



A Macro-element for Modeling the Non-linear Interaction of Soil-shallow Foundation under Seismic Loading

Van Quan Huynh ^a, Xuan Huy Nguyen ^{a*}, Trung Kien Nguyen ^a

^a *University of Transport and Communications, Hanoi, Viet Nam.*

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Abstract

This paper presents a macro-element for simulating the seismic behavior of the soil- shallow foundation interaction. The overall behavior in the soil and at the interface is replaced by a macro-element located at the base of the superstructure. The element reproduces the irreversible elastoplastic soil behavior (material non-linearity) and the foundation uplift (geometric non-linearity) at the soil- foundation interface. This new macro-element model with three degrees-of-freedom describes the force-displacement behavior of the footing center. The single element is restrained by the system of equivalent springs and dashpots. The footing is considered as a rigid body. It is solved by a suitable Newmark time integration scheme and implemented in Matlab to simulate the nonlinear behavior of soil-shallow foundation interaction under seismic loading. A reduce scaled soil-foundation system has been tested on a shaking table at the University of Transport and Communications, Hanoi, Vietnam. Five series of earthquake motions were used with maximum acceleration increased from 0.5 m/s^2 to 2.5 m/s^2 . The comparison of numerical results obtained from the simulation and experimentations shows the satisfactory agreement of the model. The proposed macro-element can be used to predict the seismic behavior of a wider variety of configurations.

Keywords: Macro-element; Soil–foundation Interaction; Non-linearity; Seismic Loading.

1. Introduction

In structural design, soil–foundation interaction (SFI) is an important phenomenon that should be taken into account. However, simulating SFI often needs complex models for the soil and the foundation with a great number of degrees-of-freedom (DOF) which requires significant computational costs. That is why various simplified modeling strategies have been recently developed. The macro-element concept consists in condensing all nonlinearities into a finite domain and works with generalized variables (forces and displacements). The macro-element model replaces the soil-foundation system with a single element that allows simulating the behavior of foundations in a simplified way. This approach helps to reproduce the non-linear behavior of foundations considering material and geometric non-linearity. The macro-element reduces the computational effort significantly while preserving the essential dynamic responses of system.

The term “macro-element” was initially introduced by Nova and Montrasio for studying the settlements of shallow foundations on sand [1]. After that, they proposed an elastic-plastic macro-element for strip and circular footings under quasi-static monotonic loading [2]. Based on this model, Paolucci proposed a numerical tool for studying the response of simple structures under seismic loading. Paolucci’s work took into account the coupling between the non-linear response of SFI and the response of superstructures [3]. Cremer et al. [4] developed the first application of the

* Corresponding author: nguyenxuanhuy@utc.edu.vn

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macro-element for purely cohesive soils with no resistance to tensile stresses. The macro-element could be used not only for static loading but also for dynamic (seismic) loading, considering the plasticity of the soil and the uplift of the foundation. Macro-element models for seismic SFI problems are also available in the literature, such as those proposed by Cremer et al. [5], Chatzigogos et al. [6], Grange et al. [7, 8], Figini et al. [9], Prisco et al. [10], Khebizi et al. [12], Jin [13] Barari et al. [14], Paroissien et al. [15] and Gorini et al. [16]. For shallow foundations, these authors used the macro element of quasi-static monotonic loading for analyzing the structures under seismic loading. According to Paolucci [11], the superstructure was represented by a single-degree-of-freedom mass and the macro-element introduced nonlinear effects in the calculation of soil reactions through a failure criterion and plastic flow rule. This model is developed further by Figini [9] by considering the uplift-plasticity coupling. However, these models weren't suitable for earthquake loads because they changed in time and fully recovered once the seismic loadings eccentricity come back. Recently, Khebizi et al. [12] proposed a macro- element to examine the response of shallow foundations under monotonic and cyclic loads. Jin [13] investigated on the response of a caisson foundation in sand with a novel macro- element developed under the framework of hypo plasticity. In the work of Barari et al. [14], the effects of foundation geometry, horizontal load eccentricity and various soil conditions were examined by means of an alternative macro-element model.

