



An Improved CTM Model for Urban Signalized Intersections and Exploration of Traffic Evolution

Arlinda A. Rrecaj^{1*}, Vlera Alimehaj², Marija Malenkovska³, Cvetko Mitrovski³

¹ University of Prishtina, Faculty of Mechanical Engineering, Department of Traffic Engineering, 10000 Prishtina, Kosovo.

² University of Prishtina, Faculty of Electrical Engineering, Department of Computer Engineering, 10000 Prishtina, Kosovo.

³ University St. Kliment Ohridski, Faculty of Technical Sciences, 70000 Bitola, North Macedonia.

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Abstract

In this paper is going to be proposed a Cell Transmission Model (*CTM*), its analysis and evaluation with a case study, which addresses in a detailed way the aspect of merging and diverging operations on urban arterials. All those few *CTM* models that have been developed so far, to model intersections, have some limitations and drawbacks. First, unlike the simple composition road networks, such as highways, urban arterials must include some complex parts called merge and diverge, due to the fact of vibrational values of reduced capacity, reduced saturation flow rate, etc. In order to simulate an urban network/arterial it is not possible to neglect the traffic signal indication on the respective time step. The objective of this paper is to highlight the difference between the results of the original *CTM* and our proposed *CTM* and to provide evidence that the later one is better than the old one. The proposed and formulated model will be employed through an algorithm of *CTM* to model a segment- arterial road of Pristina (compound from signalized intersections). For the functionalization and testing of the proposed model is build the experimental setup that is compatible with the algorithm created on *C#* environment. Results show that the proposed model can describe light and congested traffic condition. In light traffic conditions, in great mass traffic flow is dictated by the traffic signal status, while in medium congestion is obtained a rapid increase of the density to each cell. Fluctuations of the density from the lowest to the highest values are obvious during the first three cycles to all cells of the artery in a congested traffic state.

Keywords: Cell Transmission Model; Urban Traffic; Merge and Diverge; Intercell Flow; Traffic Congestion.

1. Introduction

During history of seeking various analytical traffic engineering techniques, lots of traffic simulation tools have been proved to have potential solutions in traffic problems. Related to the traffic control of intersections, whatever they were, un-signalized or signalized, adequate techniques were adopt for analysis and identifications of various problems. These simulation techniques have evolved with traffic flow models that describe traffic flow by different aspects, and as the most popular classification of them is the microscopic and macroscopic traffic flow [1].

In microscopic models the process of traffic flow is described on the level of the individual entities or driver units (vehicles) and the interactions between these units explicitly modelled. The traffic flow process is the collective

* Corresponding author: arlinda.alimehaj@uni-pr.edu

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behavior of all the units together while macroscopic models consider the traffic as the continuum as fluid flow by the characteristic quantities such as characteristics as, speed traffic density and traffic flow or volume at point and time respectively. The earliest macroscopic model was proposed by Chen et al. (2015) and Lighthill and Whitham (1955); referred as LWR model [2, 3].

Some other forms of traffic flow modeling are discretized models when the main traffic variables of *LWR* model are discretized in time length dimensions. In the frame of the discretized models one and very extensively used on the two past decades is the cell transmission model (CTM). This dissertations purpose is the development of an enhanced CTM model that will be able to model traffic conditions particularly on the urban arterials, in both free and saturated flow state. CTM model was first developed by Daganzo (1993 and 1995) [4, 5], but since that time due to the fact of its simplicity of expression of traffic parameters, has become a popular model to researchers of the modelling disciplines. Author has found the new way to overcome the difficulties of partial differentiating by partitioning the road segments and adopting the fundamental diagram of flow and density.

So, models that derived from the first general CTM hopefully have served as robust tools for addressing traffic problems although their perfections is not achieved yet. We are witness to a lot of researches based on CTM, since 1994, when Daganzo [4] has developed it in a simple link with no convergences. The models based on CTM, in the most cases, have proved to provide satisfied results on modelling the different traffic conditions as well as efficient tools for analysis, estimation and prediction of traffic parameters. The nowadays Intelligent Transportation Systems (*ITS*) require on line information of traffic parameters and during the development of the *CTM* models many benefits are achieved from them toward this aspect. In following unit is given a brief review by intentionally ordering, firstly the researches related to highways model and then the arterials, because the first ones have date immediately after the first version of *CTM*, following by researches with regard to complex nodes-intersections of urban arterials.

1.1. Highway Models

Daganzo (1994) [6] presented a simple representation of traffic on highway with a single entrance and exit. The one way road was divided in homogeneous sections (cells), with length that are set equal to the distances travelled in light traffic in a time step as the author decided to be shown with a clock tick. The paper provides an analogue presentation of the equation of CTM model with the discrete approximation of the LWR hydrodynamic model with a density-flow relationship of the trapezoid form and then used to predict the traffic parameters. Munoz et al. (2003) [7] developed a switching-mode model (SMM) for a short highway section where suggested that the upstream and downstream flows are measured and under the assumption of the triangular diagram of flow density relationship, the inter cell flows are determined as in the simple example of CTM model.

The mentioned authors applied a mixture Kalman Filter [8], which is a recursive data processing algorithm that uses only the previous time-step's prediction with the current measurement, in order to make an estimate for the current state [8], to estimate the densities on the sections with unmeasured traffic flows. Sun (2005) [9] in his dissertation developed a ramp-metering control algorithm using the linear quadratic controlling in a modified cell transmission model of a freeway segment by developed by Munoz et al. (2003) [7]. Waller et al. (2007) [10] have developed the validation of a CTM based highway segment model which is used to predict the occupancy using coverage detectors on different parts of it.

Accuracy of the results concluded that the CTM model can well be suited in the areas of forecasting through different advanced techniques as neural network, Kalman filter etc. Aligawesa (2009) [11] proposed a model and algorithm which applied to various scenarios of congestion incidents and validated with real traffic data that show satisfied results. Author proposes a switched CTM model that contains five modes namely FF, FC₁, FC₂, CF and CC, whereby each letter represents either a free flow (F) or congestion (C) status for a cell respectively, and identified the state of each cell particularly based on a proposed distribution of the probabilities of the cell state.

Chen et al. (2010) [12] tried to give an improvement to the original CTM model extending it by defining various shapes of fundamental diagram with the aim to reproduce more types of traffic conditions including capacity drops, lane-by-lane variations, non-homogenous propagation of backward waves. Author proposed different diagram for each of five lanes of a freeway segment in California for which poses the collected data by loop-detectors. For lanes 1 and 2 adopted a fundamental diagram with capacity drop, for lane 3 and 4 a triangle density-flow relationship diagram and for the lane 5 a trapezoid form fundamental diagram. Tests have shown that this modified promises good results, compared to the field data.

1.2. Intersection and Arterial Models

Easily can be guessed from the literature review that cell transmission model evolution of the complex nodes, has followed after a considerable time from the cell transmission model of the simple composition of roads, as highways and freeways segments. Since the nature of the intersections in general and some specific cases of nodes in particular, differ from the highways that in most of cases are treat as one way, with some entries and exits, considerable modifications and modalities to the original cell transmission model have seen as necessary.

Wang (2010) has developed a conditional cell transmission model CCTM in her dissertation research [13], by proposing the adding a so called conditional cell in the center of the intersection which stores the vehicles which cause blockades. With an added conditional cell, author tended to simulate the blockades and spillbacks in the intersections. With one word, when the left turn vehicles face a conditional cell on their target entry, could be blocked by this cell. The flow reduction increases if the occurrence of this conditional cell happens frequently, and the decreases if its occurrence happens rarely. This occurrence is expressed in probability percentage.

The overall inter-cell flow laws and capacity determinations of the individual lanes, are same as the cell transmission model for simple links. CCTM flows also accounted for traffic signal indication. For a red signal the value is zero and for a green signal a constant that accounts for flow takes value one. Nevertheless, the mentioned conditions are well reflected in the output results of the mode, from my point of view, the model has lead an excessive approximation of the blockades rather than in real presentation of the conditions. In the midst of scientific works toward efforts to create a suitable cell transmission model that is capable to present inclusion of different movements on the phases of traffic signal plans of controlled intersections, it is worth to mention the algorithm of CTM-URBAN [14], developed by Huang (2011). Algorithm proved to overcome limitations of the original cell transmission model in the application of urban intersection by implicating generate changeable turning ration factors, which on the past had static values. Thorough comparative analysis between this new algorithm and a validated model of Vissim [15], have been achieved optimistic results in the aspect of representing permitted flow by incorporated parameters, benefits of full blockage factor and accuracy of the queue length estimation. Xie et al. (2013) gave their contribution on the application of the cell transmission model in estimating traffic congestions/jams and collisions, with explicit analysis of the effect of traffic light through a gridlock plan of an urban network [16]. Through a parameter named CG, they presented the collision and gridlock that take values from zero to one, with statement, higher is the value, heavier is the collision impact. With approximately similar idea of presenting the blockages on the intersections by inserting additional cells on particular lanes as Wang (2010) [13], is appeared with his research Papapanagiotou et al. (2013), but with difference referring to them as virtual cells. Model serves also to calculate number of stops delays [17]. As a base for estimating queue length, by first predicting the density of each cell and then the tail of length, CTM [18]. Identification of the tail of length was done through a density evolutions table which changes over time and space. The main criteria from, which the tail of queue is identified is that if the cell i and all its downstream cell densities are larger than the critical density and if the upstream cell densities are lower than the critical density, the queue tail locates in cell i or cell $i-1$. Further, to analyze whether the queue tail still stays in cell i or moves to cell $i-1$, the demanded space from cell $i-1$ and the supplied space from cell i should be compared. If all the last time interval vehicles of cell $i-1$, $n_{i-1}(k-1)$, flow into cell i in this time interval, it suggests that the traffic flow is free and the queue tail is still in cell i . Otherwise, the tail of the queue is located in cell $i-1$. Thus, it can be summarized that the queue tail is located in cell [13]. Cell transmission model was successfully used for traffic signal optimization also. Adacher and Tiriolo (2016) [19] presented a distributed algorithm for optimization of coordinated traffic signals. As the objective function of the optimization they laid the sum of delays that were obtained from e urban cell transmission model simulation. Although, CTM was used as a layout for optimal strategies of traffic signal control by not addressing well the discharge flow rate through different times of the cycle, i.e., green and red interval, leading in overestimated signal planning splits. Chow et al. (2015) in a cell transmission model of signalized intersection defined the red and green durations of the traffic light is set to be 30 (sec) and 30 (sec) respectively [20].

