Free Vibration of Tall Buildings using Energy Method and Hamilton's Principle

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Abstract

In a framed-tube tall building, shear wall systems are the most efficient structural systems for increasing the lateral load resistance. A novel and simple mathematical model is developed herein which calculates the natural frequencies of such tall buildings. The analyses are based on a continuous model, in which a tall building structure is replaced by an idealized cantilever beam that embodies all relevant structural characteristics. Governing equations and the corresponding eigen-problem are derived based on the energy method and Hamilton's principle. Solutions are obtained for three examples; using the separation of variables technique implemented in MATLAB. The results are compared to SAP2000 full model analysis; and they indicate reasonable accuracy. The computed natural frequencies for structures 50, 60 and 70 storey buildings were over-estimate 7, 11 and 14 percent respectively. The computed errors indicate that the proposed method has acceptable accuracy; and can be used during the initial stages of designing of tall buildings; it is fast and low cost computational process.

Keywords: Tall Building; Framed Tube; Shear Wall; Free Vibration; Natural Frequency.

1. Introduction

Tall building developments have been rapidly increasing worldwide. One of the most critical issues in tall buildings is choosing proper structural form to resist lateral loads. Lateral deformation must be severely controlled, that inhabitant feels comfort and to prevent damages to second-grade structural elements. Another vital point in tall buildings' design is the dynamic analysis of these structures that is very important because of their more flexibility and consequently increases of vibrational amplitude and the fact that the dynamic characteristic of structures is mainly governed by their natural frequencies [1-2]. Therefore, dynamic parameters calculation of tall buildings is essential for primary designing. Dynamic parameters such as vibrational frequencies and mode shapes can be calculated by numerical methods such as finite element. While these numerical methods are used for final designing, approximate methods are very effective for primary designing. Approximate methods can help the designer in cases such as initial design when dimensions of some constructional members are not specified, comparison of achieved results with more advanced numerical methods, and finally specifying of structural dynamic behaviour which leads to better designing.

One of the most ordinary approximate methods for dynamic parameters calculation of tall buildings is "continuum method" in which the tall building's structure is substituted by a continuum beam, adopting Euler–Bernoulli or Timoshenko beam theory as the design tool [3]. Considering different kinds of parameters in the substituted beam can help the designer to achieve natural frequencies and mode shapes with more accuracy. For resistant of high-rise

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buildings subjected to lateral loadings, framed tube, rigid frame, braced frame, shear wall or coupled shear walls can be considered. The framed tube is an economic and ordinary form for wide ranges of tall buildings. The most primary type of framed tube includes four frame panel vertical on each other; this system consists of closely spaced perimeter columns tied at each floor level by deep spandrel beams to form a tubular structure. The framed tube structure can be considered to be composed of: (1) two web panels parallel to the direction of the lateral load, (2) two flange panels normal to the direction of the lateral load. Framed tube behaviour is similar to a cantilever beam, and the columns in two parallel sides of the neutral axis function tensile and pressed [4]. Besides, frames parallel with lateral load under bending resulted from lateral loads indicate shear behaviour [5]. Tavakoli et al. [6-7] studied the seismic performance of outrigger-belt truss system subjected to the earthquake and blast load using finite element and component-mode synthesis.

Several methods have been presented to analyze framed tube structures. Coull and Bose (1975) presented a method based on elasticity theory [8]. Coull and Ahmad (1978) presented a method for the achievement of position changes of the circumferential frame [9]. Kwan (1994) by using equal orthotropic planes, energy method and elasticity theory, presented equations for determining stress in columns and also for achievement of lateral deflection of the framed tube [4]. As the most studies of tall buildings directed toward analysis, Alavi et al. (2018, a, b) proposed simplified methods which are suitable for the preliminary design of high-rise structures [10-11]. About free vibration of tall buildings, different types of research have been done by several researchers, that in most of them the vibration of the structures is modelled as the vibration of a cantilever beam [12-14]. Many researchers have studied fundamental frequencies of tall buildings [15-17]. Kaviani et al. (2008) carried out an approximate method for determining the natural periods of multistory buildings subjected to earthquake [16]. In this article, based on a continuum approach and Hamilton's principle, a simple mathematical method for calculation of natural frequencies of the combined system of the framed tube and shear wall is presented. In particular, Mohammadnejad and Haji Kazemi in several research investigated the natural frequencies of the framed tube structures in more details, considering the effects of shear lag phenomena [18-20].

