



A Novel Macroscopic Traffic Model based on Distance Headway

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Abstract

A new model is proposed to characterize changes in traffic at transitions. These changes are affected by driver response. The distance headway between vehicles is considered as it affects driver behavior. Driver response is quick with a small distance headway and slow when the distance headway is large. The variations in traffic are greater with a slow driver while traffic is smooth with a quick driver. A model is developed which characterizes traffic based on driver response and distance headway. This model is compared with the well-known and widely employed Zhang and PW models. The Zhang model characterizes driver response at transitions using an equilibrium velocity distribution and ignores distance headway and driver response. Traffic flow in the PW model is characterized using only a velocity constant. Roe decomposition is employed to evaluate the Zhang, PW, and proposed models over a 270 m circular (ring) road. Results are presented which show that Zhang model provides unrealistic results. The corresponding behavior with the proposed model has large variations in flow with a slow driver but is smooth with a quick driver. The PW model provides smooth changes in flow according to the velocity constant, but the behavior is unrealistic because it is not based on traffic physics.

Keywords: Macroscopic Traffic; Headway; Driver Response; Zhang Model; PW Model.

1. Introduction

Effective traffic control reduces congestion and creates a smooth flow [1]. This control requires realistic characterization of traffic behavior during transitions. This behavior is based on factors such as the time for alignment, distances between vehicles, and driver response. Distance headway is the distance required for traffic alignment during transitions. This distance is small during congestion and there are significant interactions between vehicles. Conversely, the distance headway is large in free flow traffic and there are few vehicle interactions. Traffic characterization should also consider the physiological driver response. An aggressive driver has a quick response so the distance headway is small while a sluggish driver responds slowly so the distance headway is large and changes in flow are greater [2]. Further, driver response varies depending on the forward conditions [3] and this should be considered in developing traffic flow models.

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Macroscopic traffic models consider the aggregate behavior of vehicles and provide good performance with low complexity. These models employ parameters such as average velocity, traffic density, and flow. The macroscopic model created by Lighthill, Whitham and Richards (LWR) is based on small changes in density which are characterized by variations in traffic velocity [4]. This simple model is commonly employed for traffic modeling [1, 5], but it ignores traffic alignment at transitions as well as driver response. Further, the traffic flow is symmetric when the capacity is 0.5 which is not realistic [6], and driver time to adjust to changes in traffic is ignored [7, 8]. It is adequate for conditions such as stop and go traffic. Payne and Whitham (PW) [9, 10] independently developed a model which considers driver response to density changes while adjusting vehicle speeds to a desired level [11]. This model assumes vehicles have similar behavior [12] and only small changes in traffic occur [13]. Further, temporal and spatial variations in speed and density are continuous. However, driver response is assumed to be constant for all forward conditions which can result in unpredictable behavior when there are large changes in traffic [7]. In some cases, the velocity can become negative which is impossible.

Del Castillo et al. [14] accurately modeled small variations in velocity and density, but large changes in traffic can produce unrealistic behavior [12]. The model developed by Aw and Rascle [15] considers driver presumption as an increasing function of density. Thus, there is greater acceleration and deceleration for a larger forward density but velocity (speed) and driver response are ignored. A macroscopic model was proposed by Berg, Mason and Woods which is known as the BMW model [16]. It employs distance headway and the spatial second derivative of density to characterize traffic. This derivative is used to reduce the changes at large transitions, but this can result in unrealistic behavior. The BMW model has been improved by including higher order terms, but the resulting models are inadequate because they ignore traffic physics [13]. Zhang (2002) [17] considered driver anticipation of changes in velocity during alignment. Thus, the PW model velocity constant is replaced with a density dependent velocity term to improve traffic characterization. Unfortunately, large changes in density can produce inaccurate results because driver response is ignored.

The PW model has been modified to incorporate the distance headway and velocity at transitions [18, 19]. This results in faster alignment with a small headway, but the relationship between distance headway and driver response is ignored. The impact of rearward vehicles considering density, velocity and time headway was considered in [20] to obtain a model which is more realistic than the Zheng and PW models. The Zheng model uses a constant rearward velocity factor for all traffic conditions to smooth the flow, so variations in these conditions are ignored. Heterogeneous traffic is difficult to characterize because lane discipline is not followed. This traffic was investigated in [21] using the sideways distance between vehicles. This is the first model to incorporate sideways distance. It was shown that this model performs better with heterogeneous traffic than the PW, Zhang and extended speed gradient (ESG) models [21]. The ESG model is inadequate because driver presumption is the same for all conditions. Traffic in adverse conditions such as ice and snow was characterized in [22] considering the friction between the tires and road. This is the first model to incorporate the impact of adverse weather on traffic.

