



Effect of Sediment Feeding on Live-Bed Scour around Circular Bridge Piers

Firat Gumgum^{1*}, Mehmet Sukru Guney²

¹ Assistant Professor, Civil Engineering Department, Dicle University, Diyarbakir 21280, Turkey.

² Professor, Civil Engineering Department, Izmir University of Economics, Izmir 35330, Turkey.

Received 03 February 2021; Revised 09 April 2021; Accepted 20 April 2021; Published 01 May 2021

Abstract

The effect of sediment feeding was investigated in the case of live-bed scour around circular bridge piers under flood waves to provide contributions for future experimental procedures. Circular piers of three different diameters were tested in a long rectangular flume containing uniform sediment layer 25 cm thick, by generating 7 different triangular hydrographs with different durations ranging between 6 and 20 minutes and the peak discharges varying from 18 L/s to 38 L/s. Experiments were first conducted without sediment feeding and total load was collected at predetermined time intervals. Then the same experiments were performed by feeding with the same amount of collected sediment. Time dependent scour depths were measured using UVP. Bed degradation was also determined by using an empirical equation existing in the literature. It was found that feeding with the rates equal to the transported ones did not significantly change the scour depth and total sediment load within the limits of the experiments. No significant bed degradation was observed, except at the upstream end. It was revealed that the sediment feeding may not be required in the experiments where temporal evolution of the scour depth is studied in a sufficiently long flume containing sufficient sediment.

Keywords: Circular Pier; Local Scour; Live-Bed; Sediment Feeding.

1. Introduction

Local scour is simply defined as “scour caused by an obstruction in the flow” and it takes place in either clear-water or live-bed conditions. If the bottom shear stress of the flow is lower than the critical shear stress of the bed material, flow cannot erode the bed while causing local scour and this state is named as clear-water condition. On the other hand, bottom shear stress of the flow may be greater than the critical shear stress of the bed material and causes bedload transport which results in sediment supply into the scour hole. This state is named as live-bed condition.

Bed degradation in the flume may cause inaccurate results on scour depth since the live-bed scour is greatly affected by incoming sediment from upstream as stated by Laursen and Toch (1956), Jain and Fischer (1979), Chiew (1984) and Kothiyari et al. (1992) [1-4]. Therefore, the bed erosion should be compensated to obtain more accurate results on scour depth. Laursen & Toch (1956), Kothiyari et al. (1992) and Melville (1984) [1, 4, 5] managed this problem by means of sediment feeding by installing a hopper or conveyor belt to their experimental set-up. Jain & Fischer (1979), Chiew (1984) and Sheppard & Miller (2006) [2, 3, 6] resorted to recirculating flume by mounting a sediment pump to their experimental set-up. In both cases, simulation of sediment supply causes additional expense and workforce.

* Corresponding author: fiatgumgum@hotmail.com

 <http://dx.doi.org/10.28991/cej-2021-03091699>



© 2021 by the authors. Licensee C.E.J., Tehran, Iran. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC-BY) license (<http://creativecommons.org/licenses/by/4.0/>).

Researchers keep their experiments going for hours, even days to reach the equilibrium. However, there can be a significant difference of flow duration between the model and the prototype according to the time scale. For instance, Yanmaz (2002) [7] stated that a 50-hours experiment in a Froude model built on a 1/50 scale in the laboratory would correspond to 354 hours in nature. In some cases, only a small part of a hydrograph near the peak discharge may induce live-bed conditions and cause sediment motion. This reduces the required duration of sediment feeding even more. Consequently, the flow may not be effective long-time enough to significantly erode the bed if the duration of hydrograph and live bed conditions are relatively short and there is enough sediment deposited in the flume.

Although sediment transport is an attractive subject, researches concerning the effect of the absence of the upstream sediment supply are scarce in literature. Marion et al. (2006) [8] investigated the effect of sediment supply on scour at bed sills under steady flow conditions and compared with the previous studies. As a result of their experiments, scour depth decreased with the increasing sediment discharge. Also, there are some other studies related to the effect of sediment supply on bedload and bed morphology [9-11]. In these studies, coarsening phenomenon resulting from nonuniform character of sediment was studied. Hong et al. (2016) [12] investigated the evolution of local pier-scour depth with dune migration under steady flow conditions. During the experiments performed in subcritical flow regime, they did not supply sediment and the bed degradation was found to be negligible. Wang et al. (2019) [13] studied the effects of no sediment supply on bed load and bed morphology under unsteady flow conditions, this study being quite similar to our study.

In this study, it was aimed to investigate experimentally the effect of sediment feeding on scour depth around bridge piers, to provide contribution for the future experimental studies which will be realized in similar conditions. Previously, temporal evolution of live-bed scour depths around circular bridge piers under unsteady flow was investigated by Gumgum & Guney (2020) [14] and brought a novel understanding to the subject by testing triangular hydrographs, interpreting the effective parameters and providing equations to predict time dependent and maximum live-bed scour depths under unsteady flow conditions. Still, there is a question needed to be answered: to what extent the scouring would be affected under no-fed condition? For this purpose, the results of the experiments carried out by using seven different hydrographs with and without sediment feeding were examined.

Experimental set-up, methods of measurement and configurations of the hydrographs are presented. Scour depths and total sediment loads collected from the experiments are presented and effects of sediment feeding on scour depth were discussed together with bed morphology calculations by using the empirical relation suggested by Wang et al. (2019) [13]. The last part of this article contains the conclusions and the suggestions.