There are compound and various structural systems for increasing efficiency of framed tube buildings. A more uniform distribution of axial stress in flange and web frames, and also a decrease in the values of deflection at the highest level of structures could be obtained using the mega bracing system [21], shear walls shear core, and also outrigger-belt trusses in the frame tube structures [22-23]. The system which is considered in this article is a combined system of the framed tube and shear wall. When framed tube and shear wall system subjected to lateral loads, the shear wall deforms in bending form with downward concavity and with maximum gradient. Interaction of forces causes that shear wall to restrain deflection of frames in bases, and framed tube is like a restrain for the shear wall above structure. Therefore, deflection of the construction decreases. In the recent decade, studies about analysis of free vibration of the frame with shear wall have been done. Kuang (2001) based on continuum method and D'Alembert's principle achieved governing differential equations of free vibration of structures with the symmetrical shear wall [24]. Wang (2005) presented an equation for computing the natural vibration of buildings with coupled shear walls which is proved to be the fourth-order Sturm–Liouville differential equation, and a hand method for determining the first two periods of natural vibration of the buildings. Also, to determine the first natural frequency of these structures, a relation has been suggested [25]. In continuance of previous studies, Bozdogan and Ozturk represented an approximate method based on the continuum approach and transfer matrix method for free vibration analysis of multi-bay coupled shear walls [26]. Kamgar and Rahgozar (2019) used energy method as a robust method to compute the roof displacement and axial forces of columns in tall buildings reinforced with a framed tube and outrigger system [27].

Although free vibration analysis of framed tube system and shear-walled frame has been studied extensively over the past few decades, there have been few research efforts related to determining vibrational characteristics of the combined system of framed tube and shear-wall system. Therefore, to fill in the gap, in this study, a simple analytical method for calculating natural frequencies of the combined system of framed tube and shear walls is presented. On the basis of the continuum approach, framed tube and shear walls are replaced by an equivalent cantilever beam located at the mass center. It should be noted that the first natural frequency of any structure has an important issue in determining the linear and nonlinear response of structures subjected to the dynamic loads. On the other hand, calculating the values of natural frequencies of structures using numerical methods is computationally expensive. Therefore, the main aim of this paper is related to calculate the natural frequency of tall buildings that consist of framed tube and shear walls using simple analytical methods. The three-dimensional structure is replaced by an equivalent beam. For this purpose, Hamilton's principle is used to obtain the governing equation of a combined system. Then the characteristic equation is obtained by applying the boundary conditions. The characteristic equation is solved to calculate the natural frequency. Several numerical examples are solved, and the results are compared with those obtained from SAP2000 and other work. Finally, the results show the ability of the proposed method in comparison with the other methods.

2. Lagrange's Equations For Combined System Of Framed Tube And Shear Wall

In this section a simple mathematical method for calculation of natural frequencies of a combined system of framed tube and shear walls is presented based on the works done by Kwan (1994); Malekinejad and Rahgozar (2010) [4, 28]. Kwan (1994) proposed a model for the analysis of framed tube structures; those following assumptions are considered for modelling the framed tube system by using equivalent orthotropic plates [4]:

- The material of the structures is homogenous, isotropic and obeys Hooke's law.
- Spacing of beams and columns are uniform throughout the building height.
- 3-The floor slabs of tall buildings are not deformable in their planes and have no motion perpendicular to their planes.
- The structure is assumed symmetric in plan and height and cannot twist.
- All beams and columns are uniforms along with the building height.

The kinetic (Equation 1) and potential energies (Equation 2) of the considered dynamic system are written as follows [29]:

$$K(t) = \frac{1}{2} \int_0^b m(x) [\dot{y}(x,t)]^2 \, dx \tag{1}$$

$$P(t) = \frac{1}{2} \int_0^b EI(x) [y''(x,t)]^2 \, dx + \frac{1}{2} \int_0^b S(x) [y'(x,t)]^2 \, dx \tag{2}$$

In which $y(x, t)$ is displacement and $S(x)$ is the shear stiffness $GA(x)$. In which G is the shear modulus, and $A(x)$ is the cross-sectional area.

The function A is in the form of L integral, between two arbitrary times of t_1, t_2.

$$A = \int_{t_1}^{t_2} H \, dt = \int_{t_1}^{t_2} (K - P) \, dt \tag{3}$$

Hamilton's principle represents that A has a stationary value expressed as $\delta A = 0$, where δ is known as the variational operator.

