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# Novel Method for an Optimised Calculation of the Cross-Sectional Distribution of Live Loads on Girder Bridge Decks

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**Abstract**

One of the main goals in the design of girder bridge deck systems is to determine the cross-sectional distribution of live loads across the different girders that make up the cross-section of the deck. Structural grillage models and current bridge design standards based on a Load Distribution Factor (LDF) provide oversized designs, as demonstrated in this paper. This research introduces a novel method that allows the cross-sectional distribution of live loads on girder bridge decks to be calculated by applying a matrix formulation that reduces the structural problem to 2 degrees of freedom for each girder: the deflection and the rotation of the deck-slab at the centre of the girder's span. Subsequently, a parametric study is presented that analyses the structural response of 64 girder bridge decks to a total of 384 load states. In addition, the authors compare the outputs of the novel method with those obtained using traditional grillage calculation methods. Finally, the method is experimentally validated on two levels: a) a laboratory test that analyses the structural response of a small-scale girder bridge deck to the application of different load states; b) a real full-scale girder bridge load test that analyses the structural response of the bridge over the Barbate River during its static load test. Based on this analysis, the maximum divergence of the proposed method obtained from the experimental structural response is less than 10%. The use of the proposed novel analysis method undoubtedly provides significant savings in material resources and computing time, while contributing to minimizing overall costs.

*Keywords:* Cross-Sectional Load Distribution; Girder Bridge Decks; Small-Scale Bridge Test; Full-Scale Girder Bridge Deck Load Test; Optimised Matrix Method.

## 1. Introduction

Girder bridge decks are a structural typology commonly used in the design of road and railway bridges and therefore any optimization in their calculation has a profound impact on the project phase. As explained in [1], the design process relies solely on the designers' experience, intuition, and ingenuity, resulting in significant material costs, time, and human effort. Structural grillage models [2-4] are commonly used to calculate the cross-sectional distribution of live loads between the different girders making up the cross section of the deck [5-7]. Another way to approach the design

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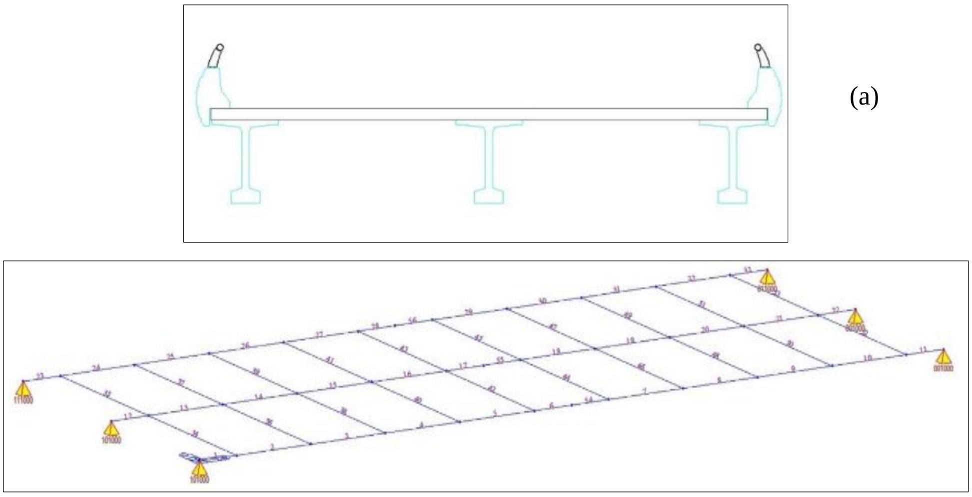
and calculation of such decks would be to apply different formulations [8-10] contained in the bridge design standards that approximate the cross-sectional distribution of the bending moment and shear stress caused by live loads through what is known as the load distribution factor (LDF) [11-13]. The LDF associated with each case study is conditioned by the type and number of girders, their spacing and length, and whether there are transverse diaphragms that provide the deck with transverse rigidity [14-16].

However, the LDF does not allow the distribution of bending stresses to be determined on all girders, and the design is therefore oversized. The need for a method to calculate the cross-sectional distribution across all types of girders, without resorting to complex structural grillage models or finite element models in specific structural calculation programs, or using approximate parametric methods based on the LDF, is one of the authors' motivations for the research work presented in this article.

A brief identification of the traditional methods used to determine the transverse distribution of live loads in girder bridge decks is presented in this article, followed by a simplified method to optimise this calculation. The result of a parametric study extended to 64 girder bridge decks and 384 load states is reported and the proposed method is validated through 2 experimental tests: a) a laboratory test on a small-scale model; b) a field test on a full-scale girder bridge deck during its static load test.

## 2. Current Methods for Girder Bridge Deck Analysis

The use of structural grillage models to analyse the cross-sectional distribution of girders began in the 1960s [2]. These models divide the girder deck into longitudinal and transverse girders (Figure 1). Longitudinal girders are responsible for providing the longitudinal bending stiffness of the deck, considering as many longitudinal girders as required to conform to the analysed girder deck. The structural section of each of the longitudinal girders is the result of the section composed of the girder analysed and the effective depth of the contributing deck excluding the latter girder [17, 18]. The cross-sectional distribution of the structural model is provided by the cross-girders and the torsional stiffness of the longitudinal girders. The structural section of the cross-girders corresponds to a rectangular section with a depth equivalent to the slab thickness and according to the discretization used in the grillage models. Analyzing such structural models involves the use of specific structural calculation programs that provide the computing power required to resolve the proposed matrix problem.



(a)

(b)

**Figure 1. Grillage discretization: (a) girder bridge deck cross section; (b) structural grillage model**

The use of structural grillage models allows the structural response of girder bridge decks to live loads to be obtained, which adequately manifest the cross-sectional distortion of the girder bridge deck and the distribution of stresses on each of the longitudinal girders making up the deck. However, such models involve complex, time-consuming analysis requiring specific structural calculation programs. As a result, simplified methods are needed for the start of the design process.

The LDF concept was first introduced by the American Association of State Highway Officials (AASHO) in 1931 [19].These methods roughly calculate the cross-sectional distribution on girder bridge decks using empirical formulas. The LDF is calculated from a series of formulations that parametrically treat the calculation of the percentage of bending moment and shear stress supported by the longitudinal girder under the most stress [20-22]. The parameters that condition the calculation of the LDF are the girder depth, span length, spacing, the number of girders, the position of the load, and the girder position. This method provides an approximate value of the maximum bending stresses on the girders but is not able to reproduce the cross-sectional distribution of longitudinal bending between all the girders that make up the deck.