2. Principles of CTM and New Approaches Toward Its Improvement

CTM relies on the conversional of trapezoidal simplification of the fundamental diagram of the flow-density relationship that on this paper will be referred as *FDR* diagram. In order to be quickly linked with the *FDR* of the *LWR* model from which has evaluated cell transmission model, let we have a brief mention of its parameters. In the diagram depicted in the Figure 1,a), we can distinguish three parameters: maximal flow-capacity, Q_{cap} [veh/s], density, ρ [veh/m], critical density, ρ_{cr} [veh/m], jam density, ρ_j [veh/m], free flow speed v_f [km/h], and backward speed or propagation wave speed, w [m/s].

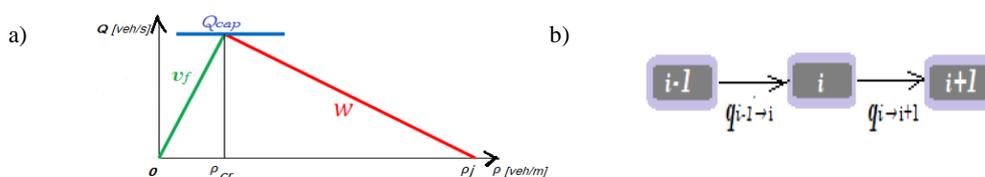


Figure 1. Fundamental diagram, a) fundamental parameters, b) Inter cell flows

Sending flow from the upstream cell and receiving flow of the downstream cell. With other words, the number of vehicles that can pass the cell $i-1$ is dictated by the empty space of the downstream cell i .

$$q_{i-1 \rightarrow i(t)} = \min\{S_{i-1}, R_i\} \tag{1}$$

Where: $S_{i-1} = \min\{v_f \rho_{i-1}, Q_{i-1}\}$ and $R_i = \min\{Q_{i-1}, w_i(\rho_j - \rho_i)\}$

Where:

$q_{i-1 \rightarrow i(t)}$ is the number of vehicles that enter in cell i during time step t ;

S_{i-1} is the sending function of cell $i-1$,

R_i is the receiving function of cell i ,

V_f is free flow speed in cell $i-1$,

ρ_{i-1} is the density of cell $i-1$,

Q_{i-1} Maximal flow or capacity of cell $i-1$,

w_i Backward speed of cell i ,

ρ_j is jam density of cell i and

ρ_i is the density of cell i .

2.1. Definitions of CTM for Urban Intersections

As were comprehensively given on the previous unit, the novel features of the proposed CTM model that guarantee accommodation of the various urban arterials, can be built by treating the variability of the characteristic parameters. Despite of original CTM when these characteristic parameters were constant at all discretized parts and during all time steps, here we face with the need of a calibration framework, from which will be obtained the main parameters depending on the dynamic of traffic conditions belonging on the parts of arterial or in any particular cell of it as well as for different evolution times, i.e. red/green interval, beginning of green, end of green, etc. In the Table (1) are presented the characteristic features of CTM of freeways and urban segments.

Table 1. Consideration of Changeable Parameters/ Calibration

	Original CTM		New CTM	
	Freeway	Intersection	Freeway	Urban Intersection
Complexity of road configuration	No	Yes Merge/diverge	No	Yes Merge/diverge
Inter Cell Flow	No	No	Yes Sending/receiving demand	Yes Sending/receiving demand
Flow Capacity	No	No	Yes Sending/receiving demand, On ramp Metering	Yes Sending/receiving demand, Signal Phase
Storage Capacity	No	No	Yes	Yes
Free Flow Speed	No	No	Yes	Yes
Backward Speed	No	No	Yes Beginning /End of Green	Yes Portion of Green
Critical Density	No	No	Yes	Yes
Queuing Discharge Headways	No	No	No	Yes Traffic Signals

2.2. Inter Cell Flows for Merge and Diverge

2.2.1. Diverge

Determination of inter cell flow of ordinary links is easier to calculate than that of the complex configuration, that *Daganzo* referred as Merges and Diverges, Figure 2. Instead of calculating the above value of only one amount of flow q_i that can be sent from the upstream cell a , we have to determine two of them, one that can be accepted from cell b and one from the cell c . These diverging flow can be easily calculated if we know the turning percentage. If we suppose we have a diverge with proportions of $\beta_b(t)$ and $\beta_c(t)$ of $S_a(t)$ going to each link, $\beta_b(t) + \beta_c(t) = 1$, the two flows that go from cell a must fulfil the preliminary condition:

$q_{a-b} = \beta_b \cdot q_i$ and $q_{a-c} = \beta_c \cdot q_i$. Secondly, the total flow $q_{a-b} + q_{a-c}$ can not be higher than the sending flow of the cell a , S_a and none of the flows q_{a-b} and q_{a-c} can exceed the respective receiving flow R_b and R_c .

Taking into account the above conditions, the main constraint equations can be written:

$$q_i = \min[S_a, R_b/\beta_b, R_c/\beta_c] \tag{2}$$

$$q_{a-b} = \beta_b \cdot q_i \tag{3}$$

$$q_{a-c} = \beta_c \cdot q_i \tag{4}$$

$$q_i \leq R_b/\beta_b \text{ and } q_i \leq R_c/\beta_c$$

After substituting the main equation for the sending and receiving functions respectively (1) in the above constraint equations, can be written the Equations 5 and 6 for flow downstream cells, *b* and *c*, respectively.

$$q_{a-b} = \min\{\beta_b \min[v_{fa}\rho_a, Q_a]; \min[Q_a, w_b(\rho_{jb} - \rho_b)]; \beta_b/\beta_c \min[Q_a, w_c(\rho_{jc} - \rho_c)]\} \tag{5}$$

$$q_{a-c} = \min\{\beta_c \min[v_{fa}\rho_a, Q_a]; \beta_c/\beta_b \min[Q_a, w_b(\rho_{jb} - \rho_b)]; \min[Q_a, w_c(\rho_{jc} - \rho_c)]\} \tag{6}$$

2.2.2. Merge

Merge can be in one the three possible regimes: Merges provides a complicated form of CTM configuration and flows must satisfy the conditions:

$$q_{a \rightarrow c} \leq S_a \text{ and } q_{b \rightarrow c} \leq S_b \tag{7}$$

$$S_a + S_b \leq R_c \tag{8}$$

When sum of sending flow of *cell a* and *cell b* Figure 2, a) is less than the receiving flow of *cell c* then the entire volumes can be absorbed by *cell c*, but in other case when sending flow is greater than the possibility of acceptance by *cell c* then the matter varies in that there are some rules of vehicle issuance according to the linear rule given below.

$$q_{a \rightarrow c} = R_c - q_{b \rightarrow c}; \quad q_{b \rightarrow c} = R_c - q_{a \rightarrow c}$$

Depending on the proportions β_a and β_b of vehicles on each cell respectively, a line can be defined so that $q_{a \rightarrow c}/q_{b \rightarrow c} = \beta_a/\beta_b$. The linear equation is:

$$q_{b \rightarrow c} = \beta_b/\beta_a \cdot q_{a \rightarrow c} \text{ and/or } q_{b \rightarrow c} = \beta_b/\beta_a \cdot q_{a \rightarrow c} \tag{9}$$

The flow originating from the cells *a* or *b* cannot be greater than their maximum flows, i.e. $\beta_a \cdot R_c > S_a$ or $\beta_b \cdot R_c > S_b$ and q_a must respect constraint to S_a and q_b to S_b . The remainder of R_c can be assigned to the other flow, i.e. $q_{a \rightarrow c} = R_c - S_b$ or $q_{b \rightarrow c} = R_c - S_a$.

The above elaboration of merge, can be defined comprehensively with the below equations:

$$q_{a \rightarrow c} = S_a, \quad q_{b \rightarrow c} = S_b \quad \text{if } S_a + S_b \leq R_c \tag{10}$$

Otherwise;

$$\begin{aligned} q_{a \rightarrow c} &= \text{mid}\{S_a, (\beta_a \cdot R_c), (R_c - S_b)\} \\ q_{b \rightarrow c} &= \text{mid}\{S_b, (\beta_b \cdot R_c), (R_c - S_a)\} \end{aligned} \tag{11}$$

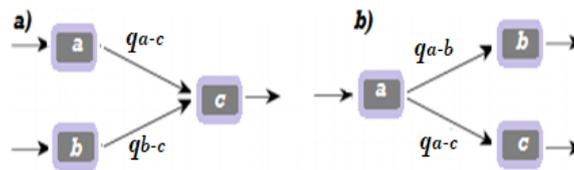


Figure 2. a) Merge and b) Diverge

2.3. Notation of Merge and Diverge Cells

We agree to make a denomination for a cell through two digits, and one letter such way that the first digit presents the segment number, the second digit presents the ranking number of cell in segment, where *n* is the most distant cell and *1* is the first cell nearby the stop line and the digits presents the turning lane r-right and l-left as in Figure 3 and choose to draw the equation of inter cell flows of diverge and merge of segment *1*. For example label *12* stands for the cell in segment *1*, ranked on the second place from the stop bar. Adding a letter *r*, we present the cell in right turn lane, nearby the sop line. The absorption cell of every approach exit has e second digit zero on its label. For example the cell *1,0* is the merge cell of segment1. Initially there are denominated the main intersection approaches as *1* and *2* and the minor approaches with *3* and *4*.