2. Experimental Set-up

Experiments were carried out in a flume with a length of 18.6 m, width of 0.8 m and depth of 0.75 m constructed in the Hydraulic Laboratory of Dokuz Eylul University Civil Engineering Department, within the scope of the project TÜBİTAK 106M274. Total length of the mobile bed was 15.6 m. Three different circular piers of diameter 4 cm (D4), 8 cm (D8) and 11 cm (D11) were used. The pier was placed at the 12th meter of the flume. The gas concrete blocks of 20 cm thickness were placed in the first 8 m and the last 5 m of the flume. After the 3rd meter of the flume, the flume was covered with non-ripple forming sediment having 1.63 mm median diameter (d_{50}) and geometric standard deviation of $\sigma = 1.303$ (uniform, because $\sigma < 1.4$) to form a bed of 25 cm. The channel bed slope was set to 0.006. Schematic view of the experimental set-up is given in Figure 1.

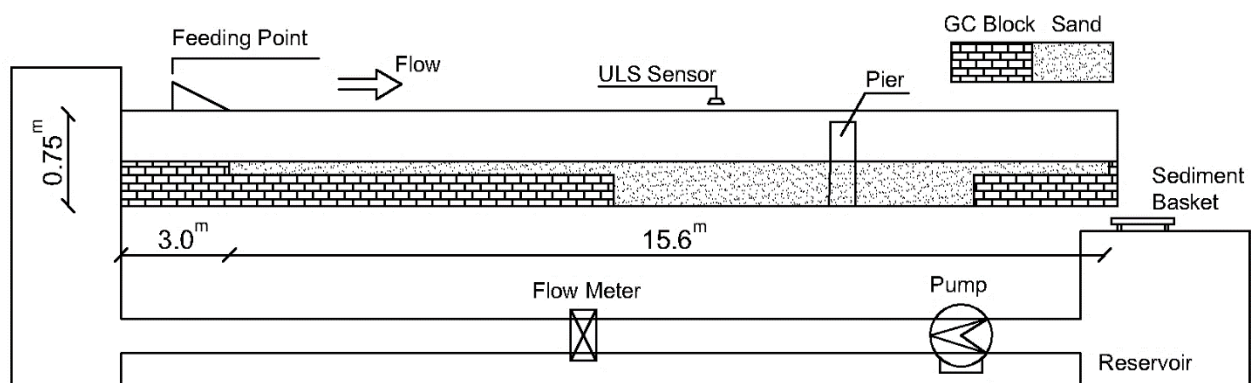


Figure 1. Schematic view of the experimental set-up

A pump with a discharge capacity of 100 L/s was used for recirculation of water. Time dependent discharge was measured by an electromagnetic flow meter mounted on the pipe before the inlet of the channel. Scour depth around the bridge pier was measured by Ultrasonic Velocity Profiler (UVP) device. As also stated in Guney et al. (2013) [15], it is possible to determine the bed morphology by using UVP which is normally designed for velocity measurement.

There are three UVP transducers used in this study. One at the upstream of the pier and two at the flanks of the pier (See Figure 4). The approaching flow depth was measured by Ultrasonic Level Sensor (ULS) which is placed 1 m upstream of the pier. Temporal lag between discharge and flow depth readings was compensated simply by overlapping discharge curve on flow depth curve.

Seven different triangular hydrographs were used, as indicated in Table 1. The name of the hydrograph (H i j - k) was defined so that the subscripts i and j denote rising and falling durations in minutes respectively and the following subscript k indicates the peak discharge of the hydrograph (L/s). A small base flow (Q_b) of 1.5 L/s corresponding to a base flow depth (h_b) = 0.012 m, was preferred to prevent scouring before the flood. It was also attempted to determine the corresponding flow rates and flood durations when the experimental setup is considered as 1/25 and 1/50 scaled physical Froude models. The so obtained prototype values are also given in Table 1, together with the model ones.

Table 1. Peak discharges and flood durations of the generated hydrographs together with 1/25 and 1/50 scaled prototypes

Hydrograph	Model			Prototype (1/25)			Prototype (1/50)		
	Peak Discharge (L/s)	Flood Duration (sec)		Peak Discharge (m ³ /s)	Flood Duration (sec)		Peak Discharge (m ³ /s)	Flood Duration (sec)	
		Rising	Falling		Rising	Falling		Rising	Falling
Hid33-20	20	180	180	63	900	900	354	1273	1273
Hid33-38	38	180	180	97	900	900	548	1273	1273
Hid55-23	23	300	300	72	1500	1500	407	2121	2121
Hid55-31	31	300	300	103	1500	1500	583	2121	2121
Hid55-38	38	300	300	119	1500	1500	672	2121	2121
Hid1010-28	28	600	600	88	3000	3000	495	4243	4243
Hid1010-33	33	600	600	103	3000	3000	583	4243	4243

First group of experiments were carried out without sediment supply. Total sediment load was collected by means of the baskets located at the exit of the flume with the interval of 2 minutes for the hydrographs having 20 mins of total duration and the interval of 1 minute for the remaining hydrographs. Collected sediment was dried and weighed. Then, the sediment graphs were formed for each hydrograph by taking into consideration the amount of collected sediment. In the second group of experiments, feeding was realized manually by means of the sediment feeding plate, designed to provide uniform feeding, according to the previously obtained sediment graphs. The flowchart describing the experimental procedure is given in Figure 2.