Traffic models should be based on realistic parameters related to traffic physics [23] to adequately characterize traffic flow. For example, driver anticipation can be used for accurate spatial traffic adjustments [24]. A new model is proposed in this paper which employs driver response. Anticipation is characterized using the distance required for vehicle alignment, velocity gradient with respect to density, and driver response time. The proposed model is compared with the PW and Zhang models considering an inactive bottleneck on a 270 m circular road. An inactive bottleneck is commonly employed to investigate traffic models because it represents very adverse conditions. The remainder of this paper is organized as follows. The PW, Zhang, and proposed models are given in Section 2. These models are evaluated in Section 3 using the Roe decomposition technique for numerical discretization. Finally, Section 4 concludes the paper.

2. Traffic Flow Models

The spatial and temporal characterization of traffic should be based on parameters such as driver response [2, 5]. Real observations have been used to determine driver response to traffic conditions [25]. In this section, a realistic traffic flow model is proposed and compared with state-of-the-art approaches [3, 22]. This improves on previous models in the literature [5, 18, 23]. The methodology employed in developing this model is shown in Figure 1. The proposed and other models are presented below.

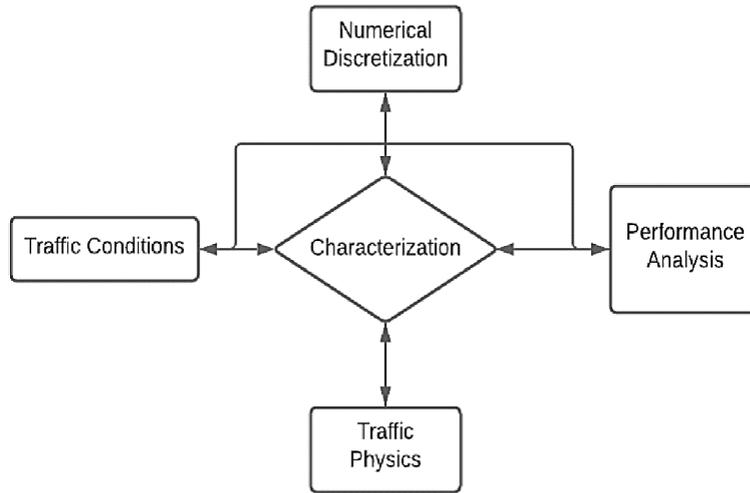


Figure 1. The methodology employed in developing the proposed model

The PW model assumes driver response to forward conditions is fixed which is unrealistic. It is given by:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial v}{\partial t} + \frac{v \partial v}{\partial x} = -\frac{C_0^2}{\rho} \frac{\partial \rho}{\partial x} + \left(\frac{v(\rho) - v}{\tau} \right) \tag{2}$$

where $v(\rho)$ denotes the equilibrium velocity distribution, $(v(\rho) - v)/\tau$ is the relaxation term which characterizes traffic alignment due to changes in velocity during time τ , and $\frac{C_0^2}{\rho} \frac{\partial \rho}{\partial x}$ is the driver presumption to forward changes with velocity constant C_0 . This constant ignores driver behavior. The Zhang model considers density dependent velocity changes at transitions and can be expressed as:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0 \tag{3}$$

$$\frac{\partial \rho v}{\partial t} + \frac{v \partial v}{\partial x} + \frac{(\rho \frac{\partial v(\rho)}{\partial \rho})^2}{\rho} \frac{\partial \rho}{\partial x} = \rho \frac{v(\rho) - v}{\tau} \tag{4}$$

Note that the Zhang and PW models have the same relaxation term [26]. Several distributions have been proposed for $v(\rho)$, but the Greenshields distribution [27, 28] is the most commonly employed and is given by:

$$v(\rho) = v_m \left(1 - \frac{\rho}{\rho_m} \right) \tag{5}$$

where ρ_m is the maximum density and v_m is the maximum velocity. With this model, the velocity is inversely proportional to the density. The anticipation term in the Zhang model is based on changes in the equilibrium velocity distribution and can be expressed as $\frac{(\rho \frac{\partial v(\rho)}{\partial \rho})^2}{\rho} \frac{\partial \rho}{\partial x}$. Note that is inversely proportional to the density. The velocity constant in the PW model is replaced with $\rho \frac{\partial v(\rho)}{\partial \rho}$. Thus, driver response depends only on the velocity profile and ignores the distance required for vehicle alignment. Therefore, the Zhang model does not adequately characterize driver response.