Hamilton's principle can be written in the following form [28]:

$$\delta A = \int_{t_1}^{t_2} \left\{ \int_0^b (\delta H + \delta y_{nc}) \, dx + \delta L \right\} dt = 0 \qquad \delta y = 0 \;\; at \;\; t = t_1, t_2 \;\; 0 \leq x \leq b \tag{4}$$

Using properties of δ operator and integration by parts, the following matters result:

- Differential equations of motion known as the Lagrange's equation,
- Boundary displacements,
- Boundary forces,
- Eigenvalue solution form.

H can be determined as follows:

$$H = \frac{1}{2} m \dot{y}^2 - \frac{1}{2} EI y''^2 - \frac{1}{2} S y'^2 \tag{5}$$

Using the Lagrange equation, Eq. (4) can be rewritten as follows:

$$\delta A = \int_{t_1}^{t_2} \left\{ \int_0^b \left(\frac{\delta H}{\delta y'} \delta y' + \frac{\delta H}{\delta y''} \delta y'' + \frac{\delta H}{\delta \dot{y}} \delta \dot{y} + F \delta y \right) dx \right\} dt = 0 \tag{6}$$

At this step, the integrand in Equation 6 should be transformed into one containing only δy. Therefore, this equation is integrated by part, both respect to space and time. After simplification, one can obtain:

$$\delta A = \int_{t_1}^{t_2} \left\{ \int_0^b \left(\left(\frac{-\partial}{\partial x} \left(\frac{\partial H}{\partial y'} \right) + \frac{\partial^2}{\partial x^2} \left(\frac{\partial H}{\partial y''} \right) - \frac{\partial}{\partial t} \left(\frac{\partial H}{\partial \dot{y}} \right) + P \right) \delta y \, dx \right) dx + \frac{\partial H}{\partial y''} \delta y' \Big|_{x=b} - \frac{\partial H}{\partial y''} \delta y' \Big|_{x=0} \right. \tag{7}$$

$$\left. + \left(\frac{\partial H}{\partial y'} - \frac{\partial}{\partial x} \left(\frac{\partial H}{\partial y''} \right) \right) \delta y \Big|_{x=b} - \left(\frac{\partial H}{\partial y'} - \frac{\partial}{\partial x} \left(\frac{\partial H}{\partial y''} \right) \right) \delta y \Big|_{x=0} \right\} dt = 0$$

By replacing Equation 5 into Equation 7, the following equation with a series of boundary conditions is derived:

$$\frac{-\partial}{\partial x}(Sy') + \frac{\partial^2}{\partial x^2}(-EIy'') - \frac{\partial}{\partial t}(m\dot{y}) = 0 \qquad 0 \le x \le b \tag{8}$$

Using the method of separation of variable and let $y(x,t) = Y(x)T(t)$, two equations can be obtained. The frequencies can be obtained from the x-dependent equation [29]:

$$-\frac{d}{dx}(SY') + \frac{d^2}{dx^2}(EIY'') - m\omega^2 Y = 0 \tag{9}$$

The Equation 9 after simplifying will be changed as follows by definition the γ and η parameters (Equations 11 and 12).

$$\frac{d^4 Y}{dn^4} - \gamma^2 \frac{d^2 Y}{dn^2} - \eta^2 \omega^2 Y = 0 \qquad 0 \le n \le 1 \tag{10}$$

$$\gamma^2 = \frac{S}{EI} b^2 \tag{11}$$

$$\eta^2 = \frac{m}{EI} b^2 \tag{12}$$

Values of *EI, S* and *m* can be determined by applying the boundary conditions Equations 13 to 16. These boundary conditions are related to the displacement value at the bottom of the structure ($Y_{(n=0)}$), the value of rotation at the bottom of the structure ($Y'_{(n=0)}$), shear force ($[Y'' - \gamma^2 Y']|_{n=1}$) and bending moment ($Y''|_{n=1}$) values at the top of the structure.

