ mm^2$$

2- Find the depth of the compression area (a);
$$a = \frac{A_s F_y}{0.85\ f_c'\ b} = \frac{942.48 \times 400}{0.85 \times 30 \times 353} = 41.88\ mm$$

3- Verifying that the steel is yielding. $(f_s = f_y)$

$$a = 41.88\ mm$$
$$c = \frac{a}{\beta} = \left(\frac{41.88}{0.85}\right) = 49.27\ mm$$
$$\epsilon_s = \left(\frac{d-c}{c}\right) 0.003 = 0.014$$
$$\epsilon_y = \frac{F_y}{E_s} = 0.002$$
$\epsilon_s > \epsilon_y$, the assumption is OK. $\therefore (f_s = f_y)$

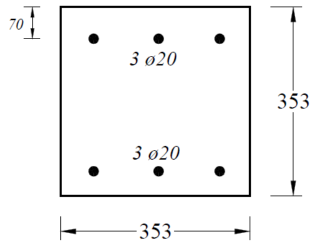


Figure 3. (g) Equivalent Square Section

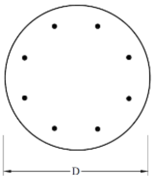
4- Flexural capacity $M_c = \emptyset_b A_s f_y \left(d - {}^{a}/_{2}\right) = 0.9 \times 942.48 \times 400 \times (283 - 20.94) \times 10^{-6}$

$$M_c = 88.9\ kN.m$$

4. Circular Beams (Group-3)

This group contains the study of circular beams. Circular beams are used but quite often in the construction Industry. Equivalent square method is used in this study to find the design moment capacity for the circular beams. In this study, the circular section (B-12) having different diameters are selected. The analytical results of these circular beams using the equivalent square method are also compared with the finite element software (SAP) and the results obtained are displayed in Table 4. Moreover, the numerical solution for finding the design capacity results for one of the circular section (diameter D=450 mm) is also shown in this study.

Table 4. Design Strength of Circular Beam

Beam ID	Circular Beams	Design Parameters					Mc (kN-m)	
		D (mm)	f_y (MPa)	f'_c (MPa)	d' (mm)	As (mm^2)	Flexural Equations	Finite Element Software
B-12		400	300	20	85	8 Φ20	81.4	88.3
		450	400	30	60	6 Φ25	140.6	146.2
		500	400	30	110	8 Φ25	212	232
		300	415	30	50	6 Φ20	66.3	70

4.1. Beam B-12 (Circular Beam)

Input Data (Figure 4a):

$A_{sT} = 8\,\emptyset 20 = 2513.28\ mm^2$
$f_y = 400\ MPa$
$f'_c = 30\ MPa$
$E = 200{,}000\ MPa$
D= 450 mm

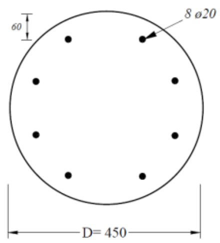


Figure 4. (a) Beam section B-11

Solution:

To solve the Circular shaped beam, it should be first converted to equivalent square shape to find the required moment capacity (Figure 4b).

1- The height for the equivalent square shape can be found as; $H_{square} = D \times 0.89$

$H_{square} = 450 \times 0.89 = 400\ mm$
$cc = 60\ mm$
$A_s = \frac{A_{sT}}{2} = 4\,\emptyset 20 = 1256.64\ mm^2$

2- Find the depth of the compression area (a);

$$a = \frac{A_s F_y}{0.85\ f'_c\ b} = \frac{1256.64 \times 400}{0.85 \times 30 \times 400} = 49.3\ mm$$

3- Verifying that the steel is yielding. ($f_s = f_y$)

$a = 49.3\ mm$

$$c = \frac{a}{\beta} = \left(\frac{49.3}{0.85}\right) = 58\ mm$$

$$\epsilon_s = \left(\frac{d-c}{c}\right) 0.003 = 0.0146$$

$$\epsilon_y = \frac{F_y}{E_s} = 0.002$$

$\epsilon_s > \epsilon_y$, the assumption is OK. $\therefore (f_s = f_y)$

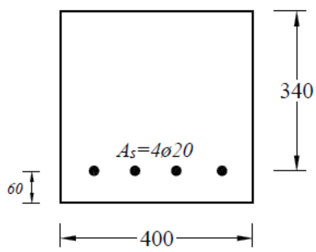


Figure 4. (b) Equivalent Square Section

4- Flexural capacity $M_c = \emptyset_b A_s f_y \left(d - {}^{a}/_{2}\right) = 0.9 \times 1256.64 \times 400 \times (340 - 29) \times 10^{-6}$

$$M_c = 140.6\ kN.m$$

5. Results and Discussion

The results obtained using simplified method for all of the three groups are compared with the computer software. The percentage difference for all of these sections are depicted in the bar charts (Figures 5 to 7). For the group-1 beam sections, the percentage difference of simplified method for beams B-1, B-2 and B-6 with the finite element software ranges below 5% while the remaining beam sections B-3, B-4 and B-5 lies within 5% to 10 % respectively.

Each beam section of Group-2 beams (B-7 to B-11) are analysed with different leg dimensions and each beam section showed promising results as the percentage difference ranges within 1% to 8%. Circular beam sections (Group-3) are analysed using the equivalent square section method. Four different circular beam sections with different diameters (B-12a – B-12d) are analysed in this group using simplified method and their results varies within a percentage difference of 5% to 10% with the finite element software.