Drivers make adjustments based on the velocity of forward vehicles [29]. The changes in velocity are greater with an aggressive behavior, but these changes depend on the traffic density. Thus, changes in velocity can be characterized using:

$$\frac{\partial v(\rho)}{\partial \rho} \tag{6}$$

Traffic alignment occurs during the relaxation time τ over the distance headway d , and the corresponding change in distance headway can be expressed as:

$$d_c = \tau \frac{\partial v(\rho)}{\partial \rho} \tag{7}$$

The distance headway will be large for a sluggish driver with a slow physiological response and vice versa for an aggressive driver with a fast response. The driver response can be expressed as:

$$\gamma = \frac{\tau_a}{\tau} \quad (8)$$

where τ_a is the response time. For a slow response, $\tau_a > \tau$ so $\gamma > 1$, while for a quick response $\tau_a < \tau$ so $\gamma < 1$. For a typical physiological response, $\tau_a = \tau$ so $\gamma = 1$. The distance headway can then be given by:

$$d = \gamma(d_s + d_c) \quad (9)$$

where d_s is the safe distance headway which should be maintained to avoid accidents. Substituting d_c from Equation 7 and γ from Equation 8 gives:

$$d = \frac{\tau_a}{\tau} \left(d_s + \tau \left| \frac{\partial v(\rho)}{\partial \rho} \right| \right) \quad (10)$$

The spatial change in headway during transitions is then:

$$\frac{\partial}{\partial x} d = \frac{\partial}{\partial x} \left(\frac{\tau_a}{\tau} \left(d_s + \tau \left| \frac{\partial v(\rho)}{\partial \rho} \right| \right) \right) \quad (11)$$

The proposed model is obtained by replacing $\left(\rho \frac{\partial v(\rho)}{\partial \rho} \right)^2$ in Equation 4 with Equation 11. The performance of the proposed, PW, and Zhang models is investigated in the next section.

3. Performance Evaluation

In this section, Roe decomposition is used to approximate the traffic flow with the proposed, PW, and Zhang models [26]. A circular road is considered with periodic boundary conditions to ensure that traffic evolves along the road throughout the simulations. The proposed and PW models are evaluated for 30 s while the Zhang model is evaluated for only 0.12 s due to the high computational complexity. Greenshields equilibrium velocity (speed) distribution given by Equation 5 is considered with $v_m = 20$ m/s. Small values of τ close to 0 result in large variations in traffic while realistic driver behavior corresponds to larger values of τ and a smooth traffic flow. Thus, $\tau = 1.0$ is suitable for transitions over small distances [30]. In this paper, $\tau_a = 1.9$ s, 0.75 s, and 1 s, are used to represent slow, quick and typical drivers, respectively. $\tau_a = 1.9$ s indicates a response time that is 90% times larger than normal, so the corresponding driver is sluggish, while $\tau_a = 0.75$ s indicates a response time that is 25% times smaller than normal, so the corresponding driver is aggressive.

The road of length $x_M = 270$ m is divided into $M = 10$ equal segments ($\delta x = 27$ m), for the proposed and PW models, and $M = 18$ equal segments ($\delta x = 15$ m), for the Zhang model. An appropriate time step for numerical evaluation must be chosen based on the road length and computational complexity. The CFL condition ensures that changes in the traffic flow are accurately captured [31]. To satisfy this condition, $\delta t = 1$ s is chosen for the proposed and PW models, and $\delta t = 0.001$ s for the Zhang model. Therefore, $t_N = 30$ s for the proposed and PW models is divided into $N = 30$ steps, while $t_N = 0.12$ s for the Zhang model is divided into $N = 120$ steps. The initial density distribution on the road at time $t = 0$ s is:

$$\rho_0 = \begin{cases} 0.2, & \text{for } x \leq 150, \\ 0.5, & \text{for } x > 150. \end{cases} \quad (12)$$

An abrupt change in density is considered to observe the traffic characterization with large changes. The behavior with small changes will be smooth. The maximum density is $\rho_m = 1$ which means that the road is 100% occupied.