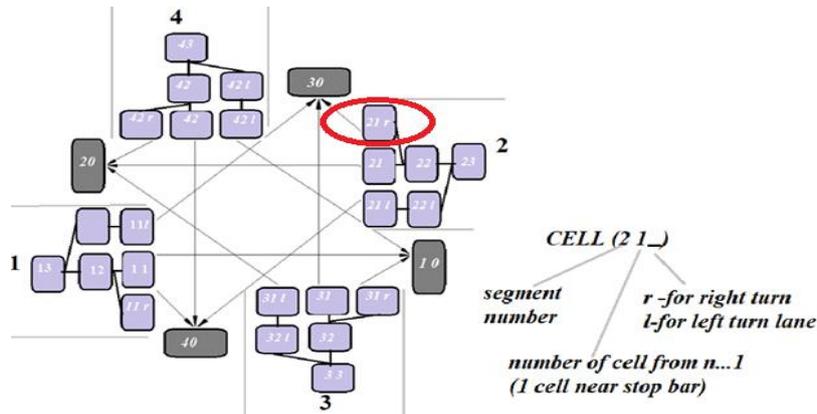


Figure 3. Merges and Diverges of an intersection-Notation

3. Demand of Nearby Stop-Bar Cell (SBC)-New Definition of Fundamental Diagram

In very early *CTM* models the demand functions of the cells next to stop (*SBC*) line are considered constant in both free flow and congested traffic flow conditions. The only difference was on defining the traffic signal phase belonging. Depending on the instant state of the traffic signal indication, the discharge flow/outflow was assumed as zero if the signal phase is red and different from zero of the signal phase is green [21-23]. As it is known, by the traffic flow nature on the signalized intersections, at the beginning of green interval the discharge rate is low, with higher discharge headways do to the reaction time and start up time of the first vehicle, which increases to the capacity flow until the queue is being dissipated.

Having the above description, the *CTM* functions do not capture the demand of the nearby-stop cell during the different portions of green interval. On this way the derived model is not capable to treat the nature of discharge features of urban arterials and may lead to non-consistency of the model and observed real traffic parameters. Aron et al. based on a discretization *LWR* model, defined that the demand on the congested traffic has a lower value than the capacity and decreases with density [24, 25].

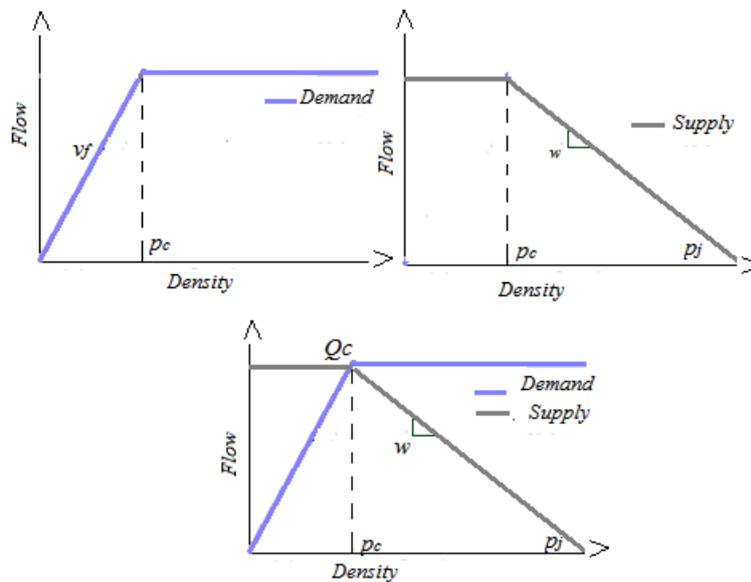


Figure 4. Illustration of demand (violet) and supply (gray) functions of the proposed CTM, on the final diagram, three new CTM parameters are presented (Q_j , ρ_j' and w')

Regarding the new one fundamental diagram *FDR*, of the proposed new *CTM* model, we can highlight three new parameters additional to the original *CTM* model. These parameters as, are depicted on the fig. are, slope of the new demand curve, w' (written same as the slope of the supply or backward speed but with a 'notation ""'), new jam density, ρ_j and jam flow rate which correspond to the new jam density, Q_j .

The density of the queued vehicles of the *SBC* is higher than the critical density which corresponds to low speeds, as the signal is indicating the first green seconds. At general cases, a role on the determination of the traffic state plays the length of the cell. A cell with a length long enough to include the entire queue, implicitly can be qualified as the congested traffic state and the cell upstream to it can be qualified as is in free flow state *FC*, Figure 5, b). In opposite

case if the SBC does not achieve to include the entire queue length, the both it and its upstream cell can be qualified as congested state or *CC*, Figure 5 a). The downstream cell, or merge cell of the entry approach of intersection is always supposed as is in free flow state. The above statement stands for the very early portion of the green interval.

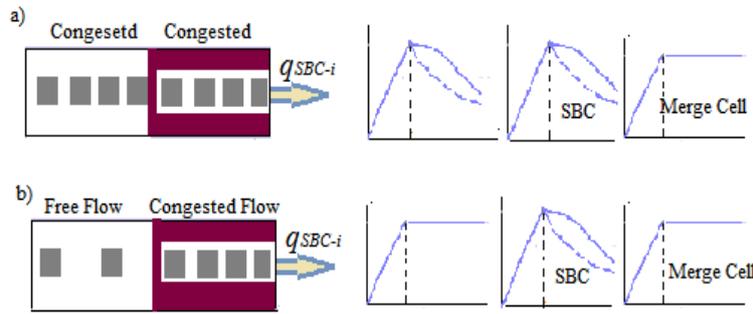


Figure 5. SBC traffic state conditions and FDR employment, a) SBC and its upstream congested and b) SBC congested and its upstream free flow

As the queue continues to dissipate through green interval, the density decreases while the flow rates increases. Since the *CTM* is a discrete model not only by length dimension but also of time, the selection of the demand functions depends on the time increment intervals. Special cases can be employed of on selection of the *FDR* diagram, based on the step time inclusion to the portion of green interval. Consequently, only during the first time step during which the vehicle density is over critical, must be employed a new demand function which is linearly decreasing with density, but on the other next time steps (surely included on green interval) we can employ a demand that corresponds to capacity flow after which, the density is under critical Figure 5, a). If the time step is almost equal to green interval, only the new *FRD* for demand is applied.

The minimum flow rate that correspond to the new demand function, jam flow, Q_j is calculated as:

$$Q_j = w' \cdot (\rho'_j - \rho_j) \tag{12}$$

From the fundamental diagram of above Figure 4 can be drawn the expression for calculations of the new parameters. The capacity flow that from the fundamental diagram was calculated as:

$$Q_c = w \cdot (\rho_j - \rho_c) \tag{13}$$

With changes of the new fundamental diagram can be calculated as:

$$Q_c = w' \cdot (\rho'_j - \rho_c) \tag{14}$$

Consequently, any intermediate value of the flow rate between the above two values $Q_j < Q_i < Q_c$, can be calculated with the formula:

$$Q_i = w' \cdot (\rho'_j - \rho_i) \tag{15}$$

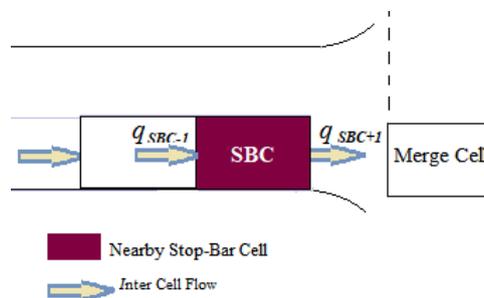


Figure 6. In flow and out flow of SBC

If we agree to denote the inflow to the *SBC*, $q_{(SBC-1 \rightarrow SBC)}$ with q_{in} and the outflow $q_{(SBC \rightarrow SBC+1)}$ with q_{out} , Figure 6 from the *CTM* fundamental diagram of relationship of flow and density we can write their equations as follows.

$$q_{in} = \min\{S_{SBC-1}, R_{SBC}\} \tag{16}$$

Where S_{SBC-1} and R_{SBC} are the sending and receiving of the *SBC-1* and *SBC* cells, respectively, calculated with Equation 17;

$$S_{SBC-1} = \min\{Q_c, v_f \rho_{SBC-1}\} \text{ and } S_{SBC} = \min\{Q_c, w(\rho_j - \rho_{SBC})\} \quad (17)$$

Since the SBC-1 is uncongested and SBC is congested, the inflow is dictated by receiving function of SBC.

$$q_{in} = \min\{Q_c, w(\rho_j - \rho_{SBC})\} = w(\rho_j - \rho_{SBC}) \quad (18)$$

The outflow $q_{(SBC \rightarrow SBC+1)}$ is calculated as:

$$q_{out} = \min\{S_{SBC}, R_{SBC+1}\} \quad (19)$$

Where S_{SBC} and R_{SBC+1} are the sending and receiving functions of the cell SBC and cell SBC+1 respectively, calculated as;

$$S_{SBC} = \min\{Q_c, v_f \rho_{SBC}\} \text{ and } R_{SBC+1} = \min\{Q_c, w(\rho_j - \rho_{SBC+1})\} \quad (20)$$

Since the SBC is congested, and SBC+1 is uncongested the outflow is dictated by demand of the SBC cell.

$$q_{out} = \min\{Q_c, v_f \rho_{SBC}\} \quad (21)$$

But as results of changes of the new fundamental diagram FDR, and new demand function of the outflow of the last equation takes the form as in equation;

$$q_{out}(t) = \min\{Q_c, v_f \rho_{SBC}\} = \min\{w' \cdot (\rho'_j - \rho_c), v_f \rho_{SBC}\} \quad (22)$$

Where Q_c may have values from its minimal value at the beginning of interval Q_j till its maximal value Q_c , based on the increasing function of the new FDR (3.12).

$$Q_{SBC} = \begin{cases} Q_j = w' \cdot (\rho'_j - \rho_j) & \text{if } t \geq 0, \\ Q_i = w' \cdot (\rho'_j - \rho_i) & \text{if } 0 \leq t \leq t_{free} \\ Q_c = w' \cdot (\rho'_j - \rho_c) & \text{if } t \geq t_{free} \\ 0 & \text{if } t < 0 \text{ (red)} \end{cases} \quad (23)$$

The aim of adopting the new one demand function was on satisfying the discharge process of queues at beginning of green through a decreasing function of flow by density.

4. Case Study

Test bed is focused on the urban road network of the city of Prishtina, capitol of Kosovo (Figure 7). It is the urban segment of boulevard “Bill Clinton” an extension of the Highway M9- “Peja League. The arterial is of length 970 meters, with lane widths 3.2, meters on the first and second intersections and 3.0 meters on the third intersection. As an entry segment of to the entire network of the city, through which spills all traffic from the respective highway, it obviously faces from congestion until full blockages of traffic.