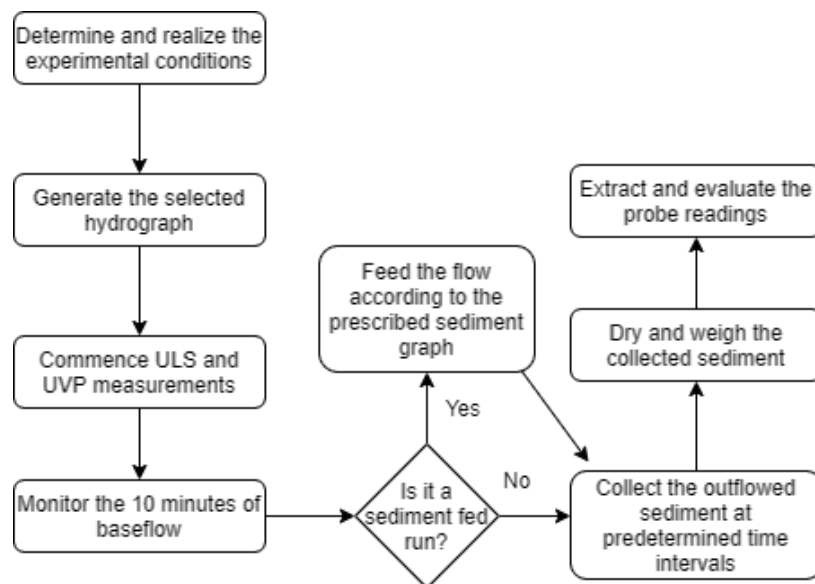
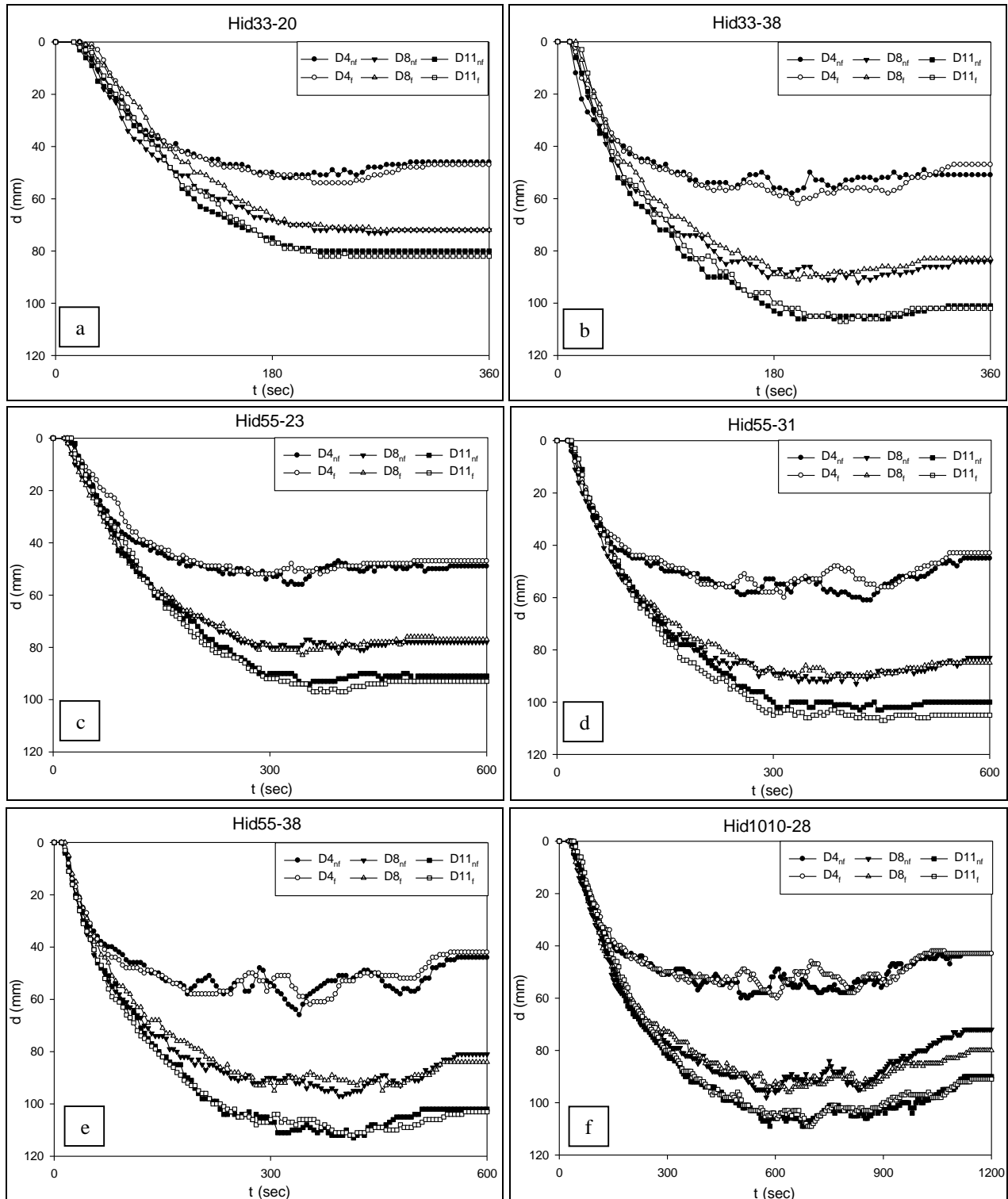


Figure 2. Flow chart of the experimental study

3. Results and Discussion

Temporal evolution of live-bed scour around bridge piers under unsteady flow was explained elaborately by Gumgum & Guney (2020) [14] and no remarkable difference was observed on scouring for the experiments without sediment feeding. However, a net bed level degradation was observed at upstream end as a prominent difference. This degradation disappeared gradually along the first 2-3 m of the test zone and no significant bed level degradation observed further. This fact led to consider the flow as “fed itself”. This issue will be discussed in the following sections.