The velocity for the proposed model over a circular road of length 270 m at 1, 10, 20 and 30 s with $\gamma = 1.9$ (slow response) is given in Figure 2. At $t = 1$ s, the velocity is 16.0 m/s from 0 to 140 m and 10.0 m/s from 160 to 270 m. At $t = 10$ s, the velocity changes from 13.8 m/s at 0 m to 12.2 m/s at 25 m, and then to 15.2 m/s at 55 m. It is 12.8 m/s at 80 m and 15.5 m/s at 110 m. The maximum velocity is 15.5 m/s whereas the minimum velocity is 11.2 m/s. At $t = 20$ s, the velocity changes from 13.5 m/s at 0 m to 12.2 m/s at 25 m, and then to 14.4 m/s at 55 m. It is 11.8 m/s at 80 m, 14.8 m/s at 110 m, and 11.2 m/s at 140 m. The velocity is then 14.8 m/s at 160 m, 11.2 m/s at 180 m, 13.0 m/s at 210 m, 12.5 m/s at 240 m, and 12.0 m/s at 270 m. The maximum velocity is 14.8 m/s whereas the minimum velocity is 11.2 m/s. At $t = 30$ s, the velocity is 11.2 m/s at 0 m, 14.2 m/s at 25 m, 12.5 m/s at 55 m, and 13.4 m/s at 80 m. It is 13.6 m/s at 110 m, 14.6 m/s at 160 m, 11.0 m/s at 180 m, 14.8 m/s at 210 m, 10.8 m/s at 240 m, and then 14.8 m/s at 270 m. The maximum velocity is 14.8 m/s whereas the minimum velocity is 10.8 m/s. These results show that with a large value of γ there are large changes in velocity over small distances. Over time, the changes in velocity grow. The maximum change in velocity is 37% from 240 m to 270 m at 30 s, while the minimum change is 1.5% from 140 m to 160 m at 10 s. Thus, the changes in velocity with $\gamma = 1.9$ are between 1.5% and 37% over a distance of 30 m. This is typical behavior during congestion as the changes are drastic. A driver takes longer to align to forward conditions and the distance between vehicles is covered slowly.

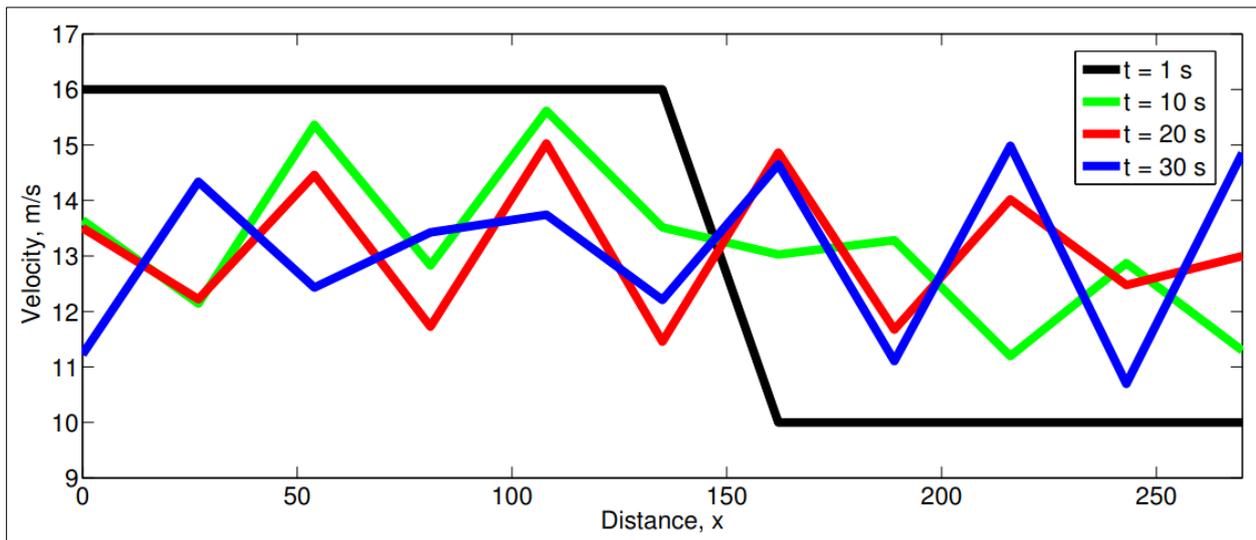


Figure 2. The velocity for the proposed model on a circular road of length 270 m with $\tau_a = 1.9$ s, $\tau = 1.0$ s, and $\gamma = 1.9$, so the driver response is 90% larger than the relaxation time, which represents a sluggish driver