Inspired by the above works, this paper proposes a macro element with the material-geometric coupling for seismic analysis of shallow foundation. The next section will describe the new macro-element where the entire soil- footing system is lumped in a single point at the footing center. The footing is considered as a rigid body. The macro-element is restrained by the system of equivalent springs and dashpots. After the mathematical presentation of the macro-element, the comparison between simulation and experiment results in Section 3 has shown the good performance of the proposed approach.

2. Description of Proposed Macro-element

2.1. The General Structure

The structure of a macro-element model will follow the scheme presented in Figure 1. The soil is divided into two fields: the close field and the far field. The close field is the area where all material and geometric non-linearity are lumped while the far field is the area where the response of the system remains linear. The entire soil-foundation system is replaced by a macro-element located at the base of the superstructure. This model has 3 DOFs (horizontal, vertical and rocking motion), describes the force-displacement behavior of the footing center (Figure 2). In this formulation, the footing is considered as a rigid body. The single macro-element is restrained by the system of equivalent springs and dashpots in order to describe the non-linear interactions between soil and foundation. There are two types of non-linearity for a macro-element. One is caused by the material characteristics, such as non-linear soil or interface behavior. Another is caused by the geometric characteristics; such as uplift of the footing accompanied by the creation of a detachment area at the soil-footing interface. These two distinct mechanisms are coupled (material-geometric coupling).

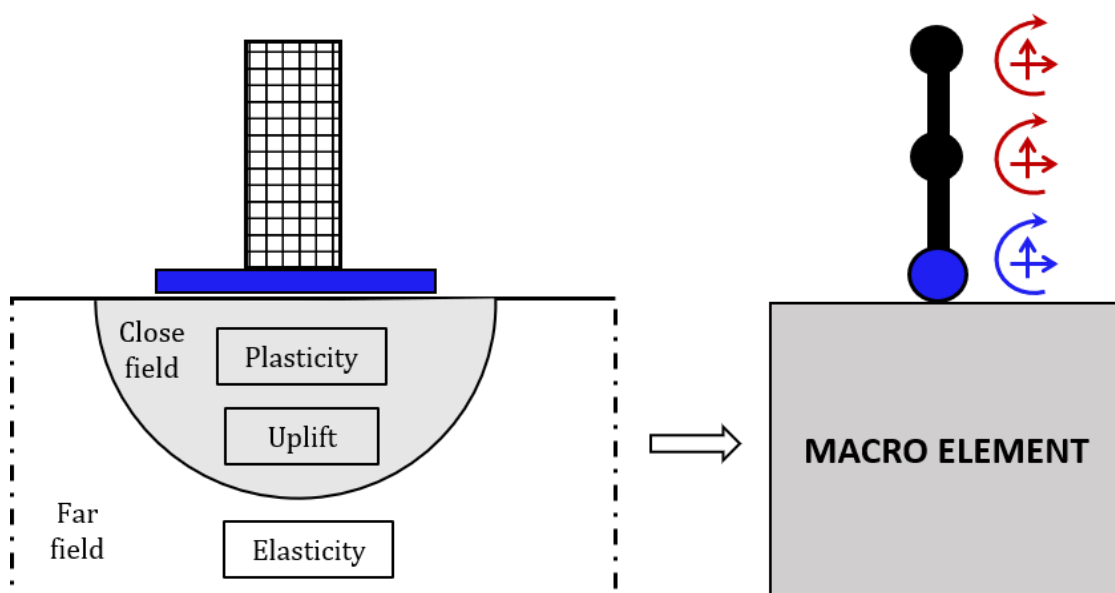


Figure 1. Macro element concept: decomposition in close field and far field

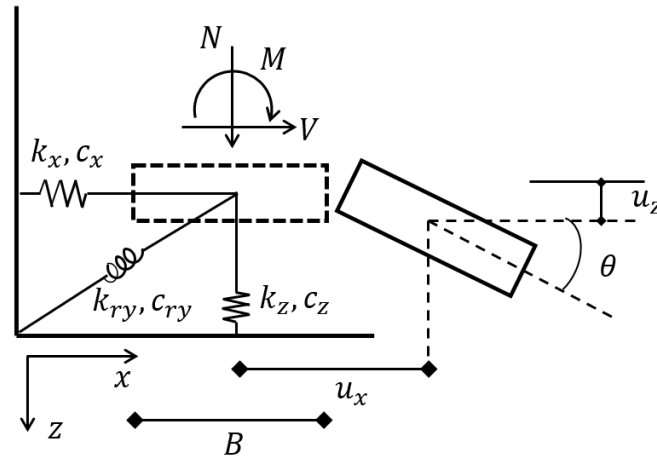


Figure 2. Macro element modeling of a soil-shallow foundation system: generalized force, displacement variables and foundation static impedances