In most studies related to the local scour around bridge piers, the use of the equilibrium scour depth as the design parameter was proposed and some relations calculating the equilibrium scour depth were suggested. There are numerous studies that describe the equilibrium state for different flow conditions [1-3, 17]. It is commonly stated that the clear water equilibrium is reached when no more than a certain percentage of scour occurs in a certain time interval, while the live-bed equilibrium is reached when the fluctuations occurring in the scour depth remain constant about an average line. However, Gumgum & Guney (2020) [14] stated that it is not practicable to study the local scour with equilibrium scour depth under live-bed floods, emphasizing that the scour depth will increase during the rising period of the hydrograph and will decrease during the falling period of the hydrograph and an equilibrium state will never be reached. Therefore, in this study the maximum and final scour depths were taken into consideration. The temporal evolutions of the scour depths are given in Figures 3a to 3g, for each generated hydrograph. Scour depth curves in this figure are obtained from the transducer located at the upstream side of the pier, where the maximum scour depths were observed. In this figure, the subscripts f and nf denote with and without sediment feeding, respectively.



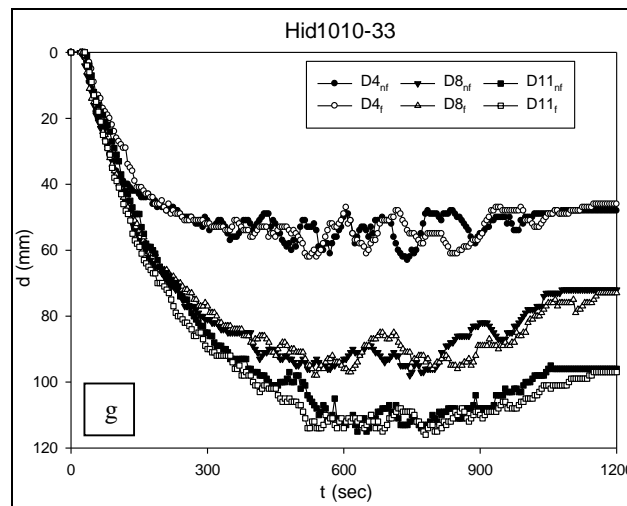


Figure 3. Temporal evolutions of the scour depths for each hydrograph

Sediment supply loses its effect on scour depth with the increased pier diameter. For instance, in the case of Hid 33-20 (Figure 3a), the scour depths concerning the piers of diameter 8 cm and 11 cm have not changed along the falling period of the hydrograph unlike the scour depth of the pier of 4 cm diameter. This effect can be seen from the other figures as well. For all the hydrographs, initial scour depths appear practically the same for a short period of time regardless of pier diameter. This phenomenon can be explained by the fact that the scour depth in shallow waters (pier width/flow depth > 5) is independent of the pier width [17]. In most of the experiments performed with or without sediment feeding, the maximum and the final scour depths are the same or very close to each other for a given hydrograph and pier. It was revealed that the difference between the maximum scour depths was not greater than 1% in 43% of the experiments carried out under fed and non-fed conditions. In 86% of the experiments, this difference was at most 5%. Similarly, in 43% of the experiments, the difference between final scour depths was not greater than 1%, while in 90% it was at most 5%. Therefore, it can be said that there was no significant change in scour depths in the absence of sediment feeding. Still, in order to support this conclusion, it is necessary to examine the change in bed morphology and bed load. Scour holes at the end of the Hid55-38 are given in Figure 4 as an illustrative example, for the piers D4, D8 and D11.



Figure 4. Scour holes at the end of the Hid55-38 for the pier a) D4; b) D8; and c) D11

The parameter of total flow work (W_k) defined as total flow work done on bed when the base flow is at threshold for incipient motion of median sediment size [18] was commonly used by Lee et al. (2004), Bombar et al. (2011) and Wang et al. (2019) [19, 20, 13] to identify the effectiveness of hydrographs on sediment transport. This parameter can be calculated from:

$$W_k = \frac{u_b^2 V_{ol}}{g y_b B} \quad (1)$$

Where u_b is the shear velocity of the base flow, V_{ol} is the volume of the water created by hydrograph through its duration and can be simply calculated from the area of the hydrograph, g is gravitational acceleration, y_b is the base flow depth, B is the channel width.

A parameter identifying the effectiveness of hydrographs is unsteadiness, proposed by Graf and Suszka (1985) [21], calculated as follows:

$$P = \frac{1}{u_b} \frac{y_p - y_b}{t} \quad (2)$$

Where y_p is peak flow depth, t is the duration of the hydrograph.