The velocity for the proposed model at 1, 10, 20 and 30 s with $\gamma = 0.75$ (quick response) is given in Figure 3. At $t = 1$ s, the velocity 16.0 m/s from 0 to 140 m and 20.0 m/s from 160 to 270 m. At $t = 10$ s, the velocity is 12.3 m/s at 0 m, 13.5 m/s at 25 m, 14.8 m/s at 55 m, and 15.7 m/s at 80 m. It is 16.2 m/s at 110 m, 15.9 m/s at 140 m, 13.4 m/s at 160 m, 11.5 m/s at 180 m, 10.9 m/s at 210 m, 10.5 m/s at 240 m, and 11.0 m/s at 270 m. The maximum velocity is 16.2 m/s whereas the minimum velocity is 10.5 m/s. At $t = 20$ s, the velocity changes from 12.0 m/s at 0 m to 12.5 m/s at 25 m. It is 13.4 m/s at 55 m, 14.0 m/s at 80 m, 14.8 m/s at 110 m, 15.0 m/s at 140 m, 14.8 m/s at 160 m, 13.8 m/s at 180 m and then 12.2 m/s at 210 m. The velocity is 11.7 m/s at 240 m and 11.5 m/s at 270 m. The maximum velocity is 15.0 m/s whereas the minimum velocity is 11.5 m/s. At $t = 30$ s, the velocity changes from 12.2 m/s at 0 m to 12.3 m/s at 25 m. It is 12.5 m/s at 55 m, 13.4 m/s at 80 m, 13.7 m/s at 110 m, 14.0 m/s at 140 m, 14.3 m/s at 160 m and then to 14.0 m/s at 180 m. The velocity is 13.9 m/s at 210 m, 12.5 m/s at 240 m, and 12.2 at 270 m. The maximum velocity is 14.3 m/s whereas the minimum velocity is 12.2 m/s. As expected, with a smaller driver response the changes in velocity are smoother. Over time, the changes in velocity decrease. The maximum change in velocity is 9.75% from 0 m to 25 m at 10 s, while the minimum change is 0.08% from 0 m to 25 m at 30 s. Thus, the changes in velocity with $\gamma = 0.75$ are between 0.08% are 9.8% over a distance of 30 m. At 10 s, the change in velocity from 0 m to 110 m is 33.3%, while at 20 s it is 23.3%. At 30 s, the change in velocity from 0 m to 110 m is 12.3%. This is typical as traffic dissemination on a highway is rapid. Drivers quickly align to forward conditions and the distance between the vehicles is covered quickly.

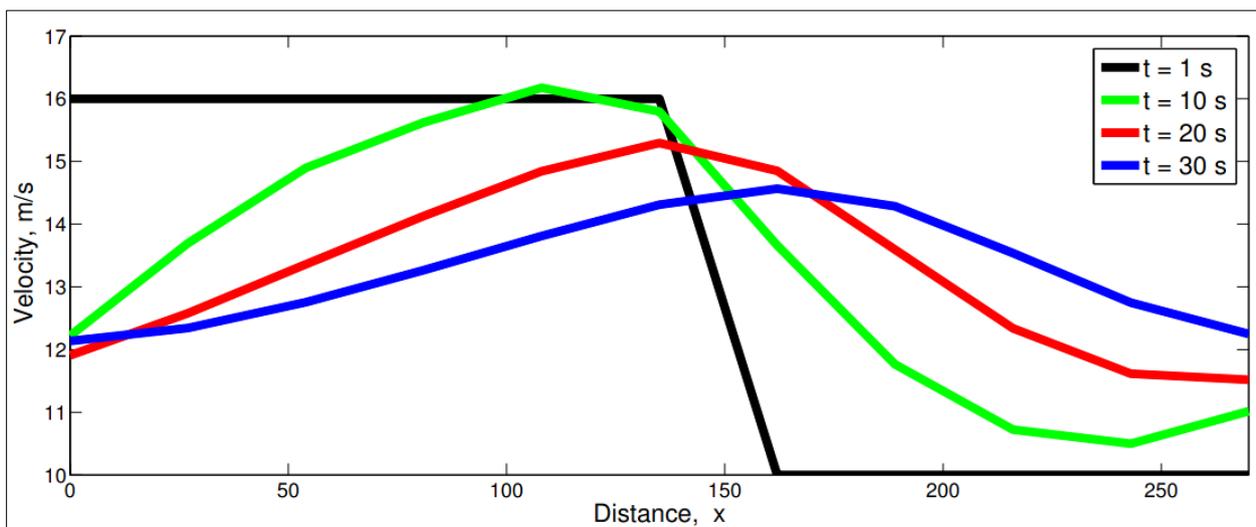


Figure 3. The velocity for the proposed model on a circular road of length 270 m with $\tau_a = 0.75$ s, $\tau = 1.0$ s, and $\gamma = 0.75$, so the driver response is 25% smaller than the relaxation time, which represents an aggressive driver