## 3. Novel Cross-Sectional Load Distribution Calculation Method for a Girder Bridge Deck

A method is proposed that allows the cross-sectional distribution of live loads on girder bridge decks to be obtained without using empirical formulas of LDFs or complex structural models involving specific structural calculation programs. This method is based on using a virtual model that reflects the transverse stiffness of the slab deck, supported on a series of springs that provide the flexural stiffness Equation 1 and torsional stiffness Equation 2 of the longitudinal girders that make up the girder bridge deck (Figure 2).

$$K_{v,i} = \frac{48 \cdot EI_i}{L^3} \tag{1}$$

$$K_{t,i} = \frac{2 \cdot GJ_i}{L} \tag{2}$$

where $EI_i$ is longitudinal bending stiffness of girder "i", $GJ_i$ = longitudinal torsional stiffness of girder “”", and L is distance between bridge supports.

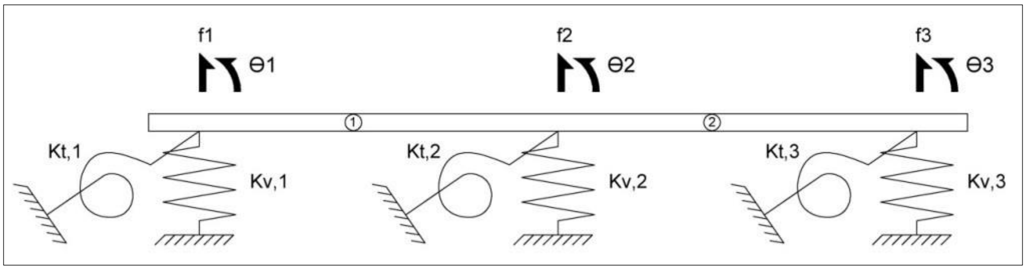



**Figure 2. Proposed method model for cross-sectional distribution on a girder bridge deck**

The proposed method considers 2 degrees of freedom for each longitudinal girder on the bridge deck: (1) the deflection and (2) the rotation of the deck-slab at the centre of the girder’s span. Figure 2 represents the structural model scheme for a girder bridge deck composed of three longitudinal girders. The matrix approach that solves the structural problem of the cross-sectional distribution of live loads between the different girders that make up the deck is set out in matrix Equations 3 to 5.

$$K_e = \frac{EI_e}{L_e^3}\begin{pmatrix} 6L & 4{L_e}^2 & -6L_e & 2{L_e}^2 \\ -12 & -6L_e & 12 & -6L_e \end{pmatrix} = \begin{pmatrix} K_{11,e} & K_{12,e} \\ K_{21,e} & K_{22,e} \end{pmatrix} \tag{3}$$

$$K_{Bi} = \begin{pmatrix} k_{v,i} & 0 \\ 0 & k_{t,i} \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} P_1 \\ M_{t,1} \\ P_2 \\ M_{t,2} \\ P_3 \\ M_{t,3} \end{pmatrix} = \begin{pmatrix} K_{11,1} + K_{B1} & K_{12,1} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ K_{21,1} & K_{22,1} + K_{11,2} + K_{B2} & K_{12,2} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & K_{21,2} & K_{22,2} + K_{B3} \end{pmatrix} \times \begin{pmatrix} f_{v,1} \\ \theta_1 \\ f_{v,2} \\ \theta_2 \\ f_{v,3} \\ \theta_3 \end{pmatrix} \tag{5}$$

Where $EI_e$ is transverse bending stiffness of element "e" of the upper deck slab, $L_e$ = length of element "e" of the upper deck slab, $f_{v,I}$ is vertical displacement experienced by longitudinal girder "i", and ϴi is transverse rotation of the bridge deck over the longitudinal girder "i".

The stiffness of the springs that represent the longitudinal bending and the torsional stiffness of the longitudinal girders that make up the model correspond to the equivalent stiffness at the centre of the span. However, the proposed method can be used to calculate the structural response of a girder bridge deck with a vertical load applied to any cross section. Using the Maxwell-Betti reciprocity theorem [23], to apply a point load of value "Q" at a distance "x" from one of the two support points of a longitudinal girder, the deflection at the centre of the span is obtained by applying the formulation reflected in Equation 6. Likewise, the proposed method can be used to calculate the transverse response of girderbridge decks to a vertical load applied at any point of the cross section. As with any matrix calculation, if the vertical load acts on a section of slab between longitudinal girders, the degrees of freedom of the structural model are locked and the reactions in the locked degrees of freedom are calculated (rigid step). Subsequently, the degrees of freedom are released and loaded with the reactions obtained in the previous step to obtain the vertical displacement of each of the longitudinal girders that make up the bridge deck (flexible step). The distribution of the maximum bending moment and maximum shear stress on the different longitudinal girders is obtained by Equations 7 and 8 respectively. The process to calculate the transverse distribution of live loads in girder bridge decks according to the method proposed by the authors is outlined in the flowchart shown in Figure 3.

$$f_{cl,Q(x),i} = f_{v,i} \cdot \sin\left(\frac{\pi \cdot x}{L}\right) \tag{6}$$

$$M_{fmáx,n} = \frac{Q \cdot (L-x) \cdot x}{L} \cdot \frac{f_{v,n}}{\sum_{i=1}^{N} f_{v,i}} \qquad x \leq L/2 \tag{7}$$

$$Q_{máx,n} = \frac{Q \cdot (L-x)}{L} \cdot \frac{f_{v,n}}{\sum_{i=1}^{N} f_{v,i}} \qquad x \leq L/2 \tag{8}$$