2.2. The Variables of Forces and Displacements

As can be seen in the Figure 2, the entire soil-footing system is lumped in a single point at the footing center. The constitutive equations of the macro-element are written in terms of generalized forces and displacement variables [6]. The vector \mathbf{F} of the generalized force variables is defined in Equation 1 as:

$$\mathbf{F} = \begin{bmatrix} V \\ M \\ N \end{bmatrix} \quad (1)$$

Whereas, N , V , and M are the vertical force, horizontal force and moment applied to the foundation, respectively.

The vector \mathbf{u} of generalized displacement variables is defined in Equation 2 as:

$$\mathbf{u} = \begin{bmatrix} u_x \\ \theta \\ u_z \end{bmatrix} \quad (2)$$

In which, u_z and u_x are the vertical and horizontal displacements of the footing center of mass; θ is its rotation angle. It is worth noting that the corresponding stiffness matrix in the elastic state can be considered as a diagonal matrix (Equation 3) because the soil -foundation system is lumped in a single point.

$$\mathbf{K}^{F0} = \begin{bmatrix} k_0 & 0 & 0 \\ 0 & k_r & 0 \\ 0 & 0 & k_v \end{bmatrix} \quad (3)$$

The terms on the diagonal represent the foundation static impedances (k_0 , k_v and k_r are the stiffness of the equivalent elastic springs to the horizontal direction, vertical direction and rotation, respectively) changed over time during the earthquake process, which can be determined from Gazetas's formulas [17].

2.3. The Yield Function and Flow Rule

Equation 4 is the yields function and flow rule which Paolucci et al. [11] employed.

$$f(\mathbf{F}) = h^2 + m^2 - v^2(1 - v)^{2\xi} \quad (4)$$

Where $h = V/(\mu N_{max})$, $m = M/(\psi B N_{max})$, $v = N/N_{max}$ and N_{max} is the ultimate bearing capacity under vertical central load. ψ is in the range between 0.35 and 0.5; $\mu = \tan \varphi$, φ is the soil friction angle; $\xi = 0.95$, in agreement with the value suggested by Nova and Montrasio [11]. The non-associative plastic flow rule from Cremer et al. [5] has been adopted (Equation 5):

$$g(\mathbf{F}) = \lambda^2 h^2 + \chi^2 m^2 + v^2 \quad (5)$$

The optimum parameters $\lambda = 4$ and $\chi = 6$ were selected.

2.4. The Stiffness Matrix

The elastic stiffness matrix of macro-elements is defined by Equation 3. The nonlinear phase of the seismic

excitation reveals that the instantaneous foundation–soil contact area changes over successive cycles of foundation rotations. This is explained by irrecoverable downward movement of soil beneath the foundation induced by severe foundation rotations during successive seismic loading, resulting in a reduction of the effective foundation width (B') that can be expressed in Equation 6 as follows:

$$B' = B(1 - \delta) \quad (6)$$

Where B is the actual width of the footing and δ can be interpreted as a degradation parameter defined in the range $0 \leq \delta \leq 1$. Initially, δ is set to zero. The parameter δ is updated throughout the seismic excitation due to accumulation of inelastic foundation tilting. Although the footing has a square shaped, reduction in the footing–soil contact in transverse direction results in a rectangular contact area. Substituting Equation 6 into the approximate static stiffness formulas for a rectangular footing, and supposing that the square-shaped foundation base is in full contact with the soil prior to the seismic excitation, the reduced stiffness factors for each vibration mode are obtained (Equations 7 to 9) as:

$$k'_0 = k_0[0.74(1 - \delta)^{0.35} + 0.09 + 0.17(1 - \delta)] \quad (7)$$

$$k'_r = k_r[(1 - 0.2\delta)(1 - \delta)^2] \quad (8)$$

$$k'_v = k_v[0.66(1 - \delta)^{0.25} + 0.34(1 - \delta)] \quad (9)$$

Where k'_0 , k'_r and k'_v are the modified stiffnesses in terms of δ ; k_0 , k_r , k_v are the equivalent elastic spring coefficients of the soil–foundation system defined in the previous section. For this purpose, we have selected the following simple degradation function (Equation 10).