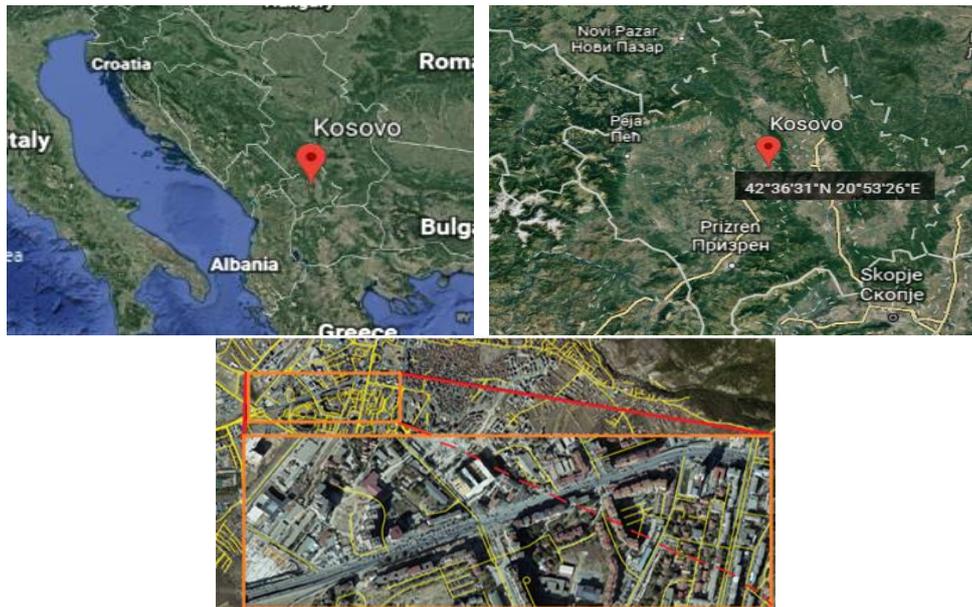


Figure 7. Urban segment road “Bill Clinton”/ M9, sources: a) Macro location, Prishtina, Kosovo, b) Micro location-Geo Portal-Kosovo map and c) CTM configuration

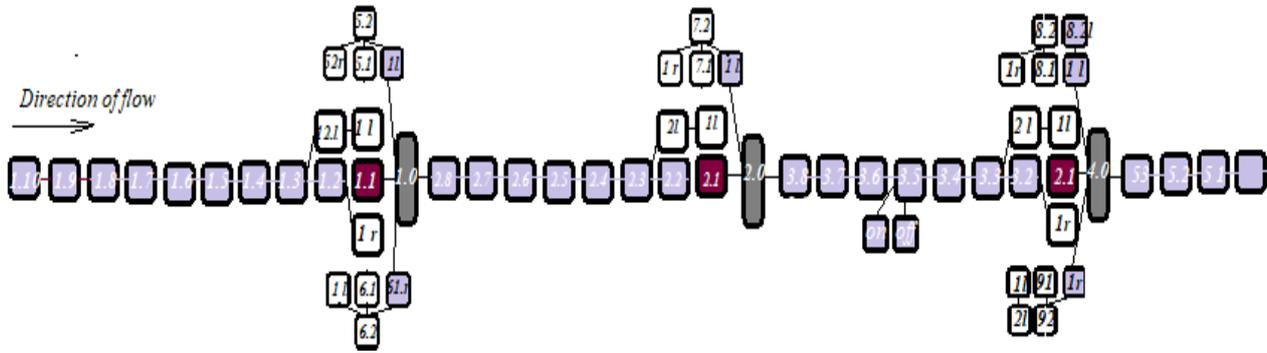


Figure 8. Urban segment road “Bill Clinton”/ M9, CTM Configuration

The actual cycle length of subjected intersections is 120 seconds, which means that one of them comprises of 52 time steps, the overall number of cycles within the simulation is 7.5 cycles, consequently the number of time steps in simulation run is 390. The beginning of greens for each cycle (as in Figure 9) are spread differently on each intersection, thus the definition of the beginning of greens intervals and their ending are very crucial in involving the cell transmission model properties. Our aim is not to perform a model simulation in separated cycles, but to obtain a continuance of the traffic state from cycle to cycle, nevertheless, the corresponding time steps to the main point as, beginning and ending of each green interval, had to be appointed on algorithm.

To find on which time step the green intervals begin and end, on respective cycle, we use the below expressions:

$$g_n^s = (n - 1) \cdot 52 + g_{0,i} \tag{24}$$

$$g_n^e = (n - 1) \cdot 52 + g_{0,i} + g_i \tag{25}$$

Where:

g_n^s is the beginning time step of the green interval on cycle n ,

g_n^e is the ending time step of the green interval on cycle n ,

$g_{0,i}$ is the beginning time of green interval on a single cycle, of Intersection i

g_i is the length of green interval of intersection i .

n is the number of cycle which can take values from $n=1, \dots, 7$. Consequently, a value one (1) of n , gives the beginning of green interval in initial cycle: $g_n^s = (1 - 1) \cdot 52 + g_{0,i} = g_{0,i}$, where value 52 corresponds to the number of time steps in a single cycle.

As it was stated through the formulation of the model algorithm, the new CTM model accounts for the variability of the parameter value related to the time steps evolution, such as flow capacity of a SBC cell that may have an increasing trend of its value on the advanced time steps, from its minimal value of Q_I at the beginning green interval (first time step), till the maximal value of capacity Q_C on the tenth time step.

The respective values of the flow capacity to first ten time step can be calculated with expression below:

$$Q_i = b + \left[a \cdot \left(t_i - (g_0 + (n \cdot 52)) \right) \right] \tag{26}$$

Where:

Q_i is the flow capacity of the time step i ,

t_i is the time step from 1 to 10,

n is the number of cycle which can take values from $n=1, \dots, 7$

$g_{0,i}$ is the beginning time of green interval on a single cycle, of Intersection i

a and b are the parameters of new demand function.

Traffic flow on the three signalized intersections is controlled with fixed-time traffic signals. Fixed-time signal control uses present time intervals that are the same every time the signal cycles, regardless of changes in traffic volumes. The duration of each phase and the phase configuration is given in the traffic signal plan in Figure 9. In particular signal phases are assigned the major through movements of the both directions separated from minor thorough movements. Left turning movement of the first and third intersections are assigned to protected phases, while those of the minor approaches are permitted on the first intersection and protected on the third one. Right turning movements are permitted plus protected with storage lane length of twenty meters on the first and third intersections. These flows of the third one are chanzalized at a radius of twenty meters.

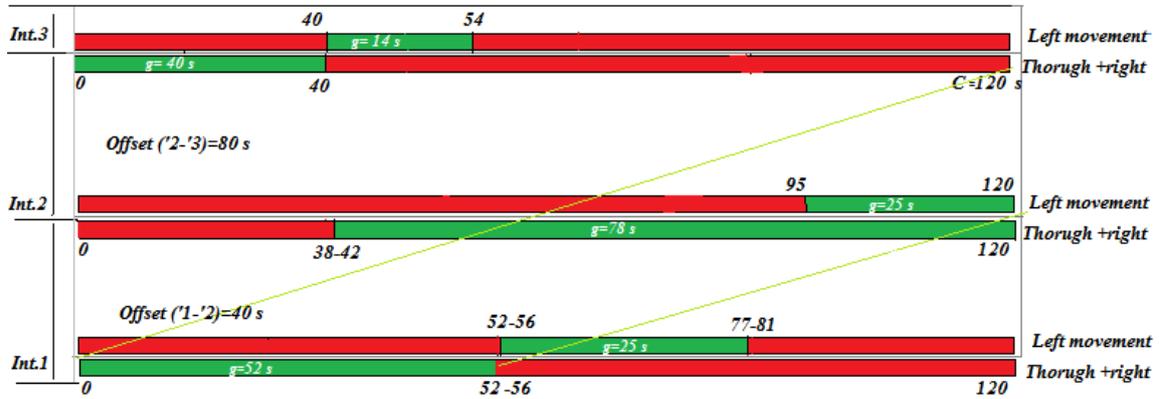


Figure 9. Traffic signal phase durations for the three intersections

Diverge flow equations of intersection 1:

$$q_{(13 \rightarrow (12,12l))} = \min\left\{S_{13}, \frac{R_{12}}{\beta_{12}}, \frac{R_{12l}}{\beta_{12l}}\right\}$$

$$q_{(13-12)} = \beta_{12} \cdot q_{(13 \rightarrow (12,12l))} \quad \text{or the expanded form:} \tag{27}$$

$$q_{(13-12)} = \min\{\beta_{12} \cdot \min[v_{f13}\rho_{13}, Q_{C13}]; \beta_{12}/\beta_{12l} \cdot \min[Q_{C13}, w_{12l}(\rho_{J12l} - \rho_{12l})]; \cdot \min[Q_{C13}, w_{12}(\rho_{J12} - \rho_{12})]\}$$

$$q_{(13-12l)} = \beta_{12l} \cdot q_{(13 \rightarrow (12,12l))}$$

$$q_{(13-12l)} = \min\{\beta_{12l} \cdot \min[v_{f13}\rho_{13}, Q_{C13}]; \min[Q_{C13}, w_{12l}(\rho_{J12l} - \rho_{12l})]; \beta_{12l}/\beta_{12} \cdot \min[Q_{C13}, w_{12}(\rho_{J12} - \rho_{12})]\} \tag{28}$$

$$q_{(12 \rightarrow (11,11r))} = \min\left\{S_{12}, \frac{R_{11}}{\beta_{11}}, \frac{R_{11r}}{\beta_{11r}}\right\}$$

$$q_{(12-11)} = \beta_{11} \cdot q_{(12 \rightarrow (11,11r))} \tag{29}$$

$$q_{(12-11)} = \min\{\beta_{11} \cdot \min[v_{f12}\rho_{12}, Q_{C12}]; \beta_{11}/\beta_{11r} \cdot \min[Q_{C12}, w_{11r}(\rho_{J11r} - \rho_{11r})]; \cdot \min[Q_{C12}, w_{11}(\rho_{J11} - \rho_{11})]\}$$

$$q_{(12-11r)} = \beta_{11r} \cdot q_{(12 \rightarrow (11,11r))}$$

$$q_{(12-11r)} = \min\{\beta_{11r} \cdot \min[v_{f12}\rho_{12}, Q_{C12}]; \min[Q_{C12}, w_{11r}(\rho_{J11r} - \rho_{11r})]; \beta_{11r}/\beta_{11} \cdot \min[Q_{C12}, w_{11}(\rho_{J11} - \rho_{11})]\} \tag{30}$$

$$q_{(23 \rightarrow (22,22l))} = \min\left\{S_{23}, \frac{R_{22}}{\beta_{22}}, \frac{R_{22l}}{\beta_{22l}}\right\}$$

$$q_{(23-22)} = \beta_{22} \cdot q_{(23 \rightarrow (22,22l))} \tag{31}$$

$$q_{(23-22)} = \min\{\beta_{22} \cdot \min[v_{f23}\rho_{23}, Q_{C23}]; \beta_{22}/\beta_{22l} \cdot \min[Q_{C23}, w_{22l}(\rho_{J22l} - \rho_{22l})]; \cdot \min[Q_{C23}, w_{22}(\rho_{J22} - \rho_{22})]\}$$

$$q_{(23-22l)} = \beta_{22l} \cdot q_{(23 \rightarrow (22,22l))}$$

$$q_{(23-22l)} = \min\{\beta_{22l} \cdot \min[v_{f23}\rho_{23}, Q_{C23}]; \min[Q_{C23}, w_{22l}(\rho_{J22l} - \rho_{22l})]; \beta_{22l}/\beta_{22} \cdot \min[Q_{C23}, w_{22}(\rho_{J22} - \rho_{22})]\} \tag{32}$$