$$Y_{(n=0)} = 0 \tag{13}$$

$$Y'_{(n=0)} = 0 \tag{14}$$

$$\left[Y''' - \gamma^2 Y' \right] \Big|_{n=1} = 0 \tag{15}$$

$$Y''|_{n=1} = 0 \tag{16}$$

The *Y* function is considered as follows to obtain a solution for the governing equation (Equation 10):

$$Y(n) = Ce^{pn} \tag{17}$$

Therefore, the solutions of the equation (Equation 10) will be as follows by considering Equation 17:

$$p^2 = \frac{\eta^2}{2} \pm \sqrt{\gamma^2 \omega^2 + \frac{\eta^4}{4}} \qquad \Rightarrow \qquad p_{1,2} = \pm B_1 \, , \, p_{3,4} = \pm i \, B_2 \tag{18}$$

In which:

$$B_1 = \sqrt{\sqrt{\gamma^2 \omega^2 + \frac{\eta^4}{4}} + \frac{\eta^2}{2}} \tag{19}$$

$$B_2 = \sqrt{\sqrt{\gamma^2 \omega^2 + \frac{\eta^4}{4}} - \frac{\eta^2}{2}} \tag{20}$$

To solve the Equation 10, one can rewrite it as follows:

$$
\begin{bmatrix} Y_{(n)} \\ Y'_{(n)} \\ Y''_{(n)} \\ Y'''_{(n)} - \alpha^2 Y'_{(n)} \end{bmatrix} = p(n,\omega) \begin{bmatrix} G \\ H \\ J \\ K \end{bmatrix}
\tag{21}
$$

$$
p(n,\omega) = \begin{bmatrix} cosh(B_1 n) & sinh(B_1 n) & cos(B_2 n) & sin(B_2 n) \\ B_1 sinh(B_1 n) & B_1 cosh(B_1 n) & -B_2 sin(B_2 n) & B_2 cos(B_2 n) \\ B_1^2 cosh(B_1 n) & B_1^2 sinh(B_1 n) & -B_2^2 cos(B_2 n) & -B_2^2 sin(B_2 n) \\ B_1 B_2^2 sinh(B_1 n) & B_1 B_2^2 cosh(B_1 n) & B_2 B_1^2 sin(B_2 n) & -B_2 B_1^2 cos(B_2 n) \end{bmatrix}
\tag{22}
$$

After substituting boundary conditions, a nontrivial solution for Equation 21 can be obtained by setting the determinant of coefficients to zero.

$$
\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & B_1 & 0 & B_2 \\ B_1^2 coshB_1 & B_1^2 sinhB_1 & -B_2^2 cosB_2 & -B_2^2 sinB_2 \\ B_1 B_2^2 sinhB_1 & B_1 B_2^2 coshB_1 & B_2 B_1^2 sinB_2 & -B_2 B_1^2 cosB_2 \end{vmatrix} = 0
\tag{23}
$$

Solving Equation 23 using MATLAB software yields:

$$
B_1^5 B_2 coshB_1 cosB_2 + B_1^4 B_2^2 sinhB_1 sinB_2 + B_1^3 B_2^3 cosh^2 B_1 + B_1^3 B_2^3 cos^2 B_2 -
$$
$$
B_1^3 B_2^3 sinh^2 B_1 + B_1^3 B_2^3 sin^2 B_2 - B_1^2 B_2^4 sinhB_1 sinhB_2 + B_1 B_2^5 coshB_1 cosB_2 = 0
\tag{24}
$$

By solving this equation, the natural frequencies are calculated. For numerical study, *EI, S, m,* parameters should be calculated. To calculate G in $S = GA(x)$ Kwan's relations are used [4].

$$
G = \frac{\dfrac{h}{st}}{\dfrac{\Delta_b}{Q} + \dfrac{\Delta_s}{Q}}
\tag{25}
$$

In which:

$$
\frac{\Delta_b}{Q} = \frac{(h-d_b)^3}{12E_m I_c} + (\frac{h}{s})\frac{(s-d_c)^2}{12E_m I_c}
\tag{26}
$$

$$
\frac{\Delta_s}{Q} = \frac{(h-d_b)}{G_m A_{sc}} + (\frac{h}{s})^2 \frac{(s-d_c)^2}{G_m A_{sb}}
\tag{27}
$$

Where I_b and I_c are moments of inertia of the beam and column respectively, A_{sb} and A_{sc} are effective shear areas of the beam and column, and finally G_m is the shear modulus of the material.