$$\delta(\theta^p) = \frac{\delta_1}{1 + \frac{1}{\delta_2 \theta^p}} \quad (10)$$

Where $\delta_1 = 0.75$ and $\delta_2 = 5000/\text{rad}$ are model parameters related to the ultimate value of δ and to the degradation speed, respectively, while is θ the cumulated plastic foundation rotation at a specified instant of time, calculated as:

$$\theta^p = \sum_n |\Delta\theta_n - \Delta M_n / k'_r| \quad (11)$$

In Equation 11, n and M_n are the increments of foundation rotation and overturning moment, respectively, calculated at the n th time step. The elastic stiffness matrix is calculated following (Equation 12):

$$K^{F'} = \begin{bmatrix} k'_0 & 0 & 0 \\ 0 & k'_r & 0 \\ 0 & 0 & k'_v \end{bmatrix} \quad (12)$$

The soil behavior is assumed to be linear visco-elastic until the failure surface in Equation 4 is reached. When the failure surface is reached, the plastic flow occurs when $f(F) \geq 0$ and $df(F) = 0$ [3]. The elastic stiffness matrix of macro-elements will be reduced by a differential value dK^F for initial value in Equation 3. This is calculated as a function of the elastic stiffness matrix K^{F0} and the derivatives of the yield and plastic potential functions (Equation 13).

$$dK^F = K^{F0} \left(\frac{\partial g}{\partial F} \right) \left(\frac{\partial f}{\partial F} \right)^T K^{F0} \left[\left(\frac{\partial f}{\partial F} \right)^T K^{F0} \left(\frac{\partial g}{\partial F} \right) \right]^{-1} \quad (13)$$

Thus, the elastoplastic stiffness matrix of the proposed macro-element in step of n^{th} is determined by the following formula (Equation 14).

$$K^F = K^{F'} - dK^F \quad (14)$$

The flowchart to summarize the defining process of a new macro-element stiffness matrix is shown in Figure 3.

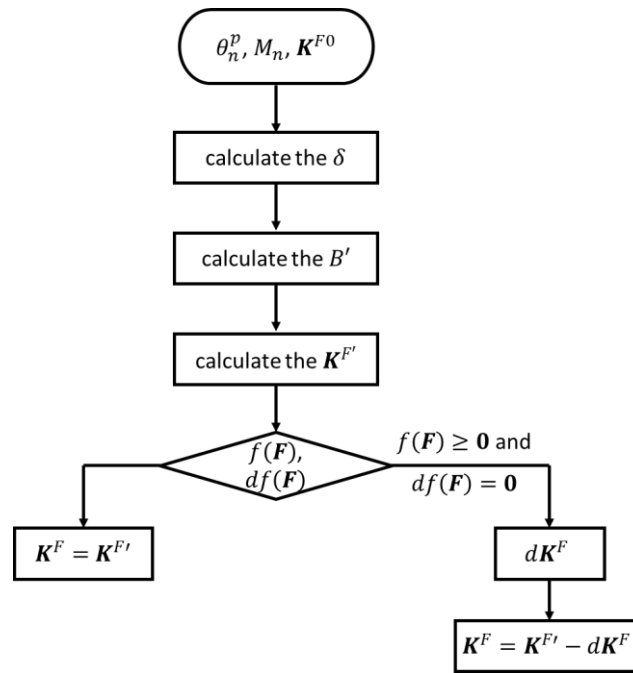


Figure 3. The algorithm for defining the new macro-element stiffness matrix

2.5. The Dynamic Equilibrium Equations

The differential equation of motion is presented in Equation 15.