Diverge flows of intersection 3:

$$q_{(33 \rightarrow (32,32l))} = \min\left\{S_{33}, \frac{R_{32}}{\beta_{32}}, \frac{R_{32l}}{\beta_{32l}}\right\}$$

$$q_{(33-32)} = \beta_{32} \cdot q_{(33 \rightarrow (32,32l))} \tag{33}$$

$$q_{(33-32)} = \min\{\beta_{32} \cdot \min[v_{f33}\rho_{33}, Q_{C33}]; \beta_{32}/\beta_{32l} \cdot \min[Q_{C33}, w_{32l}(\rho_{J32l} - \rho_{32l})]; \cdot \min[Q_{C33}, w_{32}(\rho_{J32} - \rho_{32})]\}$$

$$q_{(33-32l)} = \beta_{32l} \cdot q_{(33 \rightarrow (32,32l))} \tag{34}$$

$$q_{(33-32l)} = \min\{\beta_{32l} \cdot \min[v_{f33}\rho_{33}, Q_{C33}]; \min[Q_{C33}, w_{32l}(\rho_{J32l} - \rho_{32l})]; \beta_{22}/\beta_{22l} \cdot \min[Q_{C33}, w_{32}(\rho_{J32} - \rho_{32})]\}$$

$$q_{(32 \rightarrow (31,31r))} = \min\{S_{32}, \frac{R_{31}}{\beta_{31}}, \frac{R_{31r}}{\beta_{31r}}\} \tag{35}$$

$$q_{(32-31)} = \beta_{31} \cdot q_{(32 \rightarrow (31,31r))}$$

$$q_{(32-31)} = \min\{\beta_{31} \min[v_{f32}\rho_{32}, Q_{C32}]; \beta_{31}/\beta_{31r} \cdot \min[Q_{C32}, w_{31r}(\rho_{J31r} - \rho_{31r})]; \min[Q_{C32}, w_{31}(\rho_{J31} - \rho_{31})]\}$$

$$q_{(32-31r)} = \beta_{31r} \cdot q_{(32 \rightarrow (31,31r))} \tag{36}$$

$$q_{(32-31r)} = \min\{\beta_{31r} \cdot \min[v_{f32}\rho_{32}, Q_{C32}]; \min[Q_{C32}, w_{31r}(\rho_{J31r} - \rho_{31r})]; \beta_{31r}/\beta_{31} \cdot \min[Q_{C32}, w_{31}(\rho_{J31} - \rho_{31})]\}$$

4.1. Merges of Intersection 1

Ordinary inter cell flows are:

$$q_{(1.1-1.0)} = \min\{S_{1.1}, R_{1.0}\}$$

$$q_{(6.1r-1.0)} = \min\{S_{6.1r}, R_{1.0}, \}$$

$$q_{(5.1l-1.0)} = \min\left\{\min[n_{51l}; \frac{1}{t_f}(T - t_0 \cdot Q_{61})]; \min w/vf[Q_{10}, N_{10} - n_{10}]\right\} \tag{37}$$

Merge flow from simultaneous major through movement and protected left turning movement of minor segment.

$$q_{(5.1l-1.0)} = \min[n_{51l}; \frac{1}{t_f}(T - t_0 \cdot Q_{61})]$$

$$q_{(6.1-1.0)} = \min[n_{61}, Q_{61}]$$

if $R_{10} \geq S_{51l} + S_{61}$ *otherwise* (38)

$$q_{(5.1l-1.0)} = \min\left\{\frac{1}{t_f}(T - t_0 \cdot Q_{61}), \beta_{51l} \min[Q_{10}, N_{10} - n_{10}], \left[\min[Q_{10}, N_{10} - \min[n_{11}, Q_{11}]]\right]\right\}$$

$$q_{(1.1-1.0)} = \min\left\{\min[n_{11}, Q_{11}], \beta_{11} \min[Q_{10}, N_{10} - n_{10}], \left[\min[Q_{10}, N_{10} - \frac{1}{t_f}(T - t_0 \cdot Q_{61})]\right]\right\}$$

4.2. Merges of Intersection 2

Ordinary inter cell flows are:

$$q_{(2.1-2.0)} = \min\{S_{2.1}, R_{2.0}\} \tag{39}$$

$$q_{(7.1l-2.0)} = \min\{S_{7.1l}, R_{2.0}, \} \tag{40}$$

Merge flow from simultaneous major through movement and protected left turning movement of minor segment 7.

$$q_{(7.1l-2.0)} = S_{7.1l}$$

$$q_{(2.1-2.0)} = S_{2.1}$$

if $R_{20} \geq S_{2.1} + S_{7.1l}$ *otherwise* (41)

$$q_{(7.1l-2.0)} = \min\{S_{7.1l}, \beta_{7.1l} R_{2.0}, (R_{2.0} - S_{2.1})\}$$

$$q_{(2.1-2.0)} = \min\{S_{2.1}, \beta_{2.1} R_{2.0}, (R_{2.0} - S_{7.1l})\}$$

4.3. Merges of Intersection 3

Ordinary inter cell flow is:

$$q_{(8.1l-4.0)} = \min\{S_{8.1l}, R_{4.0}, \} \tag{42}$$

$$q_{(3.1-4.0)} = \min\{S_{3.1}, R_{4.0}\} \tag{43}$$

Merge flow from simultaneous major through movement and right turning movement of minor segment 8;

$$\begin{aligned}
 q_{(8.1l-4.0)} &= S_{8.1l} \\
 q_{(9.1r-4.0)} &= S_{9.1r} \\
 \text{if } R_{4.0} &\geq S_{8.1l} + S_{9.1r} \quad \text{otherwise}
 \end{aligned} \tag{44}$$

$$\begin{aligned}
 q_{(8.1l-4.0)} &= \text{mid} \{S_{8.1l}, \beta_{8.1l} R_{4.0}, (R_{4.0} - S_{9.1r})\} \\
 q_{(9.1r-4.0)} &= \text{mid} \{S_{9.1r}, \beta_{9.1r} R_{4.0}, (R_{4.0} - S_{8.1l})\}
 \end{aligned}$$

Merge flow of though movement that are simultaneous with left turning movement of segment 9 are:

$$\begin{aligned}
 q_{(4.1-4.0)} &= S_{4.1} \\
 q_{(8.1l-4.0)} &= S_{8.1l} \\
 \text{if } R_{4.0} &\geq S_{4.1} + S_{8.1l} \quad \text{otherwise}
 \end{aligned} \tag{45}$$

$$\begin{aligned}
 q_{(4.1-4.0)} &= \text{mid} \{S_{4.1}, \beta_{4.1} R_{4.0}, (R_{4.0} - S_{8.1l})\} \\
 q_{(8.1l-4.0)} &= \text{mid} \{S_{8.1l}, \beta_{8.1l} R_{4.0}, (R_{4.0} - S_{4.1})\}
 \end{aligned}$$

4.4. Data Collection, FDR Diagrams

Compilation of fundamental diagram is realized through surveying with smart phone video recordings. The analyzed parameters can be considered as microscopic since are evaluated for each individual entity/vehicle as a component of traffic. The length of road cells are chosen of 25 meters long. Video recordings were uploaded on computer for analysis through detailing the each vehicle’s prescription time with the aid of a smart phone application called *Stop Watch*, (Figure 10). Vehicles of each road section are analyzed and recorded. Through this application the headways with precision of one tenth are obtained and processed on excel spreadsheets (Attached in appendix of data). Through the times collected are obtained amount of flows, speeds and densities for every five second interval. Summarizing, for every single cell, are obtained 180 flow/densities data. (5second intervals of 15 minute).



Figure 10. Video recordings in “Bill Clinton” segment road and data processing with Stopwatch

As can be seen from the Table 2, for each relevant cell to the sample are obtained different values of the main traffic parameters, maximal flow, Q_c , critical density, ρ_{cr} , free flow speed, V_f backward speed, w all this because of different number of lanes (column two) which directly affects the value of the jam density, ρ_j , that on the case of three lanes has a value of $\rho_j=600$ [veh/km], while for two lanes $\rho_j=600=400$ [veh/km].