The velocity for the proposed model at 1, 10, 20 and 30 s with $\gamma = 1.0$ is given in Figure 4. At $t = 1$ s, the velocity is 16 m/s from 0 to 140 m and 10 m/s from 160 to 270 m. At $t = 10$ s, the velocity changes from 12.3 m/s at 0 m to 13.0

m/s at 25 m. It is 14.2 m/s at 55 m, 15.5 m/s at 80 m, 15.8 m/s at 110 m, 14.8 m/s at 140 m, 14.0 m/s at 160 m, 12.0 m/s at 180 m, and then 11.2 m/s at 210 m. The velocity is 11.0 m/s at 240 m and 11.4 m/s at 270 m. The maximum velocity is 15.8 m/s whereas the minimum velocity is 11 m/s. At $t = 20$ s, the velocity changes from 12.3 m/s at 0 m to 12.5 m/s at 25 m. It is 13.4 m/s at 55 m, 14.0 m/s at 80 m, 14.4 m/s at 110 m, 14.6 m/s at 140 m, and 14.4 m/s at 160 m. The velocity is 13.8 m/s at 180 m, 12.5 m/s at 210 m, and 12.1 m/s from 240 m to 270 m. The maximum velocity is 14.6 m/s whereas the minimum velocity is 12.1 m/s. At $t = 30$ s, the velocity changes from 12.7 m/s at 0 m to 12.8 m/s at 25 m. It is 13.0 m/s at 55 m, 13.2 m/s at 80 m, 13.4 m/s at 110 m, 13.8 m/s at 140 m, 13.9 m/s at 160 m, and then 13.8 m/s at 180 m. The velocity is 13.2 m/s at 210 m, 13.0 m/s at 240 m, and 12.8 m/s at 270 m. The maximum velocity is 13.9 m/s whereas the minimum velocity is 11.8 m/s. The maximum change in velocity is 9.2% from 24 m to 55 m at 10 s, while the minimum change is 0% from 240 m to 270 m at 20 s. Thus, the changes in velocity with $\gamma = 1.0$ are between 0 to 9.2% over a distance of 30 m. At 10 s, the change in velocity from 0 m to 110 m is 28.5%, while at 20 s it is 17.1%. At 30 s, the change in velocity from 0 m to 110 m is 5.5%. As expected, the changes in velocity are between those with a larger and smaller driver response. The flow is smooth with a typical driver and the changes in velocity are less than with an aggressive driver.

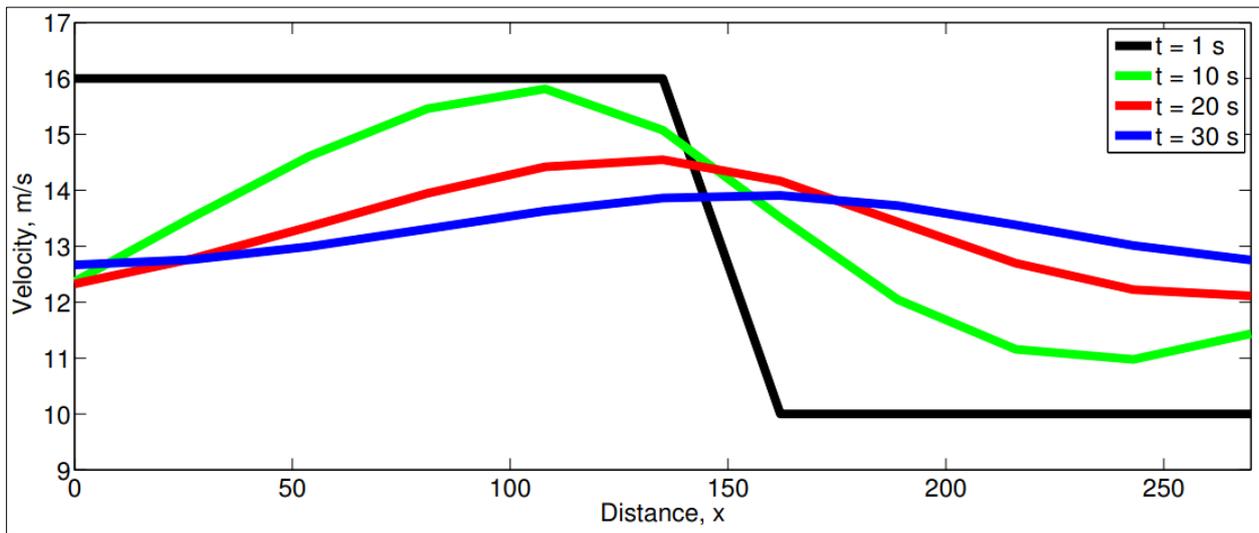


Figure 4. The velocity for the proposed model on a circular road of length 270 m with $\tau_a = 1$ s, $\tau = 1.0$ s, and $\gamma = 1.0$, so the driver response is the same as the relaxation time, which represents a typical driver