$$M\ddot{x} + C\dot{x} + F = P \quad (15)$$

Where

$$x = [x_0 \ \phi \ x_v]^T \quad (16)$$

$$P = [-m_0\ddot{x}_g \ 0 \ -(m_0)\ddot{z}_g]^T \quad (17)$$

$$F = [V^F \ M^F \ N^F]^T \quad (18)$$

$$M = \begin{bmatrix} m_0 & 0 & 0 \\ 0 & J & 0 \\ 0 & 0 & m_0 \end{bmatrix} \quad (19)$$

$$C = \begin{bmatrix} c_0 & 0 & 0 \\ 0 & c_r & 0 \\ 0 & 0 & c_v \end{bmatrix} \quad (20)$$

Where x is vector of the displacements of the basement; P is vector of base excitations; F is vector of soil reactions; x_0, ϕ, x_v are horizontal displacements of the basement, rocking motion of the basement and vertical displacements, respectively; \ddot{x}_g, \ddot{z}_g are horizontal and vertical base excitations, respectively; m_0, J are mass of the foundation and sum of the centroid moments of inertia of the foundation, respectively; c_0, c_r, c_v are equivalent dashpot coefficients of the soil-foundation system corresponding to the translational, rocking and vertical modes of vibration, respectively. The Newmark time integration scheme was used for solving the motion equations Equation 15. Acceleration and velocity are written in Newmark time-stepping method based on the following equations. The equations of motion at time $i, i+1$ and incremental equation of motion are as follows.

$$M\ddot{x}_i + C\dot{x}_i = P_i \quad (21)$$

$$M\ddot{x}_{i+1} + C\dot{x}_{i+1} = P_{i+1} \quad (22)$$

$$M\Delta\ddot{x}_{i+1} + C\Delta\dot{x}_{i+1} = \Delta P_{i+1} \quad (23)$$

Denoting by the subscript n the quantities calculated at time $t = n\Delta t$, the motion form of Newmark time integration scheme of Equation 15 can be rewritten as Equation 24.

$$\left[\frac{M}{\beta(\Delta t)^2} + \frac{C\gamma}{\beta\Delta t} \right] x_{n+1} + F_{n+1}(x_{n+1}) = p_{n+1} + M \left[\frac{1-2\beta}{2\beta} \ddot{x}_n + \frac{\dot{x}_n \Delta t + x_n}{\beta(\Delta t)^2} \right] + C \left[\left(\frac{\gamma}{2\beta} - 1 \right) \dot{x}_n \Delta t + \left(\frac{\gamma}{\beta} - 1 \right) \dot{x}_n + \frac{\gamma}{\beta\Delta t} x_n \right] \quad (24)$$

Where Δt is the time step; $\beta = 0.25$ and $\gamma = 0.5$ are the Newmark integration parameters; $F_{n+1}^F = F_n^F + K^F(x_{n+1} - x_n)$.

3. Results and Discussion

In order to validate the capability of the numerical strategy, the proposed macro-element is implemented in Matlab by using the motion equation form of Newmark time integration scheme (Equation 24). The experimental investigation was conducted at the University of Transport and Communications, Hanoi, Vietnam to study a shallow foundation lying on sand with a known density. The soil-foundation system was subjected to a series of earthquake base excitations on the shaking table. Due to the limitations of the dimension of shaking table (2×2 m), a polycarbonate box of 1.85×1.5×0.7 m was selected and fixed to the table.