The following step by step procedures shown the methodology of the proposed method

- Determining the values of mass per unit length (*m*) along height of structure;

- Determining the flexural (*EI*) and shear stiffness (*S*) of the structure;

- Calculation the γ and η parameter using Equations 11 and 12;

- Solving Equation 24 to find the B_1 and B_2 parameters and finally computing the first natural frequency of the structure using Equations 19 and 20.

3. Examples and Comparison of Results with Computer Analysis

To verify the accuracy and efficiency of the proposed approximate method, three numerical high-rises symmetric reinforced concrete buildings which consist of a framed tube and shear walls are presented for determining the natural frequencies [30]. Then, a comparison is presented between the results in order to evaluate the simplicity and accuracy of this method. Characteristics of these structures are listed in Table 1, also plan and actual system of tall building are shown in Figure 1.

Table 1. Geometrical characteristics of structures in plan and height

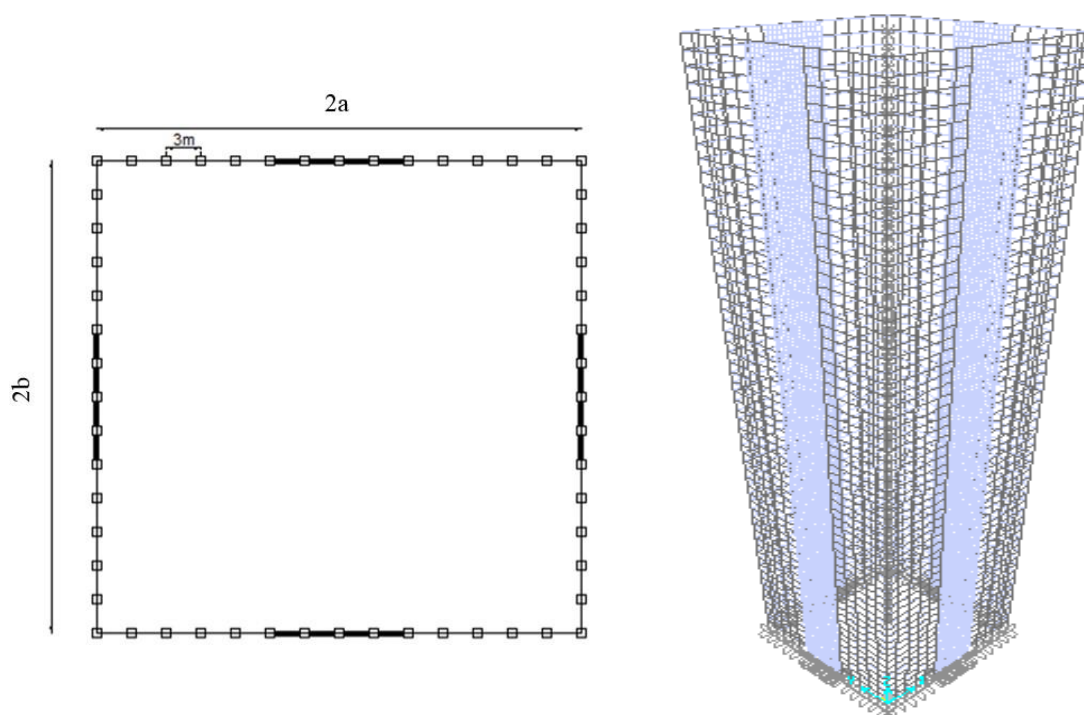| Number of stories	Story's height	Spans length		Dimensions of shear-wall			Plan's dimensions	
n	*h(m)*	$S_w(m)$	$S_f(m)$	*B(m)*	*h(m)*	*t(m)*	*2a(m)*	*2b(m)*
50	3	3	3	2.2	3	0.3	36	36
60	3	3	3	2.2	3	0.35	42	42
70	3	3	3	2.2	3	0.35	42	42

Figure 1. Plan and actual system of tall building consist of framed tube and shear walls

The elastic characteristics of materials are listed in Table 2.

Table 2. Elastic characteristics

E (*GPa*)	*G* (*GPa*)	$\rho_c(kg/m^3)$	*ν*
20	8	400	0.25

Equivalent properties of the buildings are listed in Table 3 based on Kwan's method in 1994 [4].