Wang et al. (2019) proposed the parameter ξ which is a combination of W_k and P . They stated that this expression is well correlated with data available in the literature.

$$\xi = W_k P^{0.2} \quad (3)$$

This parameter was associated with sediment yields for both uniform and nonuniform sediments and highest incision depth (Δz_0). According to their study, Δz_0 occurred at the upstream end of the test section which is consistent with the present study. The only difference observed in experiments conducted under fed and non-fed conditions was that when feeding with the collected amount of sediment, a small mound was formed at the upstream end (feeding point), and in the case of non-fed, a degradation was observed at the upstream end. Wang et al. (2019) [13] proposed an equation to predict normalized largest incision depth $\Delta z_0^* = \Delta z_0 / y_b$.

$$\Delta z_0^* = 0.2191 \xi^{0.4944} \quad (4)$$

Bed shear velocity of the base flow $u_b = (gRS_0)^{0.5}$ is 0.027 m/s which is below the critical bed shear velocity (u_c) = 0.032 m/s calculated from Shields diagram. Setting u_b low was necessary because any higher flow rate would cause undesired scouring around the bridge pier during the base flow. However, it took 0.02-0.04 t at most to satisfy $u_b > u_c$ for the incipient sediment movement. Hydrographs were readjusted by disregarding this short duration and assuming Q_b as 3.5 L/s and y_b as 0.018 m which satisfies $u_b > u_c$, only in the calculation of W_k . The so obtained values are given in Table 2 with readjusted hydrograph durations (t^*).

Table 2. Unsteadiness and total flow work parameters of the hydrographs

Hydrograph	t (sec)	t^* (sec)	y_p (m)	V_{ol} (m ³)	W_k	P (10 ⁻³)	ξ	Δz_0 (m)
Hid33-20	360	345	0.051	2.85	65.9	4.076	21.9	0.012
Hid33-38	360	353	0.074	6.09	141.0	6.480	51.5	0.018
Hid55-23	600	575	0.056	5.61	129.8	2.759	39.9	0.016
Hid55-31	600	585	0.073	8.10	187.6	3.825	61.6	0.020
Hid55-38	600	592	0.080	10.21	236.4	4.264	79.4	0.023
Hid1010-28	1200	1160	0.065	14.21	328.9	1.662	91.5	0.025
Hid1010-33	1200	1170	0.072	17.26	399.5	1.881	113.9	0.027

Calculated Δz_0 values indicate highest incision depths. The highest incision depths occurred at the upstream end and this finding is compatible with the results of Wang et al. (2019) [13] where there is a net bed degradation at downstream but considerably low comparing to Δz_0 . They indicated the presence of a transition region which was affected from unsteady flow conditions most, and downstream of this transition zone was relatively unaffected. Two of their results with relatively low W_k show no bed elevation change at downstream. Bed level degradation along the channel can be predicted according to the different ξ values by (Figure 5), where $x^* = x/L$, L is the length of the test zone.