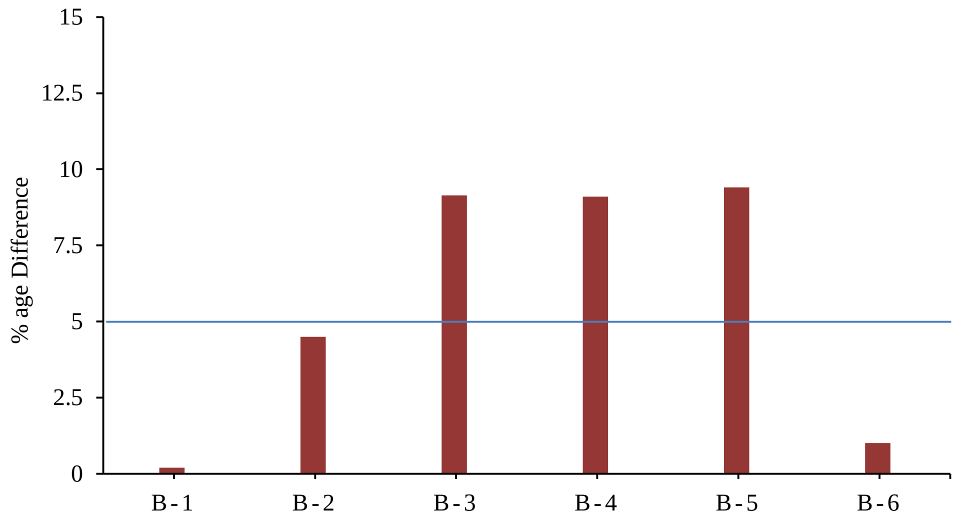


Figure 5. Percentage difference for straight edged beams (Group-1)

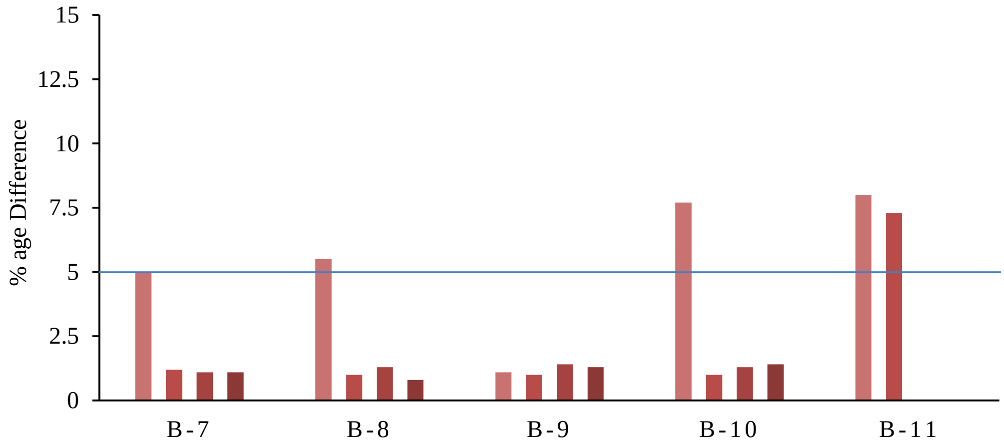


Figure 6. Percentage difference for sloped edged beams (Group -2)

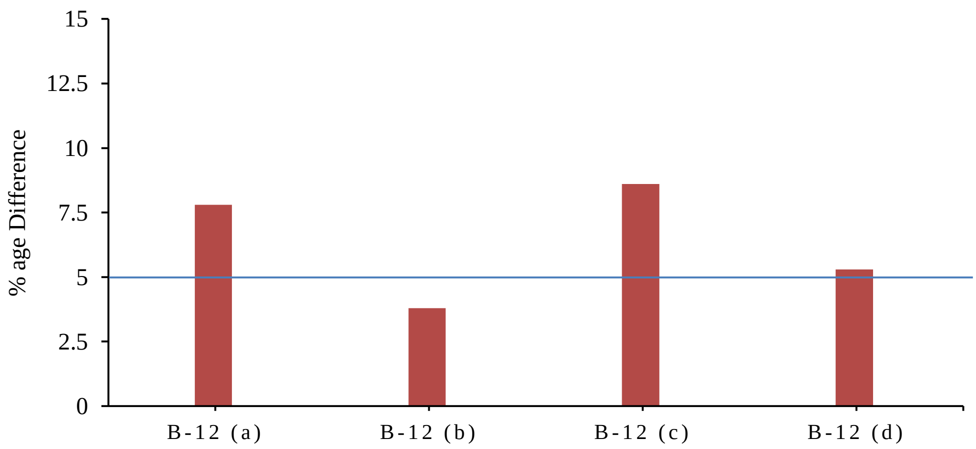


Figure 7. Percentage difference for circular beams (Group -3)

6. Conclusion

This paper presents simple method to estimate the flexural capacity of different irregular shaped beam section M_c. Three different types of the irregular beams groups (*beams with straight edges, beams with sloped edges, and circular beams*) are studied using the simplified method. This study helps in analysing the flexural capacity of all irregular shaped beams presented in this paper. The moment capacities obtained from the simplified method of these irregular shaped beams showed promising results when compared with the finite element software (SAP) with a percentage difference of 1% to 10 % respectively.

The moment capacity for the first two groups (*beams with straight edges and beams with sloped edges*) can be found by using the similar flexural equation used in case of rectangular beam with single reinforcement. For the third group (circular beams) and for the hexagonal shaped beam sections, equivalent square method is used to find the flexural capacities as this approach is quite simple to use and results obtained are quite close to the finite element software results.

7. Conflicts of Interest

The authors declare no conflict of interest.

8. References

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