Table 2. FDR parameters for each sample/cell

Sample	Relevant CELL	No. of Lanes	Q_c [veh/hr]	ρ_{cr} [veh/km]	V_f [km/hr]	w [km/hr]
[1]	(1.10-1.3)	3	2880,00	147,00	20,00	6,35
[2]	(1.2&1.1)	2	4230,00	115,00	37,00	14,84
[3]	(1.0) Merge	3	4320,00	117,00	37,00	8,94
[4]	(2.4) Middle	3	4320 (3600)	149 (113)	29 (32)	9,57(7,39)
[5]	(2.3)	3	3600,00	176,00	20,00	8,49
[6]	(2.2)	2	3600,00	164,00,	22,00	15,25
[7]	(2.1)	2	2880,00	88,00	33,00	9,23
[8]	(2.0) Merge	2	3600,00	183,00	20,00	16,58
[9]	(3.2)	2	2880 (2160)	77 (102)	37 (21)	8,91(7,24)
[10]	(3.1)	2	2280,00	115,00	20,00	10,10
[11]	(3.7-3.6)	3	2880,00	133,00	22,00	6,16

5. Experiment Performing

5.1. Initialization

Prior to experiments realization, an initialization as a foregoing process is performed in order to estimate the evolution of traffic through artery from the “zero point”. This “zero point” means the initial conditions of the whole artery, when all the cells are empty (density is zero), but the entry cell of the first segment (*Cell 1.10*) feeds the artery with average traffic flow until it get a stable state of traffic. A special survey is dedicated to the most representative cells i.e. initial cells on the segment (*Cell 1.8, Cell 2.8* and *Cell 3.8*), cells upstream to intersections (*Cell 1.1, Cell 2.1* and *Cell 3.1*) during the first cycle, consequently during first 52 time-steps. As it can be seen from the below diagrams, there is a faster increase of density (fill up with vehicles) of the cells of the first segment, that start immediately after the second time-step (Figure 11,a), while there is an obvious delayed increasing of density on the cells of the far away segments (Figure 11,b) and (Figure 11,c). The earlier increasing of density is noted by after the tenth time-step to *Cell 2.8*, which reaches the average value in the middle of cycle (around time-step 25). The most delayed increasing of density is noted to the most far away cell (*Cell 2.1*) of the segment 2.

It is important to note that the high density values are never reached during the first cycle, by the last segment (segment3: *Cell 3.8* to *3.1*), that it is understood that a single cycle cannot be considered as a “feeding” or “warm up” period during which the artery sufficiently fills up. The *Cell 3.1* positioned upstream to intersection 3, starts to fill up with vehicles by the end of cycle, at time-step 36 (Figure 11, c). An approximately equal evolution is obvious in the aspect of inter-cell flow too. Attention must be paid to the signal timing when it comes to the evaluation of the inter-cell flow. The *SBC* of first intersection (*Cell 1.1*) starts to release vehicles from the first time-step (Figure 11, d); the *SBC* of intersection 2 (*Cell 2.1*) starts to release vehicles by the time-step 23 when its interval green begins (Figure 11, f) but no single vehicle is released from the *Cell 3.1* during the first cycle. The diagrams are accompanied with the tables at the end of this section that contains numerical values of the density and inter cell flow. As can be seen, a diagonal between positive and zero values is formed, that can be interpreted as a tendency of the increasing of the parameters for the distant cells, during later time-steps.

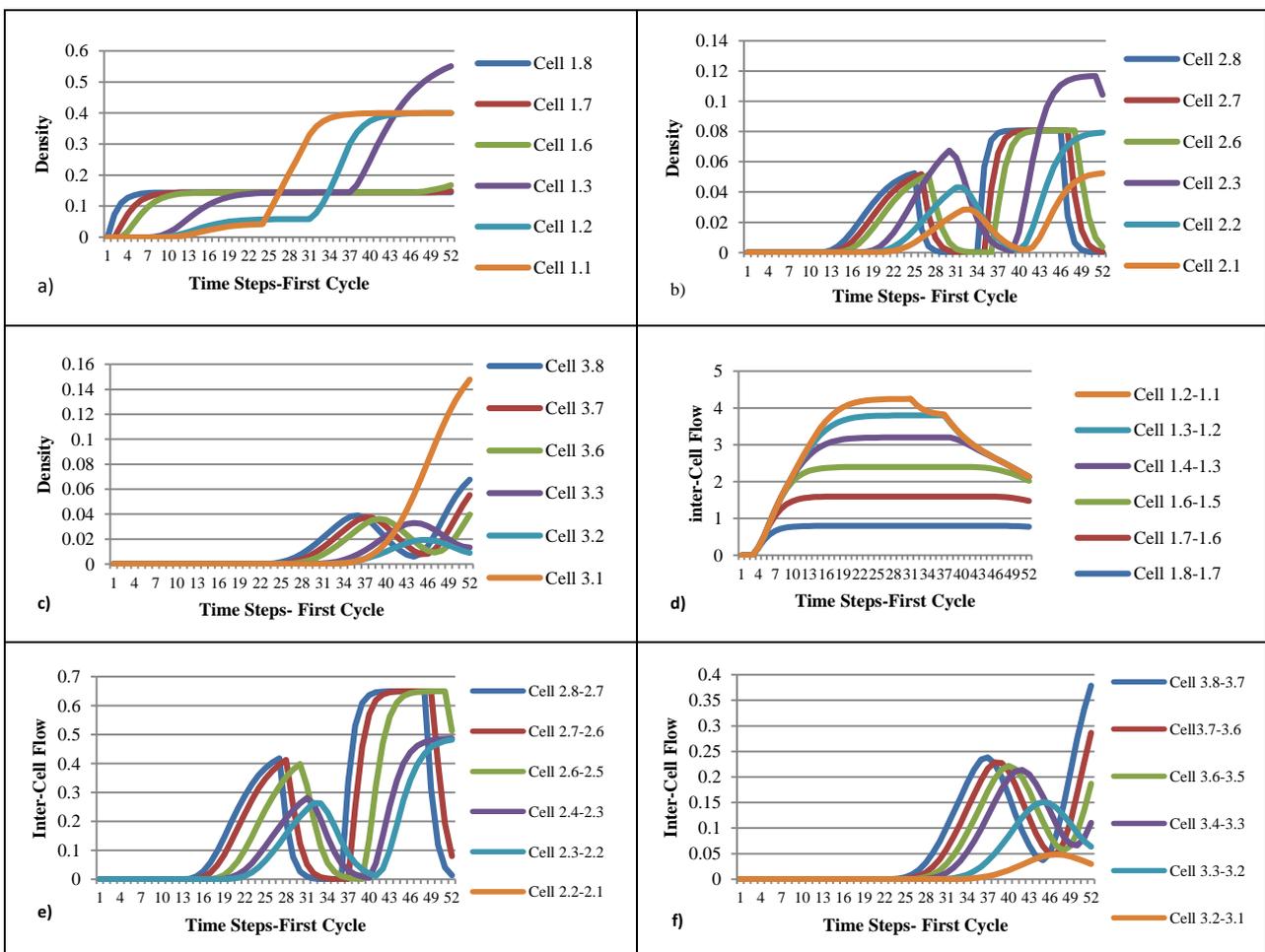


Figure 11. Traffic flow evolution during first cycle, a)Density of segment 1;b) Density of segment 2; c) Density of segment 3; d) Inter cell flow of segment 1; e) Inter cell flow of segment 2 and f) Inter cell flow of segment

5.2. Modelling of Different Traffic Conditions

5.2.1. Light Traffic Conditions

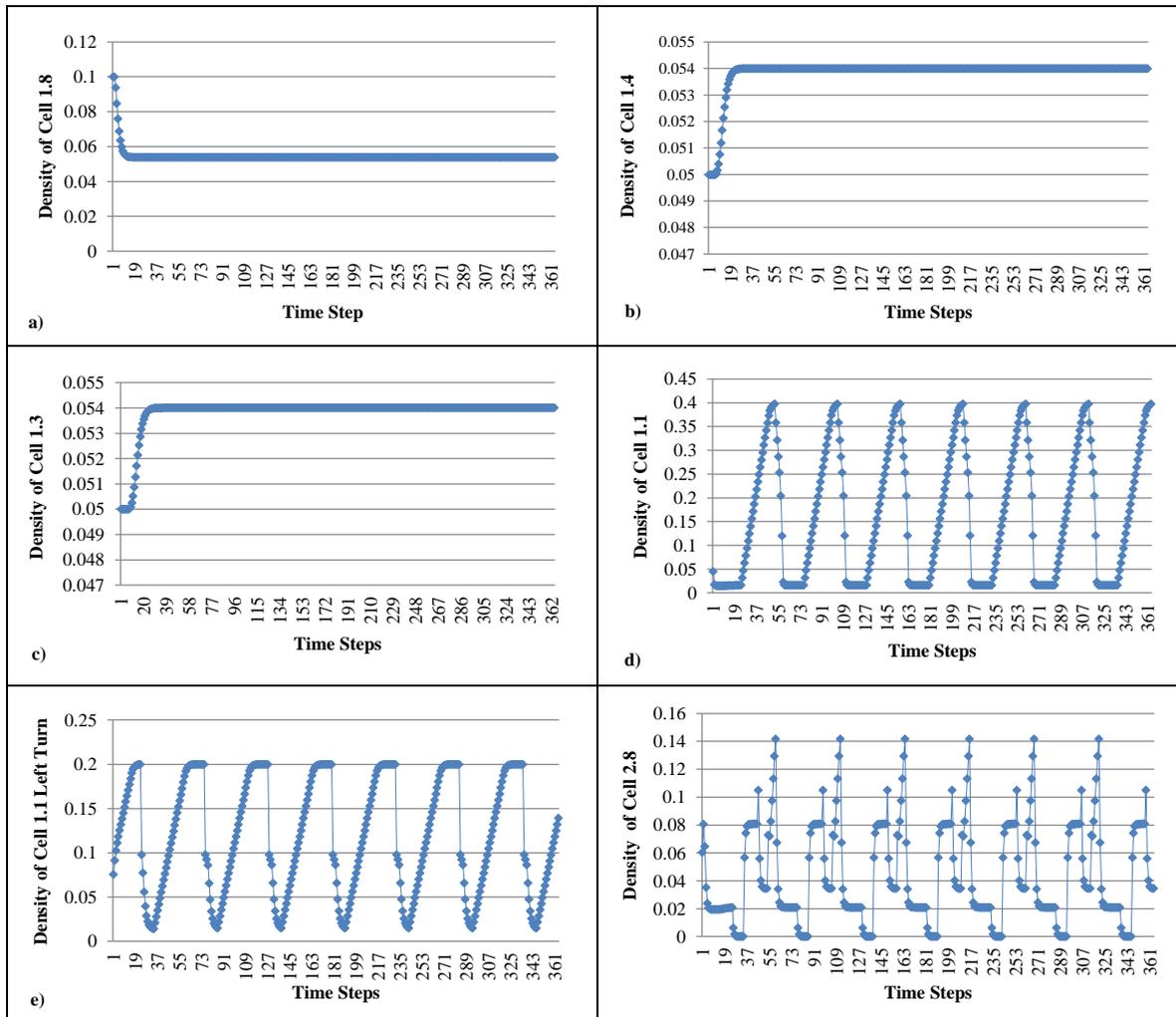
Toward testing of suitability of new *CTM* to model different traffic conditions, special emphasis is given to the definition of entering flow or demand on the first cell, and the initial values of the density on each cell. It should be noted that for following experiments, we put default value for light traffic conditions $0.3\text{veh/sec/Cell } 1.10$ (360veh/hr), for medium to congested conditions above $0.6\text{veh/sec/Cell } 1.10$ (720veh/hr). The parameter of density has been chosen for the evaluation of the traffic conditions of each cell. The evaluation is done for the most representative cells toward the subject direction of artery, i.e. initial cells on the segment (*Cell 1.8*, *Cell 2.8* and *Cell 3.8*), cells upstream to intersections (*Cell 1.1*, *Cell 2.1* and *Cell 3.1*), left turn lane cells (*Cell 1.1L*, *Cell 2.1L*, *Cell 3.1L*).