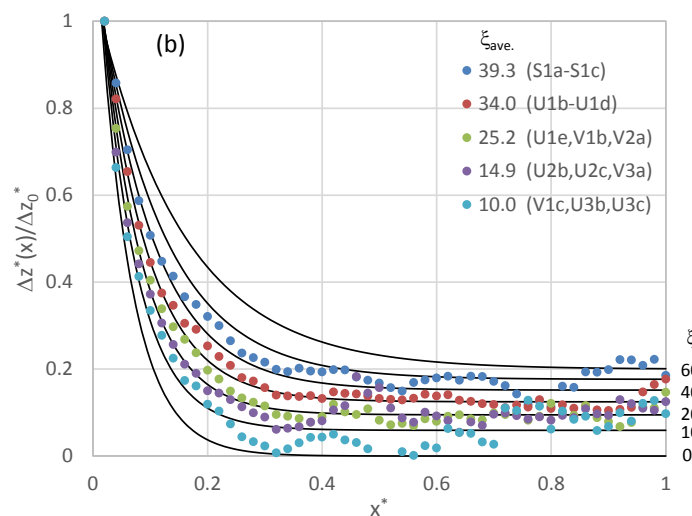


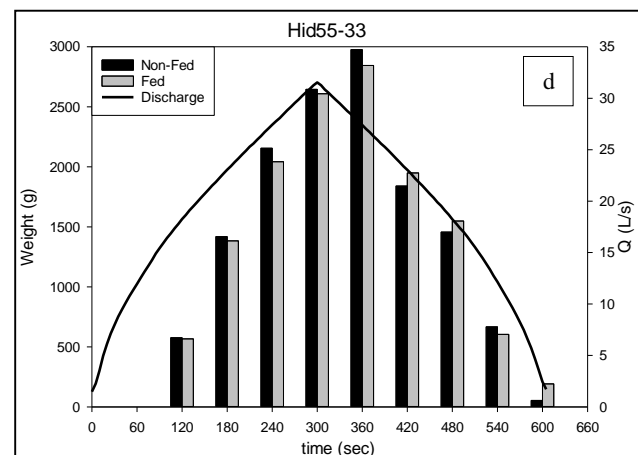
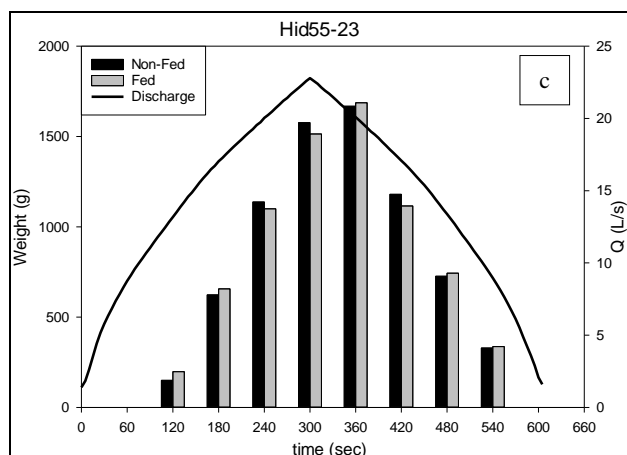
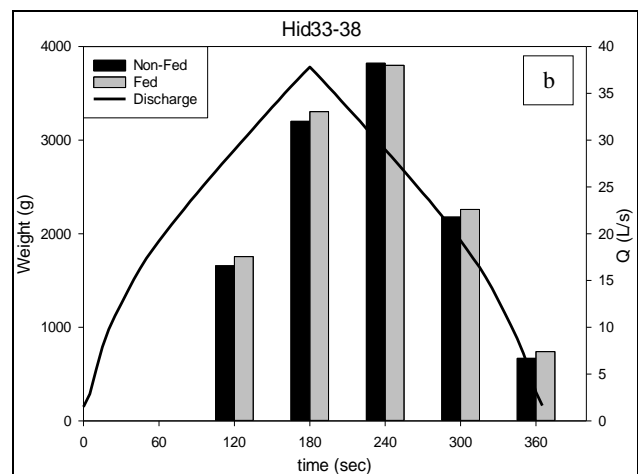
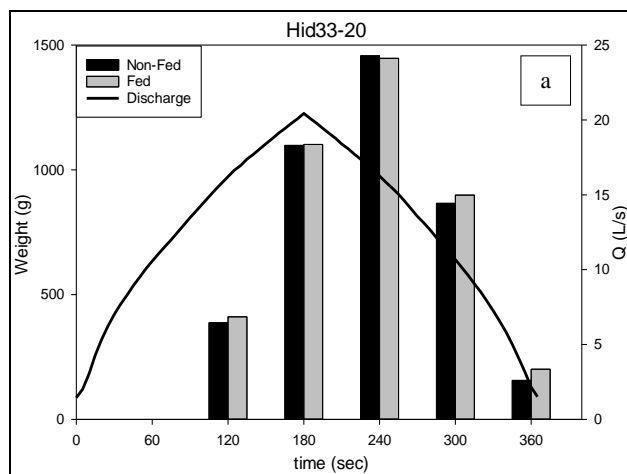
Figure 5. Bed level degradation along the channel according to the ξ

The pier is located at $x^* = 0.65$ in the present study. When the first four ξ values in Table 2 are located in Figure 5, negligibly small degradation is observed. Although the last three ξ values are beyond the limits of Figure 5, bed level degradation over the pier zone seems to be negligible. Consequently, the order of magnitude of the occurred bed degradation was so that it would not affect the size of the scour hole.

The amounts of the total sediment load collected in experiments conducted under fed and non-fed conditions are given in Figures 6a to 6g, together with the discharge curve. The comparison of the total sediment load collected in experiments performed under fed and non-fed conditions also supports this conclusion. The lag between the total sediment load and the discharge curve was not considered since it does not affect the comparison of the total sediment load for both conditions. However, first baskets of each experiments were empty (or negligibly small amount) as a result of the lag. The differences between the amounts of the sediments collected in the baskets during with and without sediment feeding experiments, ΔW , were divided to the total sediment load W . The so obtained ratios $\Delta W/W$ are given in Table 3. The difference is less than 10% for 78% of the baskets. Considering the stochastic character of sediment movement and the fact that the amount of sediment collected in some baskets under non-fed condition was higher, the effect of the sediment feeding on total sediment load for the performed experiments can be regarded as insignificant. Since the most distinctive property of live-bed scour is sediment transport, this conclusion supports the scour depth results given in Figures 3a to 3g.

Table 3. Percentages of the collected sediment differences during with and without feeding experiments

$\Delta W/W$ (%)	Baskets (%)
<1	18.52
<2	35.19
<3	35.19
<4	50.00
<5	55.56
<6	59.26
<7	62.96
<8	66.67
<9	75.93
<10	77.78