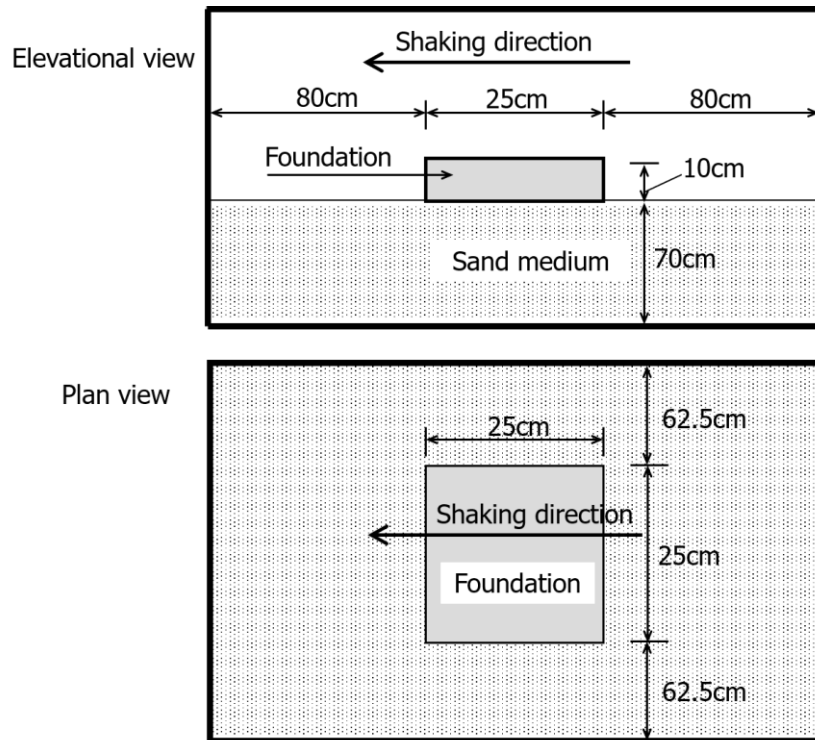


Figure 4. Dimensions of the soil- foundation system

The dry sand was filled in the polycarbonate box and compacted in layers so that nearly homogeneous soil conditions were obtained. The sand relative density D_r was 82%, the mass density $\rho = 2.66 \text{ g/cm}^3$ and the angle of internal friction $\varphi = 42.6^\circ$. The shallow foundation model with the dimension of 0.25×0.25×0.1 m (L×W×H) was placed on the sand layers. The recording system consists of 2 accelerometers placed on the foundation model. The overall view of the specimen on the shaking table is shown in Figures 4 and 5.



Figure 5. The experimental setup

The parameters of macro-element model are shown in Table 1.

Table 1. Numerical model parameters used in dynamic analyses

k_0 (N/m)	k_r (Nm/rad)	k_v (N/m)	c_0 (Ns/m)	c_r (Ns/m)	c_v (Ns/m)	m_0 (kg)
$202.68 \cdot 10^6$	$201.74 \cdot 10^5$	$338.48 \cdot 10^6$	$1.34 \cdot 10^5$	$1.26 \cdot 10^3$	$2.42 \cdot 10^5$	15
J (kgm ²)	N_{max} (kN)	μ	ψ	ξ	λ	χ
$90.625 \cdot 10^{-3}$	28.05	0.682	0.43	0.95	4	6

The tests were conducted in one horizontally direction (Figure 4). The motion records were derived from the Tolmezzo earthquake. Five earthquake motions, denoted from T1 to T5, were used for the experiments with maximum acceleration increased from 0.5 m/s^2 to 2.5 m/s^2 (0.5 m/s^2 , 1.0 m/s^2 , 1.5 m/s^2 , 2.0 m/s^2 and 2.5 m/s^2).