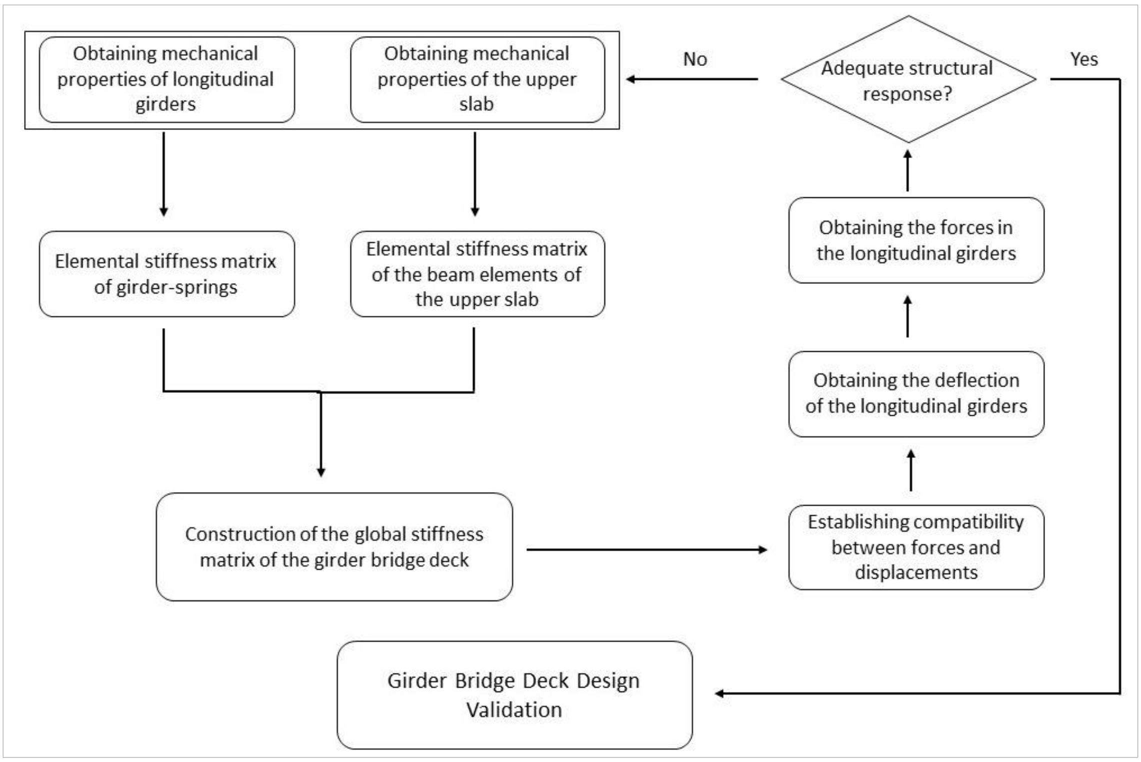



**Figure 3. Flowchart of the process to calculate the transverse distribution of mobile loads and design validation of the girder bridge deck**

## 4. Parametric Study and Discussion

### 4.1. Parametric Study

The cross-sectional response of girder bridge decks is influenced, among other factors, by the length, depth and number of longitudinal girders. A parametric study is presented (Table 1) that analyses the structural response of a total of 64 girder bridge decks. This parametric study compares the structural response obtained using traditional grillage calculation methods with respect to the structural response obtained from the proposed method.

**Table 1. Variables analysed in the parametric study**

| Length (m) | Depth (m) | Number of girders |
|---|---|---|
| 20 | 1.3 | 3 |
| 25 | 1.5 | 4 |
| 30 | 1.7 | 5 |
| 35 | 1.9 | 6 |

The parametric analysis reflects an adequate convergence of the structural response obtained with the calculation methods analysed. Following the analysis of 64 calculation models and 384 load states, the maximum divergence of structural models is less than 10%, with the average divergence of all load states analysed being less than 5% (Figure 4). The divergence between the structural response obtained by the model proposed by the authors and that obtained by traditional structural grillage methods is calculated by dividing the difference between the value obtained by one method and the value of the structural response obtained by the traditional structural grillage methods. Figure 4 shows the structural response of a six-girder bridge deck to two load states: an overload of 300 KN applied in the centre of the span of girder 1 and the same overload of 300 KN applied in the centre of the span of girder 3. The structural response obtained by applying the traditional structural grillage methods and applying the method proposed by the authors is contrasted in the graphs in Figure 4.

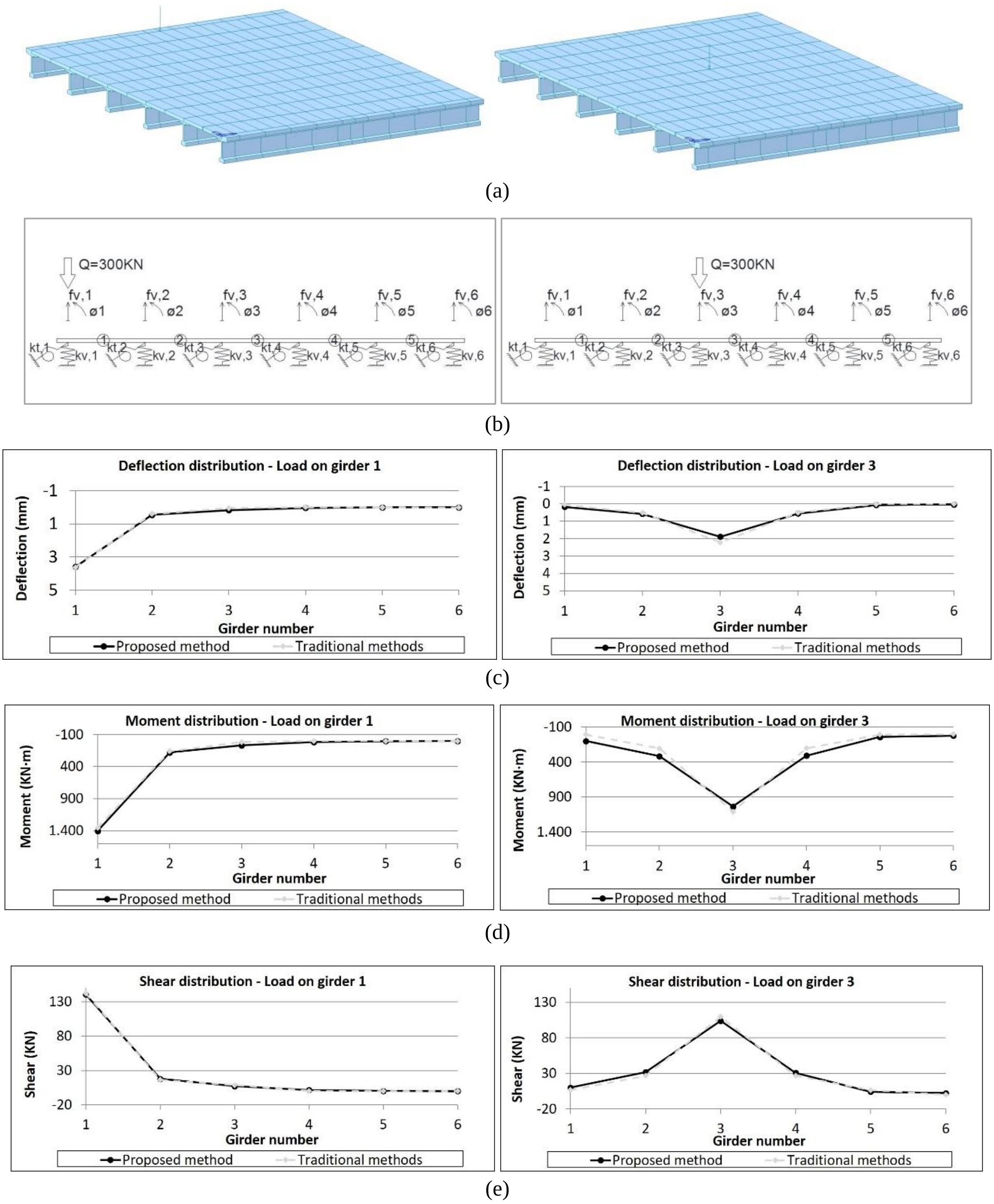



**Figure 4. Analysis of bridge with 6 longitudinal girders, a span of 20 metres and a depth of 1.9 metres: (a) structural grillage model; (b) proposed structural model; (c) deflection distribution on longitudinal girders; (d) moment distribution on longitudinal girders.**

## 4.2. Discussion

Figure 5 shows the evolution of the divergence between the proposed calculation model and the traditional structural grillage methods with the different variables analysed in the parametric analysis. The points reflected in the graphs in Figure 5 represent the divergence in each of the girder bridge deck models analysed in the parametric analysis. This divergence has been calculated as the ratio between the difference in the value of the analysed variable obtained by the proposed method and the corresponding value obtained by conventional calculation methods, with respect to the value of the variable obtained using conventional calculation methods. The following can be concluded from the analysis of the evolution of the divergence with each of the analysed variables: (a) the divergence increases with the length of the girder bridge deck; (b) the divergence decreases with the depth of the girder bridge deck; (c) the divergence decreases with the number of girders that make up the bridge deck.