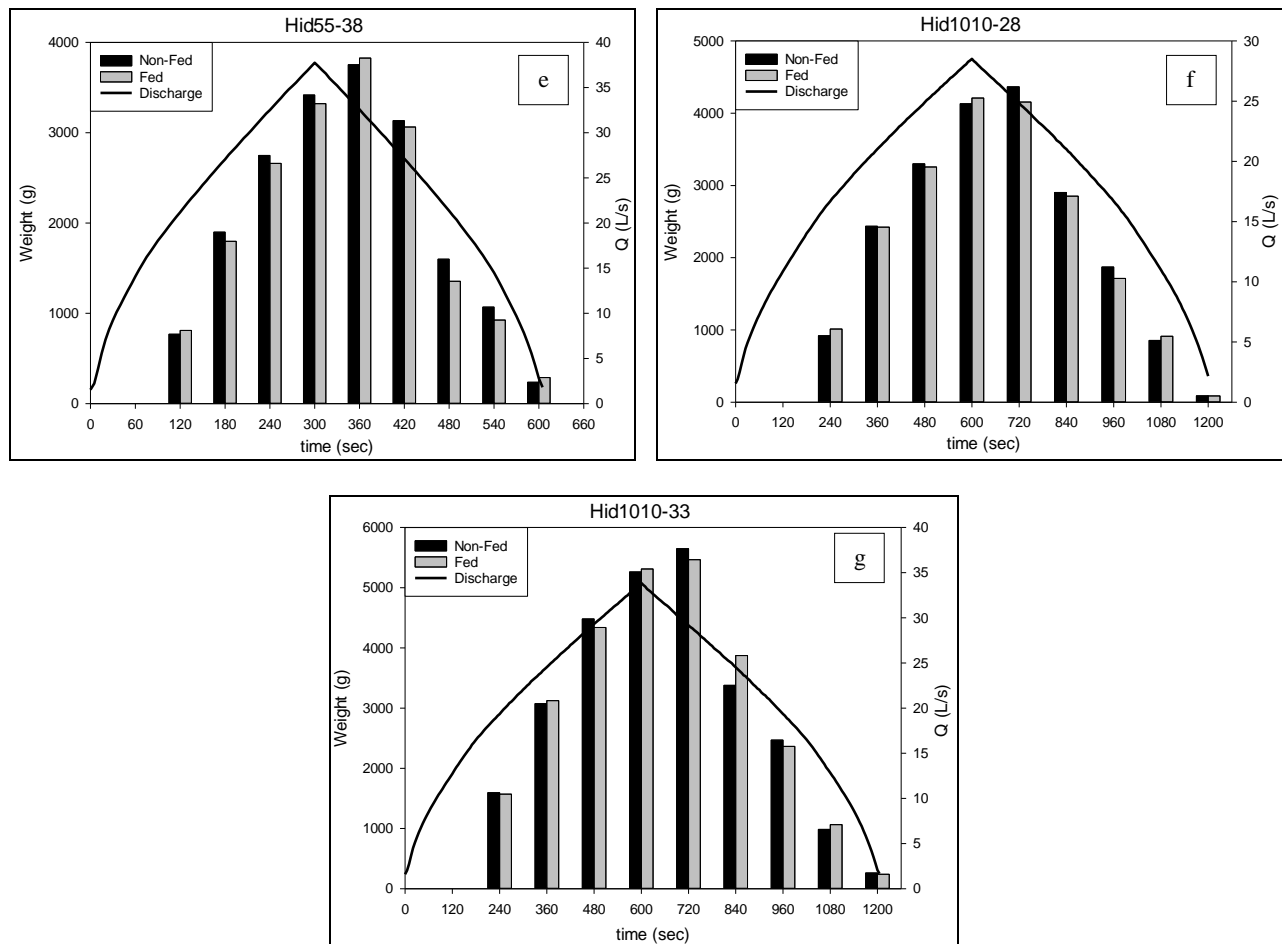


Figure 6. The amount of the total sediment load and discharge curve for each hydrograph

4. Conclusion

A large-scale experimental program of 42 runs was conducted to investigate the effects of sediment feeding on live-bed scour depth to provide contributions for the future experimental procedure. In the experiments with and without sediment feeding, the difference between the maximum and final scour depths were found negligibly small. In 86% of the experiments, the difference between the maximum scour depths were lower than 5%. In 90% of the experiments, the difference between the final scour depths were lower than 5%. The difference between the total sediment loads were found to be negligibly small as well. The major difference was the bed degradation at upstream and this degradation disappeared gradually in relatively short distance without affecting approaching flow parameters. Bed degradation at downstream was found to be negligible. These results are in accord with the findings of Wang et al. (2019) [13].

Although it is known that sediment feeding is necessary if the equilibrium scour depth is investigated, this may not be required in some experiments concerning temporal evolution of the scour depth. It can be thought that it is not necessary to feed the flow when there is sufficient sediment in a long enough flume because the flume will be able to feed itself from the upstream sediment deposit. In other words, a long enough flume with sufficient sediment may not require sediment feeding for an experiment that will take certain time. The predetermination of the flume's "self-feeding" capacity within the limits of the experiments can avoid undertaking the experiments with sediment feeding which is more difficult and time consuming. Further studies worth being performed by using different hydrographs and various bed materials to clarify the upstream feeding capacity of the flume. It would also be interesting to carry out some experiments with different feeding rates in order to determine the "self-feeding" concept.

5. Declarations

5.1. Author Contributions

Conceptualization, F.G. and M.S.G.; methodology, F.G. and M.S.G.; software, F.G.; validation, F.G. and M.S.G.; formal analysis, F.G.; investigation, F.G.; resources, F.G.; data curation, F.G.; writing—original draft preparation, F.G.; writing—review and editing, M.S.G.; visualization, F.G.; supervision, M.S.G.; project administration, M.S.G.; funding acquisition, M.S.G. All authors have read and agreed to the published version of the manuscript.

5.2. Data Availability Statement

The data presented in this study are available on request from the corresponding author.

5.3. Funding

The authors thank Dokuz Eylul University for the financial support provided through the scientific research project BAP (Project Number 2016.KB.FEN.003).

5.4. Conflicts of Interest

The authors declare no conflict of interest.