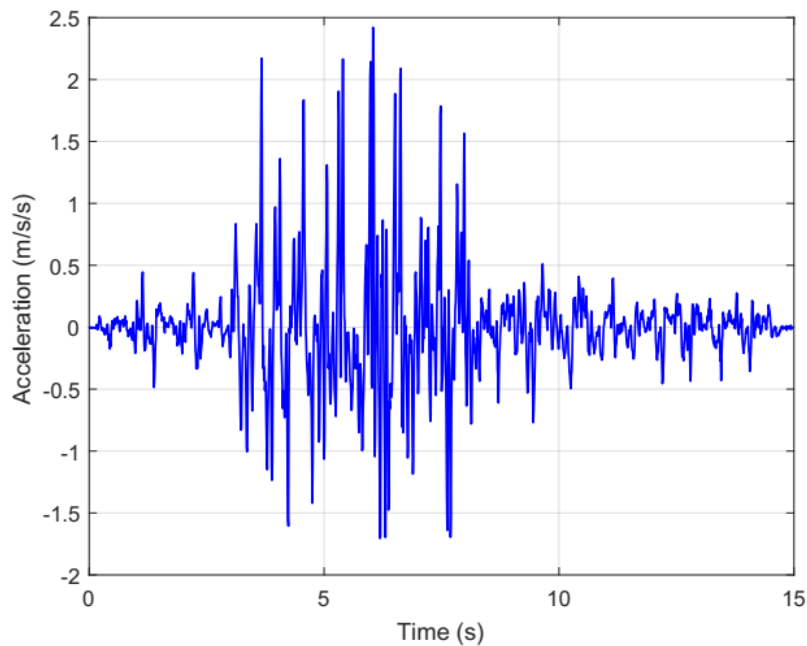


Figure 6. Tolmezzo earthquake acceleration

These five motions were applied to the soil- foundation system in chronological order. They have been launched independently in the macro- element model. Figures 7 to 10 shows the comparison of the acceleration at the foundation for all tests, except for test T1 where the values were too small. As can be seen, the simulation results were generally close to the experimental results. No significant shift existed between the simulation and experiment curves.