Light Traffic

Input values:

- Traffic demand: 0.3veh/sec ,
- Signal timing plan as in Figure 12,
- Initial density 0.05veh/m .

In light traffic conditions, in great mass traffic flow is dictated by the traffic signal status, consequently we have a harmonious move of the density curve that follows the signal status. An increasing trend of density during green intervals and in opposite and a decreasing trend of density during green interval for each cell is noted. By the time-step 17 the initial cells (*Cell 1.10*, *Cell 1.9*, *Cell 1.8* and *Cell 1.7*) of segment 1 reach the value of initial density (0.05veh/m) and maintain that value during the overall running time, (353 time-steps, equal; to seven cycles) (Figures 12,a, b and c). The highest values of density are observed in the cells approaching the intersections, as a result of the influence of signal status. *SBC* of the first segment reach faster (at time-step 17) the density value 0.4veh/m that is equal to 10veh/SBC ($3,3\text{veh/lane}$), while the those of the next two segment reach the same value slowly (at time-step 33), (Figures 12,d, f and k). A rapid increasing of density to overcoming of higher values is not observed during the whole simulation time during light traffic conditions



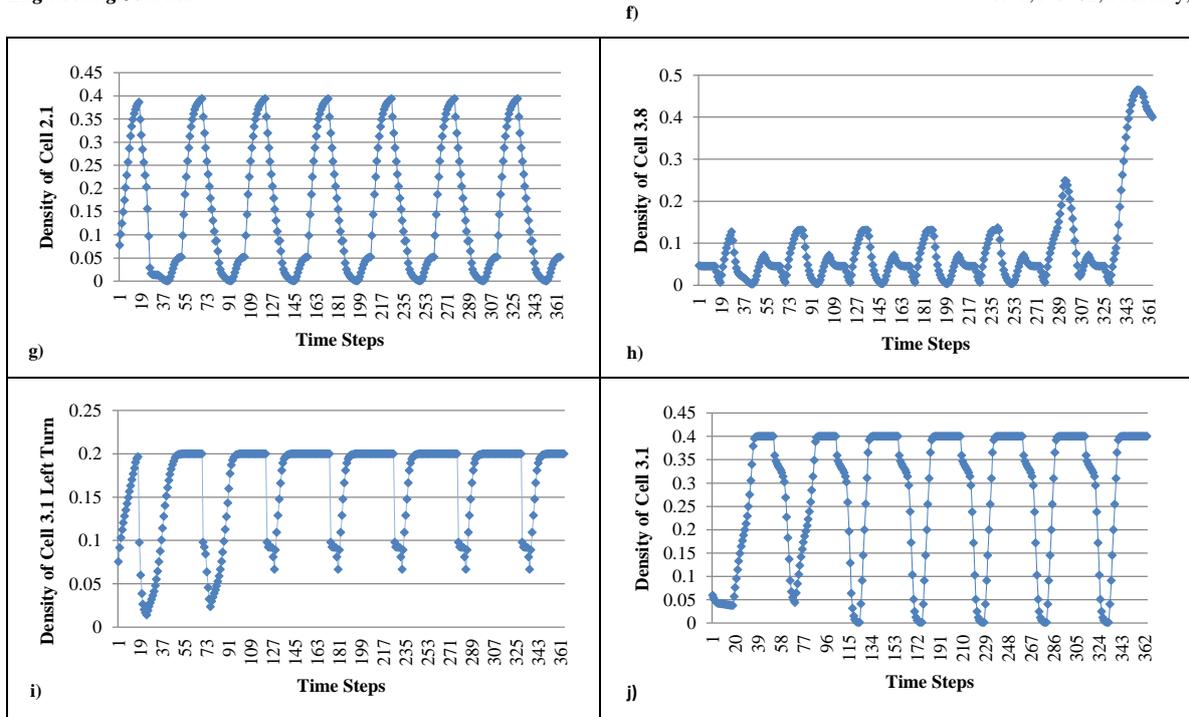


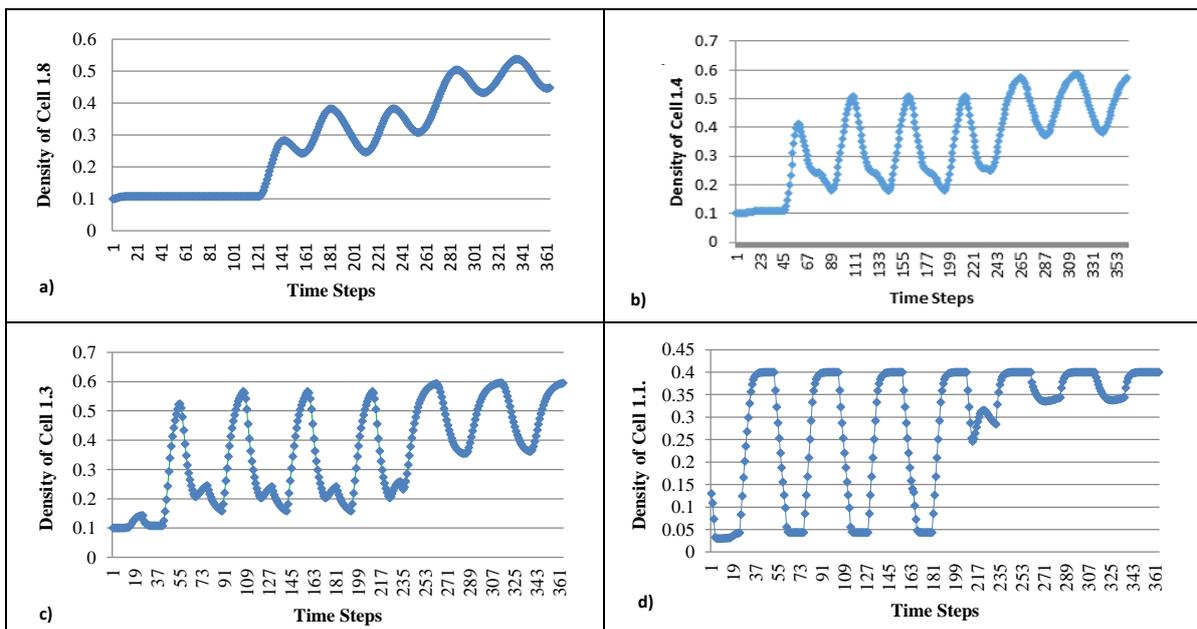
Figure 12. Evolution of flow and density on characteristic Cells during light traffic flow conditions

5.2.2. Congested Traffic Conditions

- Traffic demand: 0.6 veh/sec,
- Signal timing plan as in Figure 12,
- Initial density 0.1 veh/m.

Explication of results (Evolution of Densities)

For creation of medium congestion of traffic flow, in this experiment is imposed a higher number of released vehicles to the first cell. Beside the higher traffic flow values, a double initial density is set to the CTM calculator. Unlike the light conditions, in medium congestion is obtained a rapid increasing of the density to each cell. Fluctuations of the density from the lowest to the highest values are obvious during the first three cycles to all cells of the artery (Figures 13, a, b, c and d). The lowest is zero and the highest takes values till 0.4veh/m, 0.5veh/m or 0.6veh/m, depending on the number of lanes that cell covers. After the first three cycles, density never get back to the lowest values but their fluctuations are between 0.4veh/m to 0.6veh/m (Figures 12, d, f, g and h).