6. References

- [1] Laursen, Emmett M., Toch, Arthur. Scour around bridge piers and abutments. Vol. 4. Ames, IA: Iowa Highway Research Board, (1956).
- [2] Jain, Subhash C., Fischer, Edward E. Scour around circular bridge piers at high Froude numbers (No. FHWA-RD-79-104 Final Rpt.), (1979).
- [3] Chiew, Yee-Meng. Local scour at bridge piers (Doctoral dissertation, ResearchSpace@ Auckland), (1984).
- [4] Kothyari, U. C., K. G. Ranga Raju, and R. J. Garde. "Live-Bed Scour around Cylindrical Bridge Piers." *Journal of Hydraulic Research* 30, no. 5 (September 1992): 701–715. doi:10.1080/00221689209498889.
- [5] Melville, Bruce W. "Live - bed Scour at Bridge Piers." *Journal of Hydraulic Engineering* 110, no. 9 (September 1984): 1234 – 1247. doi:10.1061/(asce)0733-9429(1984)110:9(1234).
- [6] Sheppard, D. Max, and William Miller. "Live-Bed Local Pier Scour Experiments." *Journal of Hydraulic Engineering* 132, no. 7 (July 2006): 635–642. doi:10.1061/(asce)0733-9429(2006)132:7(635).
- [7] Yanmaz, Ali M. "Köprü Hidroliği" METU Press, (October, 2002)
- [8] Marion, Andrea, Matteo Tregnaghi, and Simon Tait. "Sediment Supply and Local Scouring at Bed Sills in High-Gradient Streams." *Water Resources Research* 42, no. 6 (June 2006). doi:10.1029/2005wr004124.
- [9] Singer, Michael Bliss. "Downstream Patterns of Bed Material Grain Size in a Large, Lowland Alluvial River Subject to Low Sediment Supply." *Water Resources Research* 44, no. 12 (December 2008). doi:10.1029/2008wr007183.
- [10] Ferrer-Boix, Carles, and Marwan A. Hassan. "Influence of the Sediment Supply Texture on Morphological Adjustments in Gravel-Bed Rivers." *Water Resources Research* 50, no. 11 (November 2014): 8868–8890. doi:10.1002/2013wr015117.
- [11] Pfeiffer, Allison M., Noah J. Finnegan, and Jane K. Willenbring. "Sediment Supply Controls Equilibrium Channel Geometry in Gravel Rivers." *Proceedings of the National Academy of Sciences* 114, no. 13 (March 13, 2017): 3346–3351. doi:10.1073/pnas.1612907114.
- [12] Hong, Jian-Hao, Yee-Meng Chiew, Po-Hung Yeh, and Hsun-Chuan Chan. "Evolution of Local Pier-Scour Depth with Dune Migration in Subcritical Flow Conditions." *Journal of Hydraulic Engineering* 143, no. 4 (April 2017): 04016098. doi:10.1061/(asce)hy.1943-7900.0001261.
- [13] Wang, Le, Alan Cuthbertson, Gareth Pender, and Deyu Zhong. "Bed Load Sediment Transport and Morphological Evolution in a Degrading Uniform Sediment Channel under Unsteady Flow Hydrographs." *Water Resources Research* 55, no. 7 (July 2019): 5431–5452. doi:10.1029/2018wr024413.
- [14] Gumgum, Firat, and Mehmet Sukru Guney. "Time Dependent Live-Bed Scour Around Circular Piers under Flood Waves." *Periodica Polytechnica Civil Engineering* (January 1, 2020). doi:10.3311/ppci.14664.
- [15] Guney, M. Sukru, Gokcen Bombar, Aysegul O. Aksoy, and Mustafa Dogan. "Use of UVP to Investigate the Evolution of Bed Configuration." *KSCE Journal of Civil Engineering* 17, no. 5 (June 28, 2013): 1188–1197. doi:10.1007/s12205-013-0131-5.
- [16] Melville, Bruce W., and Yee-Meng Chiew. "Time Scale for Local Scour at Bridge Piers." *Journal of Hydraulic Engineering* 125, no. 1 (January 1999): 59–65. doi:10.1061/(asce)0733-9429(1999)125:1(59).
- [17] Melville, Bruce W. "Pier and Abutment Scour: Integrated Approach." *Journal of Hydraulic Engineering* 123, no. 2 (February 1997): 125–136. doi:10.1061/(asce)0733-9429(1997)123:2(125).
- [18] Yen, Chin-lien, and Kwan Tun Lee. "Bed Topography and Sediment Sorting in Channel Bend with Unsteady Flow." *Journal of Hydraulic Engineering* 121, no. 8 (August 1995): 591–599. doi:10.1061/(asce)0733-9429(1995)121:8(591).
- [19] Lee, Kwan Tun, Yi-Liang Liu, and Kai-Hung Cheng. "Experimental Investigation of Bedload Transport Processes Under Unsteady Flow Conditions." *Hydrological Processes* 18, no. 13 (August 27, 2004): 2439–2454. doi:10.1002/hyp.1473.
- [20] Bombar, Gökçen, Şebnem Elçi, Gokmen Tayfur, M. Şükrü Güney, and Aslı Bor. "Experimental and Numerical Investigation of Bed-Load Transport Under Unsteady Flows." *Journal of Hydraulic Engineering* 137, no. 10 (October 2011): 1276–1282. doi:10.1061/(asce)hy.1943-7900.0000412.
- [21] Graf, W. H., Suszka, L. "Unsteady flow and its effect on sediment transport." 21st IAHR Congress, (1985).