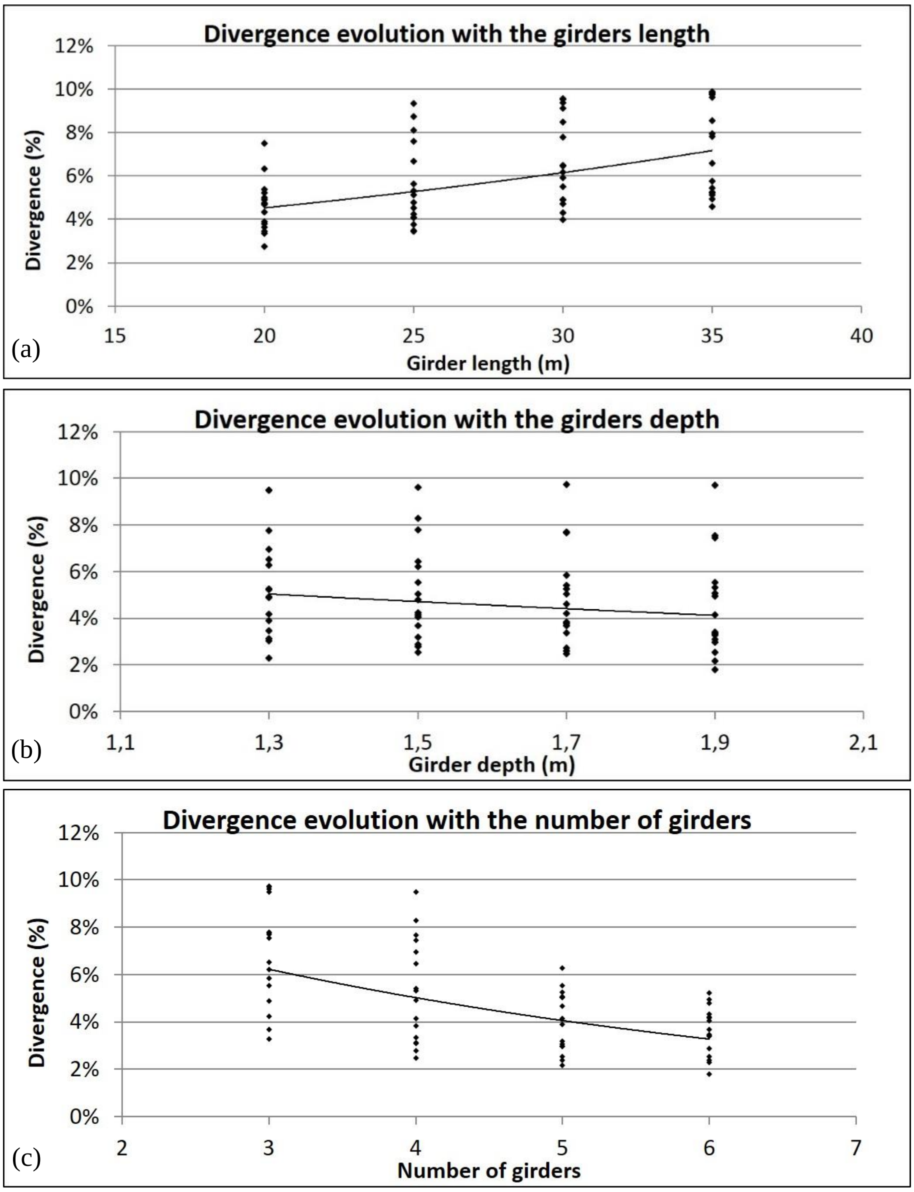



**Figure 5. Divergence evolution with each of the analysed variables: (a) girder length; (b) girder depth; (c) number of girders**

## 5. Validation

In this study, two tests are used to validate the method proposed by the authors to calculate the cross-sectional distribution of live loads in girder bridge decks.

### 5.1. Small-Scale Model Test in Laboratory

The authors used a small-scale model of a girder bridge deck (Figure 6) for a new validation, in this experimental case, of the proposed method for calculating the cross-sectional distribution of live loads in this type of bridge decks. The small scale model analysed consists of five girders with a depth of 86 mm, a span length of 1,560 mm, and a spacing of 150 mm. The model is made of methacrylate, as the relatively low elastic modulus of this material (E=3,300 MPa) facilitates the loading operation and allows the bearing devices of the structural model to be simplified. To empirically characterise the structural response of the small scale model, 2 groups of sensors are installed (Figure 7): (a) five potentiometer displacement transducer units to determine the value of the deflection experienced at mid span section; b) five strain gauge units to determine the deformation experienced [24-26] by the lower fibre of the girders at their mid span section. The acquisition, recording, and monitoring of the data provided by the sensors is done using a Structural Monitoring System (SMS) composed of the following elements: (a) a modular central data acquisition and processing unit (MCDA&PU) model NI-CDAQ-9188 with the capacity to simultaneously manage the signal from up to eight Data Acquisition Units (DAU); (b) a NI-9205 voltage DAU used to process analogue signals from potentiometric displacement transducers; (c) an extensometer DAU model NI-9237 that powers the Wheatstone Bridge's electronic assembly of extensometers and processes the analogue signal from these sensors [26, 27]; (d) a Power Supply (PS) that provides the voltage needed to power potentiometric displacement transducers; (e) a workstation responsible for communicating with the MCDA&PU and recording and viewing data provided by sensors through a Data Acquisition and Monitoring Program designed and programmed by the authors (Figure 8).