Table 3. Equivalent properties of tall buildings

G (*GPa*)	*t* (*m*)
1.37	0.21

Flexural (*EI*) and shear stiffness (*S*) of the framed tube system and shear walls are calculated as follows:

50 storey $\Rightarrow$ $EI = 2 \times 10^9 \times 6418.41 + 2 \times 10^9 \times 2799.42 = 18435.66 \times 10^9$ $kg.m^2$

$\quad\quad\quad\quad\quad S = 20.59 \times 10^8 + 63.36 \times 10^8 = 83.95 \times 10^8$ $kg.m^2$

60 storey $\Rightarrow$ $EI = 2 \times 10^9 \times 10372.57 + 2 \times 10^9 \times 5556.72 = 31858.58 \times 10^9$ $kg.m^2$

$\quad\quad\quad\quad\quad S = 24.16 \times 10^8 + 73.92 \times 10^8 = 98.08 \times 10^8$ $kg.m^2$

70 storey $\Rightarrow$ $EI = 2 \times 10^9 \times 15386.45 + 2 \times 10^9 \times 8975.68 = 48724.26 \times 10^9$ $kg.m^2$

$\quad\quad\quad\quad\quad S = 28.52 \times 10^8 + 85.32 \times 10^8 = 113.84 \times 10^8$ $kg.m^2$

Where *m* is mass per unit height of the buildings, which is derived as follows:

50 storey $\quad \Rightarrow \quad m = \dfrac{76550400}{150} = 510336$ kg / m

60 storey $\quad \Rightarrow \quad m = \dfrac{118056960}{180} = 655872$ kg / m

70 storey $\quad \Rightarrow \quad m = \dfrac{179576640}{210} = 855127$ kg / m

By substituting the values of *EI*, *S* and *m* for each of the structure into the Equations 11 and 12, γ and λ can be calculated. By substituting their values into Equations 19 and 20, B_1 and B_2 can be found. Finally, by using Equation 24, natural frequencies are calculated based on a computer program which has been developed in MATLAB for three numerical examples 50, 60 and 70 storey tall buildings. Comparison of computer analysis results (SAP2000) with the proposed method are listed in Table 4.

Table 4. Comparison of natural frequencies between SAP2000 and proposed approximate method

Number of stories	γ	λ	$\omega(rad/s)$		Percent of Error in ω
			Proposed method	SAP2000	
50	4.25	3.74	1.93	1.80	7
60	3.15	4.65	1.53	1.37	11
70	2.85	6.12	1.27	1.09	14

The calculated natural frequencies for structures 50, 60 and 70 storey tall buildings have over estimate 7, 11 and 14 percent differences with results of computer analysis (SAP2000). The main source of errors between the proposed approximate method and SAP2000 may be lead from followings: all closely spaced perimeter columns tied at each floor level by deep spandrel beams are modelled as a tubular structure, the equivalent elastic properties for *GA* and *EI* and neglecting the effect of shear lag in the approximate method have been used.

Also the results for 60 and 70 storey tall buildings with shear walls are compared with the research carried out by Rahgozar et al. using B-spline functions [30]. As shown in Table (5), the natural frequencies calculated by the proposed approximate method are overestimated by 15 and 9 percent for 60 and 70 storey building respectively.

Table 5. Comparison of natural frequencies between proposed approximate method and Rahgozar et al. [30]

Number of stories	$\omega(rad/s)$		Percent of Error in ω
	Proposed method	Rahgozar et al. [30]	
60	1.53	1.30	15
70	1.27	1.15	9

4. Conclusion

Natural frequencies and mode-shapes play an important role in structural design of tall buildings. Especially the first natural mode; since it is the dominant component in response of a tall building to earthquake or wind loading. In this article, an approximate method for free vibration analysis of the combined system of framed tube and shear walls was presented. In the proposed method, the structure is modelled as a cantilever hollow box with equivalent structural characteristics. The governing differential equation was derived by energy method and Hamilton's principle. Applying appropriate boundary conditions, natural frequencies of the combined system of framed tube and shear walls were obtained. Comparing to results from comprehensive finite element models; the proposed method overestimate the first natural frequency by 7% for the 50-storey, 11% for the 60-storey, and 14% for the 70-storey building. Differences are within acceptable ranges for a quick estimate. Hence, the proposed method may reliably be used for free vibration

analysis of framed tube tall buildings reinforced by shear walls. The proposed method is simple, accurate, economical, reliable, and especially suitable for use during the preliminary design; where a large number of structures with different features need to be analyzed.

5. Conflicts of Interest

The authors declare no conflict of interest.

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