The velocity for the Zhang model on a circular road of length 270 m over 0.12 s is given in Figure 5. The maximum and minimum values are -8000 m/s and 1800 m/s, respectively, which indicates that the Zhang model performance is unrealistic.

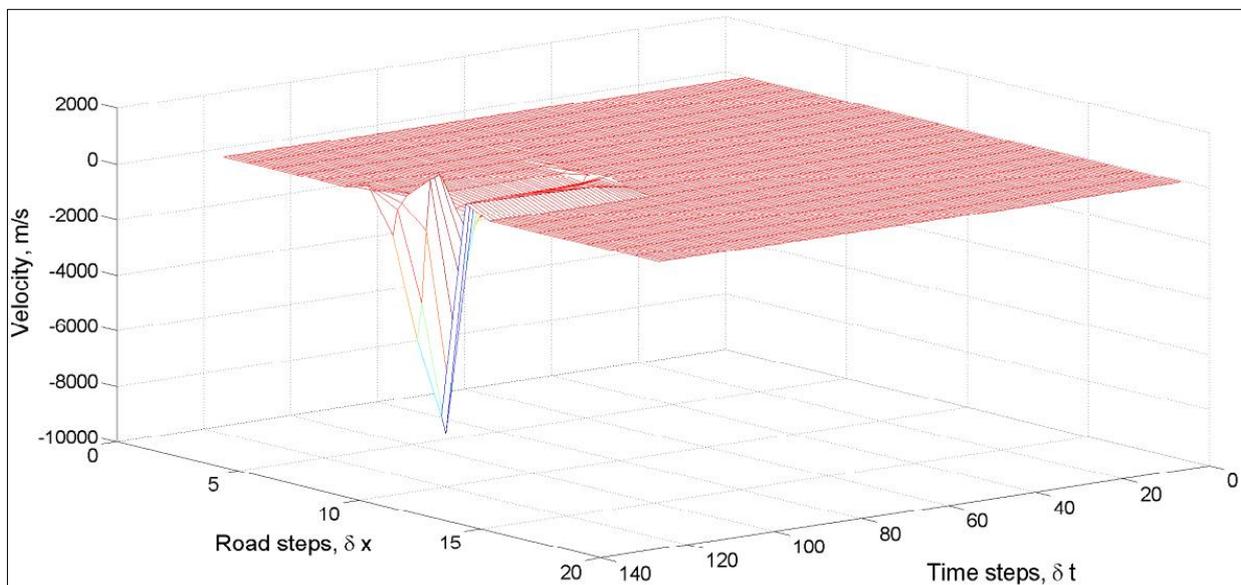


Figure 5. The velocity for the Zhang model on a circular road of length 270 m with $\tau = 1.0$ s

The velocity for the PW model on a circular road of length 270 m at 1, 10, and 30 s with $C_0 = 5$ m/s and 20 m/s is given in Figure 6. At 10 s, with $C_0 = 5$ m/s the maximum velocity is 15.8 m/s at 160 m while the minimum is 10.0 m/s at 245 m, and with $C_0 = 20$ m/s the maximum velocity is 14.9 m/s at 140 m while the minimum is 11.6 m/s at 270 m. The differences in maximum and minimum velocity are 0.9 m/s and 1.6 m/s, respectively. At 20 s, with $C_0 = 5$ m/s the maximum velocity is 15.0 m/s at 220 m while the minimum is 11.2 m/s at 0 m, and with $C_0 = 20$ m/s the maximum velocity is 14.0 m/s between 187 m and 204 m while the minimum is 12.6 m/s at 50 m. The differences in maximum and minimum velocity are 1.0 m/s and 1.4 m/s, respectively. At 30 s, with $C_0 = 5$ m/s the maximum velocity is 14.5 m/s at 270 m while the minimum is 12.0 m/s at 55 m, and with $C_0 = 20$ m/s the maximum velocity is 13.5 m/s at 0 m while the minimum is 13.0 m/s at 100 m. The differences in maximum and minimum velocity are 1.0 m/s.