**Figure 6. Small-scale girder bridge model loaded at one third of the span of girder 1**

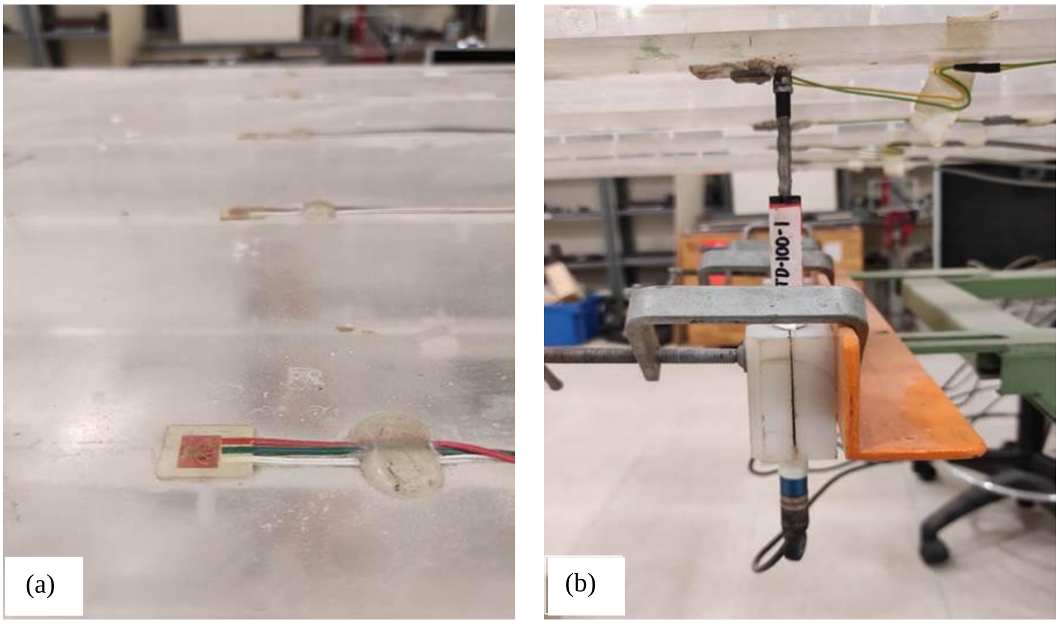
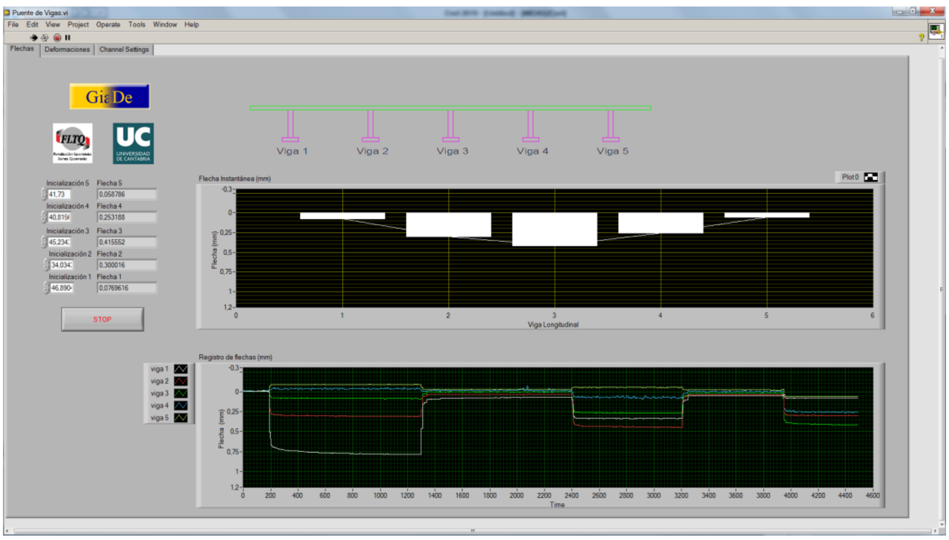



**Figure 7. Small-scale model sensor system: (a) strain gauge; (b) potentiometer displacement transducer**

(a)



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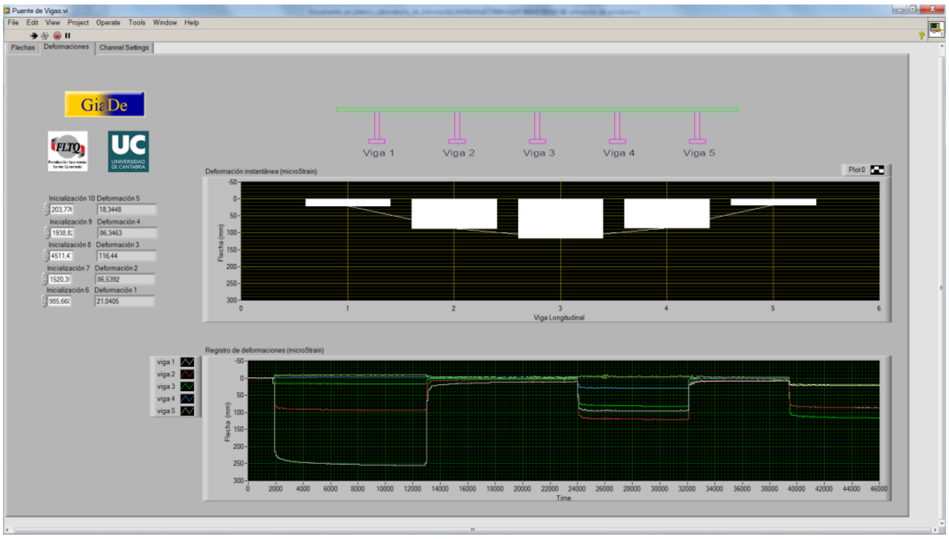
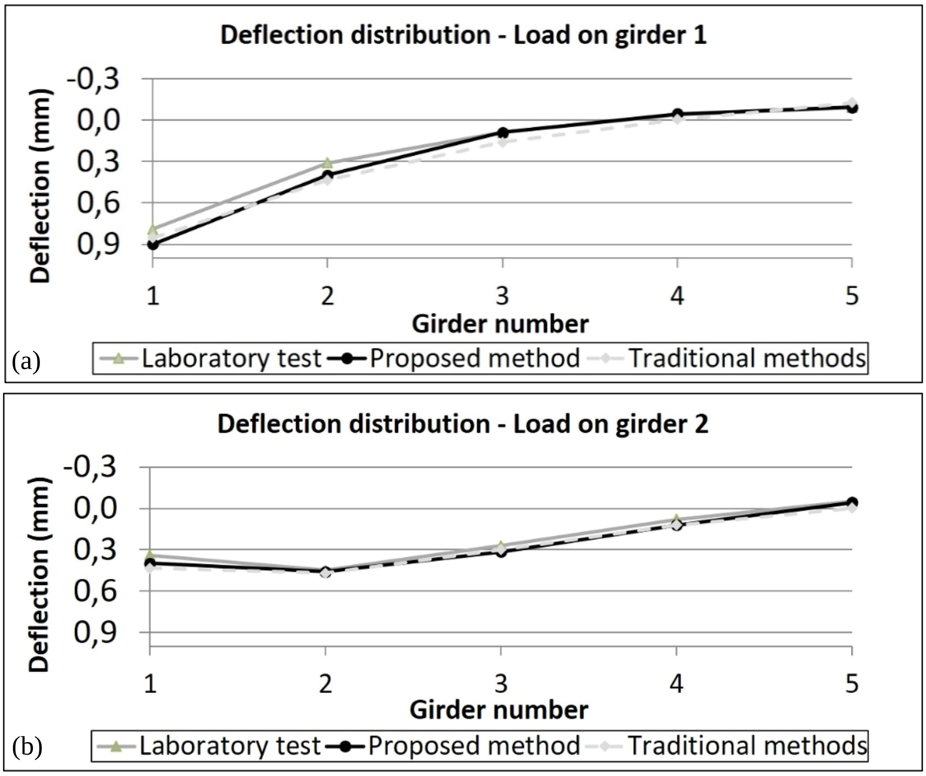