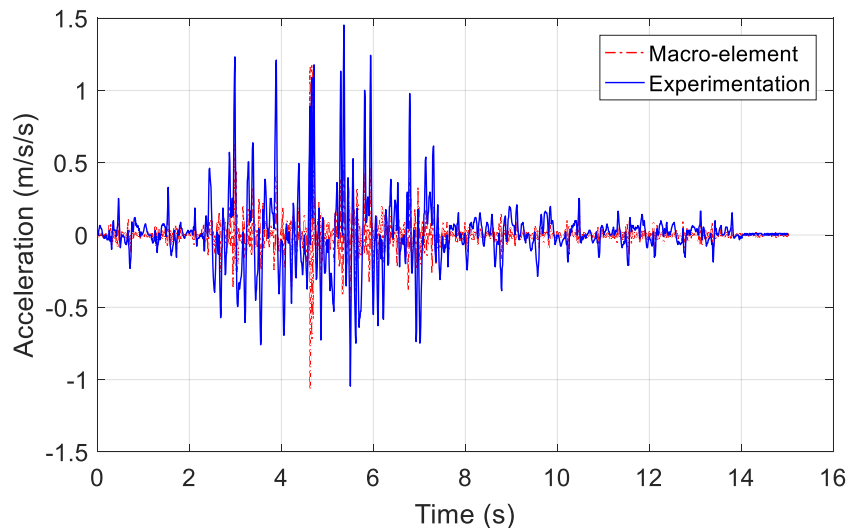


Figure 7. Horizontal acceleration at the center of foundation– Test T2

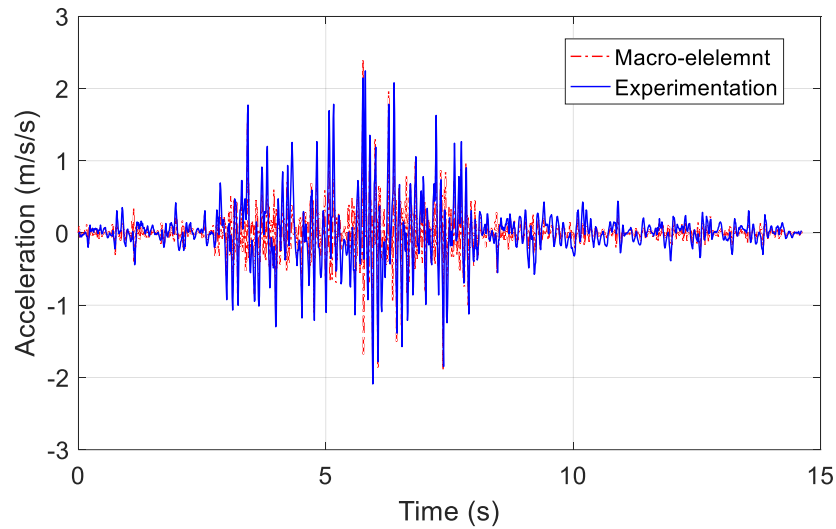


Figure 8. Horizontal acceleration at the center of foundation– Test T3

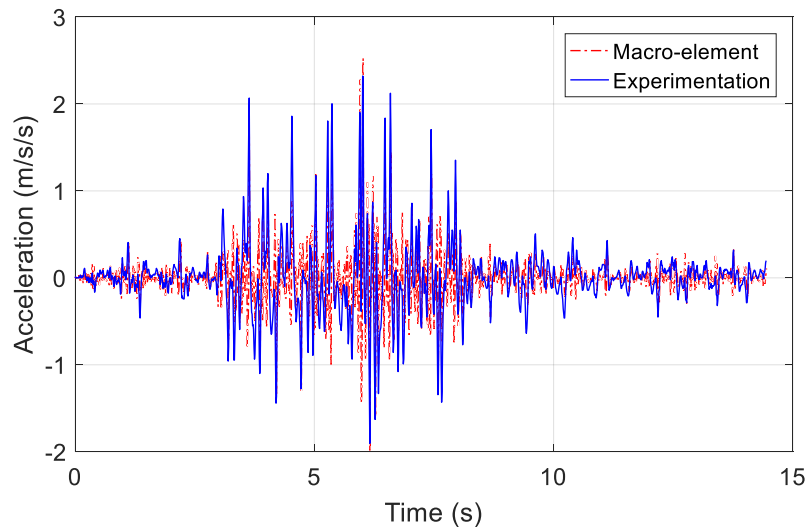


Figure 9. Horizontal acceleration at the center of foundation– Test T4

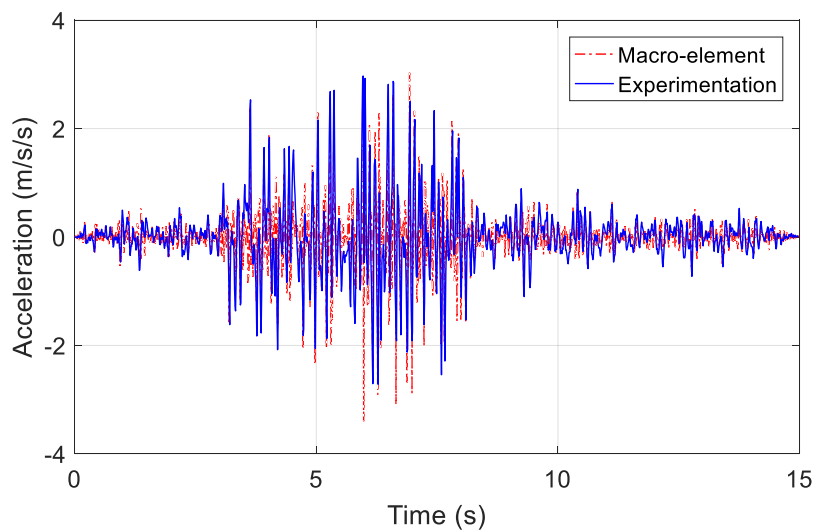


Figure 10. Horizontal acceleration at the center of foundation– Test T5

Table 2 represents the maximum values of acceleration at foundation obtained from test T2 to test T5. It was found that the macro-element was able to simulate correctly the behavior of foundation. The percentage errors reported

permit an assessment of the accuracy of the numerical model. It is noteworthy that the agreement for the T3 and test T4 is in general better than the T2 and T5.

Table 2. Maximum values of acceleration at foundation

Test	Experimentation (m/s^2)	Macro-element (m/s^2)	Error (%)
T2	1.452	1.173	-19.2%
T3	2.244	2.414	7.6%
T4	2.323	2.525	8.7%
T5	2.970	3.398	14.4%

The computational effort involved in the macro- element is orders of magnitudes less than that of 3D FEA analysis by Huynh et al. [18]. The present solutions, implemented in non-compiled MATLAB script files, took between 15-30 seconds to conduct each test. In comparison, the 3D FEA under Cyclic TP software took about 5 hours on the same computer.

4. Conclusion

In this paper, a new macro-element for modeling the behavior of soil-shallow foundation interaction under seismic loading has been represented. The proposed macro-element considered simultaneously the effect of material and geometric nonlinearities on the response of foundation. It used a 3-DOF macro-element and the differential equations of system's motion. A reduce scaled of soil-foundation system was tested on a shake table to investigate the seismic behavior of this type of interface. The comparison between simulation and experiment results show that this model suited for simulating a couple of material and geometric behaviors of a shallow foundation under seismic loading. The macro-element can be used to investigate numerically the behavior of a wider variety of configurations which are difficult to study experimentally. In the future, possible improvements of the proposed macro-element are necessary, especially concerning the interaction of soil- structure.

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6. Conflicts of Interest

The authors declare no conflict of interest.

7. References

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