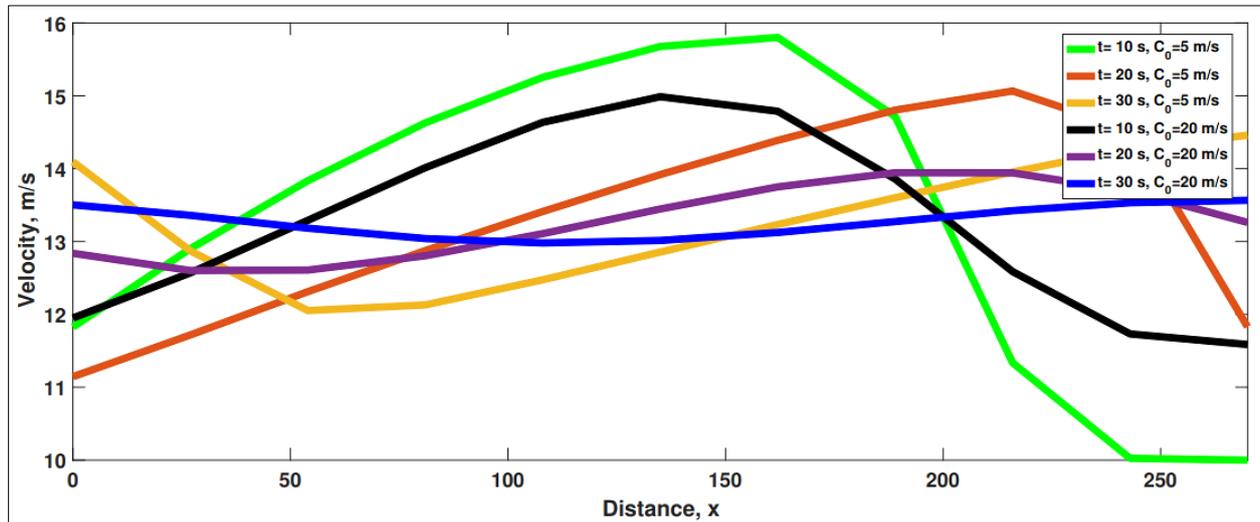


Figure 6. The velocity for the PW model on a circular road of length 270 m with $\tau = 1.0$ s and $C_0 = 5$ and 20 m/s

The PW traffic model employs a velocity constant C_0 which is not based on driver response. This constant is used to smooth changes in the flow and vehicles are assumed to have similar behavior, which is inadequate and unrealistic. Conversely, the proposed model explicitly considers driver response. In particular, the distance headway and time to align to forward vehicles is used to characterize traffic flow. There are larger changes in velocity with a slow driver response ($\gamma > 1$) than with a quick driver response ($\gamma < 1$). With a slow driver large changes in velocity occur over 30 m. With an aggressive driver, there are smaller changes over a distance of 110 m between 10 and 30 s than with a typical driver. The velocity with the Zhang model goes well beyond the maximum and minimum values, which is impossible. This model cannot adequately characterize traffic behavior because it ignores parameters such as driver response.

4. Conclusion

In this paper, results were presented which show that the traffic behavior with the well-known Zhang model can be unrealistic. For example, oscillations in the velocity were observed and values are negative and above the maximum. This is because the model considers density dependent velocity rather than parameters such as driver response. The proposed model provides a response that is realistic and within the minimum and maximum values because it incorporates driver response based on distance headway and driver physiology. Larger variations in velocity occurred with a slow driver response compared to a quick driver response, as expected. Further, the PW model employs a velocity constant to characterize traffic behavior rather than driver response, so it is not based on real traffic behavior. The proposed model can be used to improve public safety and reduce congestion and pollution by predicting traffic flow. Further, it can be employed in planning the deployment of traffic infrastructure.

5. Declarations

5.1. Author Contributions

Conceptualization, Z.H.K. and T.A.G.; methodology, Z.H.K.; software, K.S.K. and Z.H.K.; validation, Z.H.K., T.A.G., and K.S.K.; formal analysis, Z.H.K.; investigation, Z.H.K.; writing—original draft preparation, Z.H.K.; writing—review and editing, Z.H.K.; supervision, T.A.G.; project administration, T.A.G.; funding acquisition, T.A.G. All authors have read and agreed to the published version of the manuscript.

5.2. Data Availability Statement

Data sharing is not applicable to this article.

5.3. Funding and Acknowledgements

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5.4. Conflicts of Interest

The authors declare no conflict of interest

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