**Figure 8. Real-time visualisation of the data provided by the sensors: (a) real-time monitoring of the vertical deflection in the centre of longitudinal girders span; (b) real-time monitoring of deformation in the bottom fiber of longitudinal girders**

Three load tests were simulated, each consisting of applying a point load of 138 N at a distance corresponding to one third of the total length of longitudinal girders "1", "2", and "3" of the girder bridge. The girders "1", "2" and "3" of the bridge correspond to one of the end girders, its adjoining girder, and the central girder. In each of the load states analysed, a theoretical/empirical comparative analysis was carried out by comparing the experimental values obtained in the laboratory with those obtained from the analysis of 2 theoretical calculation models: a) the proposed method; b) a structural grillage model. The comparative analysis comprises the analysis of the deflections and the longitudinal deformation experienced by the lower fibre at mid-section for all girders.

### 5.1.1. Deflection Analysis at Girder Span Mid-Section - Laboratory Test

The comparative analysis of the structural response of the small-scale bridge model reflects an adequate adjustment of the structural response obtained from the theoretical models (Figure 9). The proposed method has a maximum deviation of less than 10% from the structural response of the laboratory model, while the structural response obtained through the application of traditional methods has a maximum deviation from the reduced model of 20%. Based on a theoretical/empirical comparative analysis, the structural response of the proposed method is well suited to the structural response of girder bridge decks.



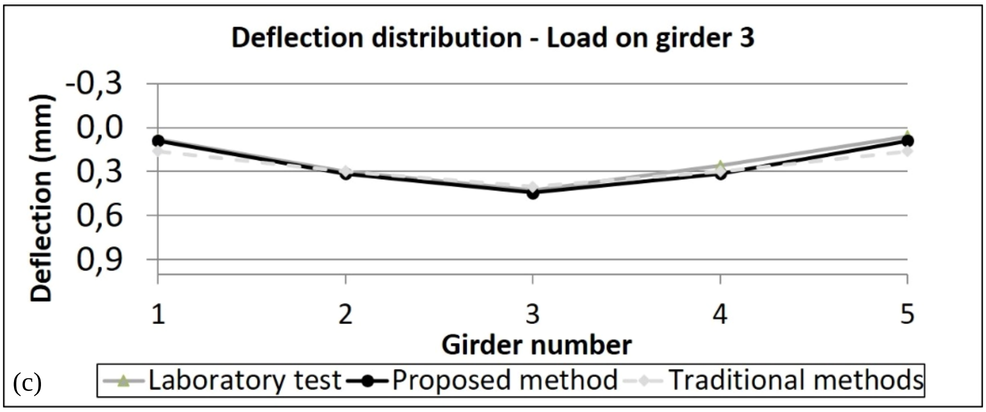


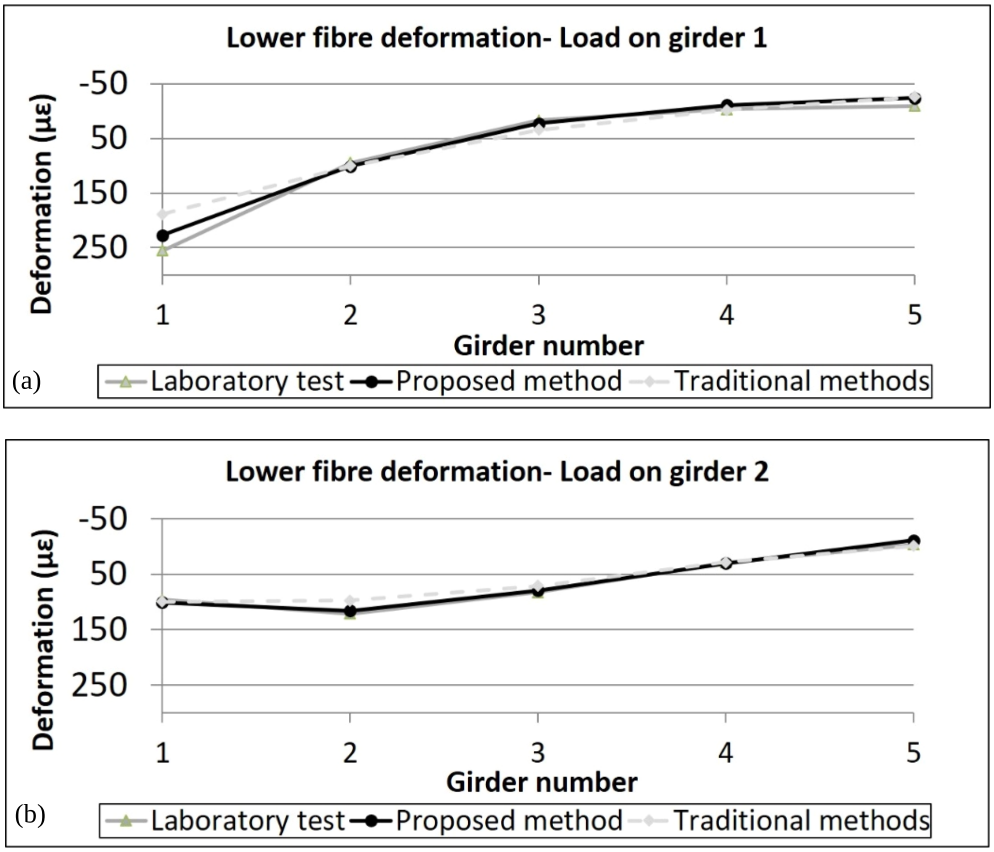

**Figure 9. Laboratory test. Theoretical/empirical comparative analysis. Deflection distribution: (a) load on girder 1; (b) load on girder 2; (c) load on girder 3**

### 5.1.2. Analysis of Longitudinal Deformation Experienced by the Bottom Fiber at Span Mid-section

The deformation experienced by the bottom flange at mid-section girder is obtained from the formulation expressed in Equation 9.

$$\varepsilon_n = \frac{Q \cdot L \cdot c'}{6 \cdot EI_n} \cdot \frac{f_{vn}}{\sum_{i=1}^{N} f_{vi}} \tag{9}$$

Where c´ is Distance from the barycentric fibre to the bottom fibre of longitudinal girder "n". The results obtained from the analysis of the theoretical models and those provided by the small-scale bridge monitoring system have an adequate adjustment (Figure 10). Specifically, the maximum deviation applying the proposed method is less than 10%, while traditional methods have a maximum deviation closer to 20% from the deformations experienced by the small-scale bridge.