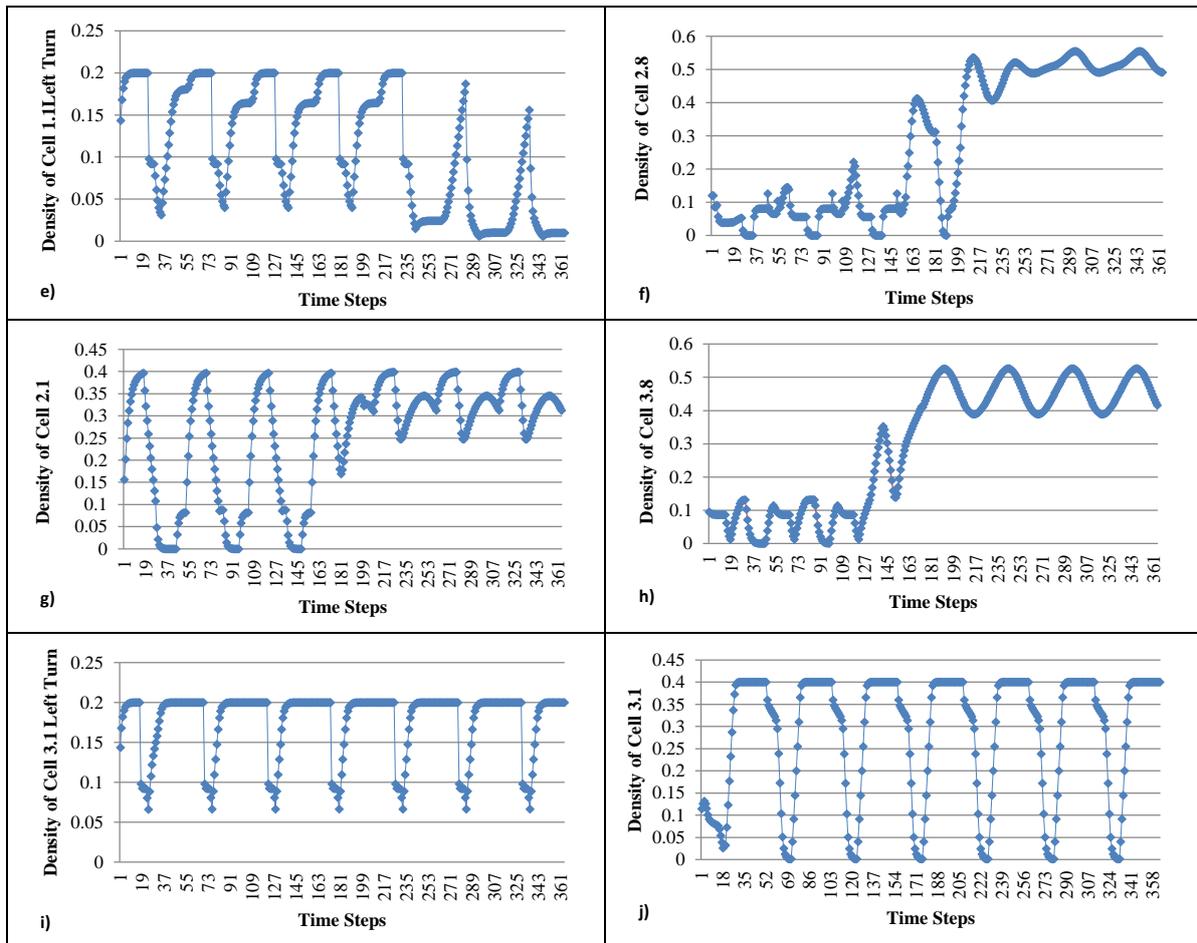


Figure 13. Evolution of flow and density on characteristic Cells during congested traffic flow conditions

Beside the possibility to update the densities of cells through time step, the new CTM model offers the estimation of evolution of the velocities which may be an important factor of challenging real time prediction and control strategies.

6. Hypothesis Testing

A t-statistic, statistical significance indicates whether or not the difference between two groups’ averages reflect a real difference in the population from they were sampled. *T-test* can be computed based on the important statistical concept of *CLT (Central Limit Theorem)* that states that given a large sample size from population with a level of variance, the mean of all samples from the population will be approximately equal to the mean of the population. [46, 47 and 48]. The main concepts of the *CLT* is the confidence interval and the critical values calculated with expressions (46) and (47), respectively.

$$P(-t_{(1-\alpha/2)} \leq \frac{(\bar{\mu} - \mu)}{\sigma\sqrt{n}} \leq t_{(1-\alpha/2)}) \approx (1-\alpha) \tag{46}$$

$$T = \frac{\bar{\mu} - \mu}{S_n/\sqrt{n}} \tag{47}$$

Where: $\bar{\mu}$ is the sample mean ($\bar{\mu} = (\sum x)/n$, $\{x_1, x_2, \dots, x_N\}$ are random variables) and σ is the standard deviation (variance) ($\sigma^2 = \sum(x_i - \bar{\mu})^2 / (n - 1)$), P is probability of data lie in a confidence interval between critical values (i.e.90 % if $\alpha=0.1$ or 95% if $\alpha=0.05$). $t_{(1-\alpha/2)}$ and $-t_{(1-\alpha/2)}$ are the critical values that define the confidence interval (i.e. for $\alpha/2 = 0.025$, $|t_{(1-\alpha/2)}| = 1.96$, T is the test Statistic value and S_n is the pooled standard deviation.

The main objective, as stated on the objective unit of this dissertation is to highlight the difference between the results of original *CTM* and our proposed *CTM* and to provide evidence that the later one is better than the old one. The results of the new *CTM* will be analyzed according to the effect of the new function demand. The outflow from the *SBC* cells, directly has an effect to the delays of these cells. For the purpose of elaboration of the hypothesis upon the differences between both models, the delays of the second and the third *SBC* of the artery during the first ten time

steps of green intervals are compared. In order to have a better representation of data, the delay data of the last three cycles were used only. Average values of the delays on three cycles of each *SBC* are involved into the statistic test. In this dissertation the delay is calculated as the difference between numbers of vehicles in a cell *i* during time step *T* and the number of vehicles that leave the same cell and enter to next cell: $D_{i(T)} = \sum_{T=1}^{10} (n_{i(T)} - q_{i \rightarrow i+1}(T))$

The delay difference between two models can be analyzed by formulating the problem through specification of null hypothesis. A statistical hypothesis is an assumption or a statement about one or two parameters of one or more populations. There can be decided, based on data in a sample, or samples, whether the stated hypothesis is true or not. A statistic is computed for a selected level of confidence, and if the difference between the two means is less than that statistic, then the null hypothesis is **accepted** and it is concluded that there is insufficient evidence to prove that the one model is better than the other, otherwise if the difference is higher the null hypothesis is **rejected** and it is concluded that two models are different. Through a number of repetitions (four to ten runs for all alternatives, according to recommendations of *Traffic Analysis Toolbox III*), in order to accept or reject the hypothesis, based on a confidence interval. In this dissertation are done ten repetitions of runs in order to avoid the possibility of change of results from random chance. Table of delay results is given in Table 3.

Table 3. Average value of delays for ten time steps of cycle $D_{i(T)}$

Orig. CTM	54.7	60.0	54.7	40.3	55.0	40.3	52.4	57.7	40.00	82.8
New CTM	65.7	60.9	65.7	60.0	60.6	65.7	50.3	72.60	65.70	60.1

The statistical hypothesis is:

Null Hypothesis: $H_0: \mu_{oCTM} - \mu_{nCTM} = 0$
Against: $H_1: \mu_{oCTM} - \mu_{nCTM} \neq 0$ (48)

Where μ_{oCTM} and μ_{nCTM} are delay means of the runs for original *CTM* and our proposed *CTM*.

Procedure Step I: Pooled standard deviation $\bar{\sigma}_p$. Let n be the number of runs for each model. and σ_{oCTM} and σ_{nCTM} the standard deviations of the delay parameters of two models, respectively. Pooled standard deviation $\bar{\sigma}_p$ is calculated:

$$\sigma_p^2 = \frac{(n-1)\sigma_{oCTM}^2 + (n-1)\sigma_{nCTM}^2}{n+n-2} \text{ For } n=10 \text{ sample of data } \sigma_p^2 = \frac{(9 \cdot 33.95 + 9 \cdot 161.50)}{18} = 97.72 \tag{49}$$

Procedure step II: Choose of confidence interval. As usually, the confidence interval for hypothesis testing is often chosen a 95% confidence level. So, $\alpha = 1 - 95\% = 1 - 0.95 = 0.05 \Rightarrow \alpha/2 = 0.025$.

Procedure step III: Difference between two means $\bar{\mu}_{oCTM} = 62.18$ and $\bar{\mu}_{nCTM} = 53.79$. Procedure step IV: *T*-Statistic, degrees of freedom: $n+n-2$, determine $t_{0.05}$ from Statistic Table 3;

$$T = \frac{(\bar{\mu}_{oCTM} - \bar{\mu}_{nCTM}) - 0}{\sigma_p \cdot \sqrt{\frac{1}{n} + \frac{1}{n}}} \text{ By replacing the values, we have: } T = \frac{(62.18 - 53.79) - 0}{\sqrt{97.72} \cdot \sqrt{\frac{1}{10} + \frac{1}{10}}} = 2.040 \tag{50}$$

Procedure step IV: From table of Student’s distribution for $18^{th}(2n-2)$ of freedom, we find that $t=1.73$. Check if the test statistic belongs to rejection region and decide to accept or reject H_0 . From $|T| > t_{0.05}$ or $-T > t_{0.05}$, reject H_0 , otherwise accept. At our case $T=2.04 > t=1.73$ and finally H_0 rejected, in favour of the proposition that difference between original *CTM* and *CTM* is statistically significant.

7. Conclusions

Through this is shed light on the disadvantages and flaws of the existing *CTM* models so far. *CTM* was used mostly to model one way traffic networks, with simple composition of the connections as freeway, with or without on ramping and off ramping segments. Obvious limitations are overcome in our model by taking in considerations:

- Physic features/geometric features: Complex composition of urban segment approaching to intersections, providing merge and diverge cells. Basic principles (*the flow from cell i-1 to cell i is the smallest value between the sending flow from the upstream cell or receiving flow of the downstream cell and capacity flow*) of the *CTM* models such fully adapted to the merge and diverge configurations.
- Dynamic traffic of urban segments: In the original *CTM*, besides the special node complexities regarding to diverging and merging, was neglected the definition of out flow from the nearby stop-bar cell (*SBC*). As it is known, by the traffic flow nature on the signalized intersections, at the beginning of green interval the discharge rate is low, with higher discharge headways do to the reaction time and start up time of the first vehicle, which increases to the capacity flow until the queue is being dissipated. Implication of new demand function makes

new CTM model capable emphasize the difference of the outflow rate from the SBC on different consequential portions of green interval.

Model can describe light traffic, medium and congested traffic condition. In light traffic conditions, in great mass traffic flow is dictated by the traffic signal status, consequently we have a harmonious move of the density curve that follows the signal status. An increasing trend of density during green intervals and in opposite and a decreasing trend of density during green interval for each cell is noted. Unlike the light conditions, in medium congestion is obtained a rapid increasing of the density to each cell. Fluctuations of the density from the lowest to the highest values are obvious during the first three cycles to all cells of the artery. The lowest is zero and the highest takes values till 0.4, 0.5 or 0.6 *veh/m*, depending on the number of lanes that cell covers. After the first three cycles, density never get back to the lowest values but their fluctuations are between 0.4 to 0.6 *veh/m*. Hypothesis test concludes that finally H0 rejected, in favour of the preposition that difference between original CTM and CTM is statistically significant.

8. Declarations

8.1. Data Availability Statement

The data presented in this study are available in article.

8.2. Funding

Department of Traffic Engineering, University of Prishtina “Hasan Prishtina”, Kosovo.

8.3. Conflicts of Interest

The authors declare no conflict of interest.

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