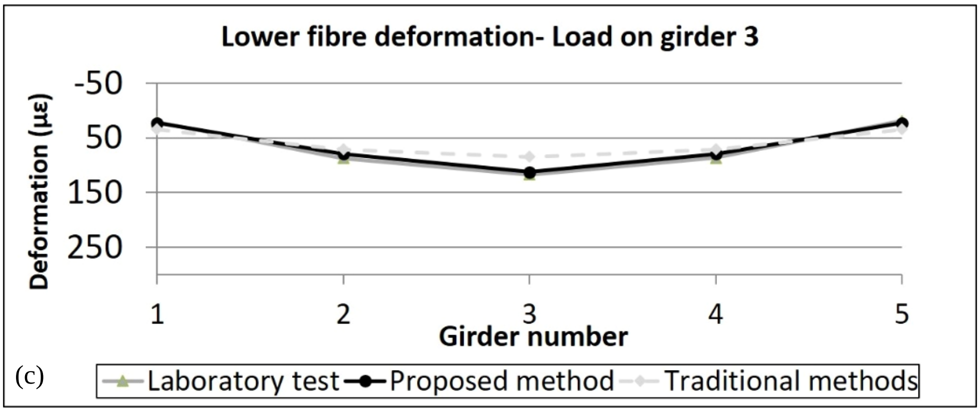



**Figure 10. Laboratory test. Theoretical-empirical comparative analysis. Lower fiber deformation: (a) load on girder 1; (b) load on girder 2; (c) load on girder 3**

### 5.2. Full-Scale Girder Bridge Load Test

The authors have used the static load test of an actual girder bridge deck as a second experimental validation of the proposed method to analyse the structural response of this type of bridge decks. This analysis studies the structural response of a road bridge located in Benalup (Cadiz, Spain) over the Barbate River during its static load test. The bridge analysed consists of 3 spans of 19.00 + 19.00 + 19.00 with a total length of 57.00 m (Figure 11). The deck originally consisted of three "double T" girders that are 1 m deep, and a 6 m wide and 0.26 m thick slab. The bridge is longitudinally isostatic and transversely hyperstatic. The deck has been enlarged by adding two 1 m deep "double T" girders at the lateral ends. With this expansion, the slab has a total width of 10.14 m, maintaining a thickness of 0.26 m.

**Figure 11. Full-scale Girder Bridge load test**

The objective of the static load test carried out is to verify the deformation and traction behaviour of the structure to the overload defined in the recommendations for carrying out acceptance load tests on road bridges defined by the Spanish Ministry of Public Works [28]. The type of truck used has a maximum total weight of 442 kN distributed over five axles of 66.6, 88.2, 98.6, 90.0 and 98.6 kN respectively from the cab to the rear. The truck has a total width of 2.47 m. The longitudinal separation between axles is 3.85 m between first and second, 3 m between second and third, 1.37 m between third and fourth, and 1.37 m between fourth and fifth. The total length of the truck is 12.19 m, with 1.30 m remaining between the first axle and the front part, and 1.3 m between the fifth axle and the rear of the truck. A total of two load hypotheses are proposed, using two or four trucks (Figure 12): (a) in scenario 1, two eccentric trucks are arranged on the bridge span analysed. Transversally, the trucks are located 0.50 m from the curb and 1.07 m from the edge of the deck, while longitudinally they are centred with respect to the central section of the span under study; (b) in scenario 2, the intention is to verify the section subjected to maximum positive bending of the span under study. For this, another row of two trucks is located at a transversal distance of 1.00 m with respect to the first row of trucks. To measure the deflections, a total of five displacement transducers with a precision of 0.01 mm are used simultaneously, which allow the deflections in the central section of the span of the bridge under study (Figure 13) to be obtained.

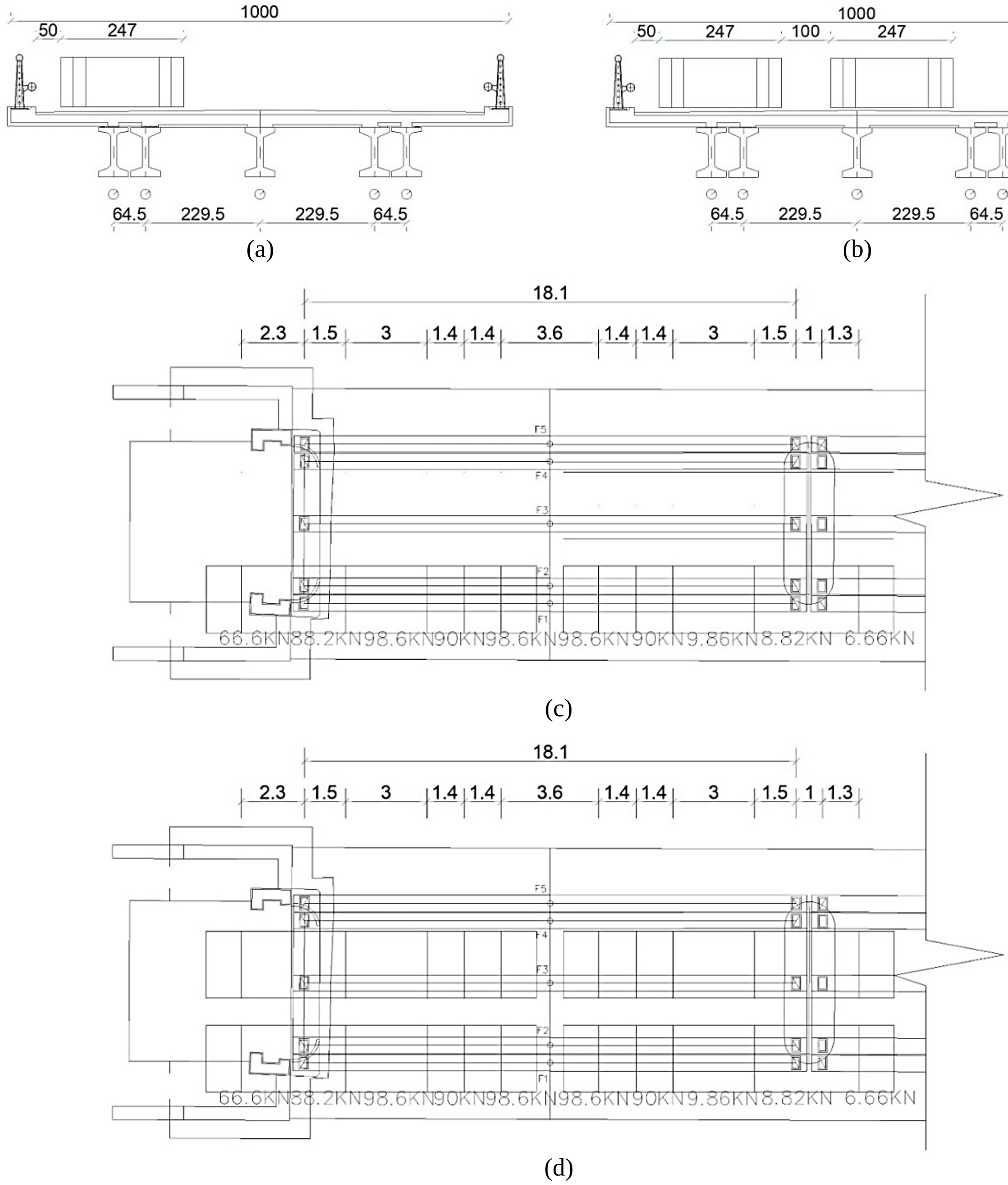



**Figure 12. Definition of the load hypotheses in the girder bridge: (a) distribution of trucks in the cross section in scenario 1; (b) distribution of trucks in the cross section in scenario 2; distribution of trucks along the bridge span under study in scenario 1; (d) distribution of trucks along the bridge span under study in scenario 2.**

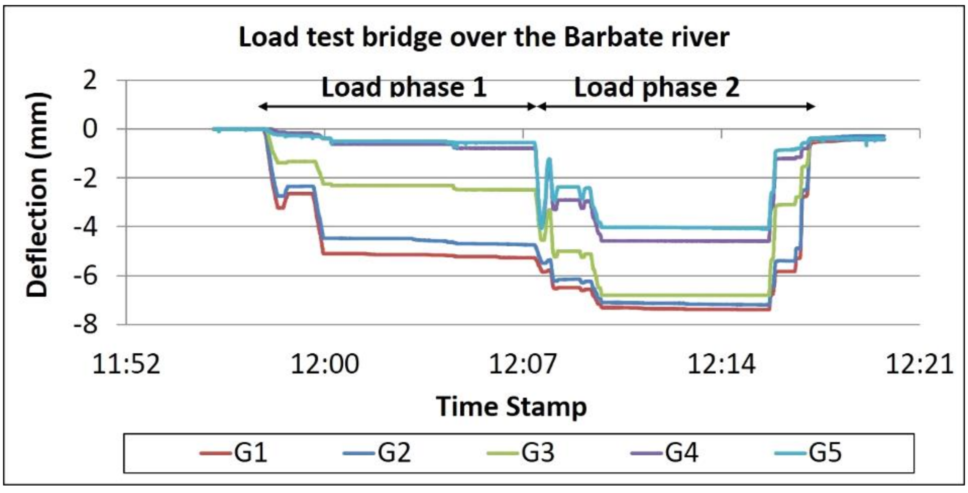
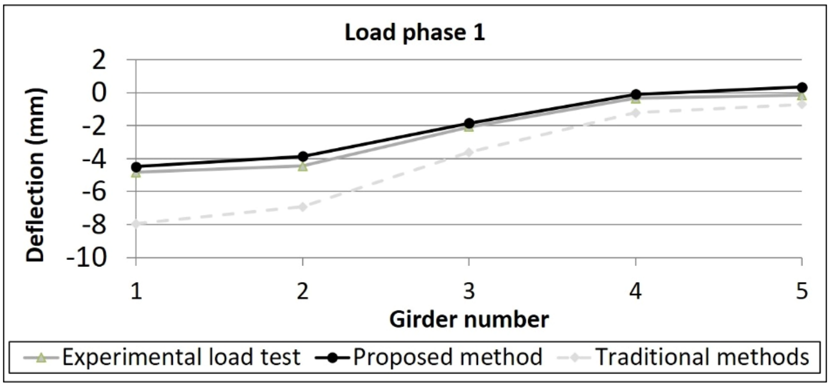
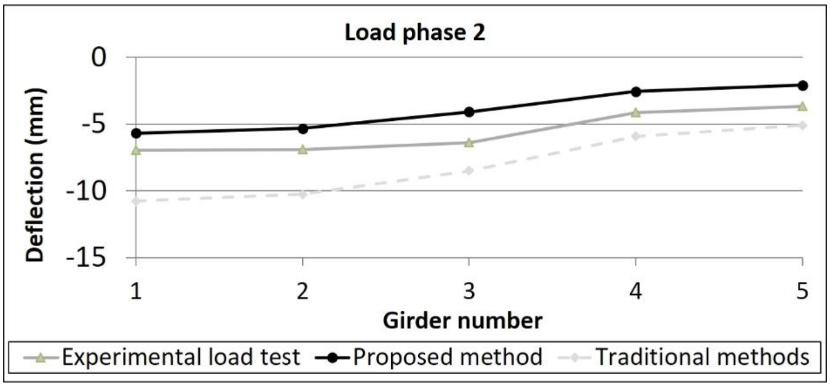



**Figure 13. Deflection recorded during the load test of the bridge over the Barbate River**

Figure 14 shows the contrast between the structural response obtained experimentally when interpreting the signals provided by the potentiometric displacement transducers installed along the structure, the theoretical structural response obtained through traditional calculation methods, and that obtained through the method proposed by the authors. Despite the uncertainty in load tests on road bridges induced by the errors made in the correct positioning of the trucks, correctly weighing the trucks, and the actual elastic modulus of the bridge deck at the moment in which the load test is executed, the divergence between the deflection of the most loaded girder obtained experimentally and that obtained theoretically applying the method proposed by the authors is less than 10 percent.




(a)




(b)

**Figure 14. Full-scale Girder Bridge load test. Theoretical-empirical comparative analysis: (a) deflection in girders of the bridge deck in scenario 1; (b) deflection in girders of the bridge deck in scenario 2**

## 6. Summary and Conclusions

The proposed method has been presented and validated parametrically, and the divergence of the method with traditional methods has been analysed. The parametric analysis considers the length, number, and depth of girders. The structural response of 64 girder bridge decks to a total of 384 load states is analysed. The variables analysed are the maximum deflection, shear stress, and bending moment on each of the girders. The results of the parametric analysis reflect a maximum divergence between the proposed method and traditional methods of 10% and an average divergence of less than 5% (Figure 5). The method proposed by the authors makes it possible to obtain both the deflection and the stresses experienced by each of the longitudinal girders that make up the girder bridge deck, while the use of traditional LDF methods can only obtain an estimate of the maximum stresses in the girder bridge deck, and it only distinguishes between exterior and interior longitudinal girders. Gheitasi and Harris [8] show an overestimation of the stresses obtained when using the LDF quantified as 125% for interior girders and greater than 300% for exterior girders. Likewise, Huang, Semendary and Razzaq [10-22] quantify the overestimation of the longitudinal bending stresses experienced by the girders obtained using the LDF method as 150%.

Moreover, the results of the proposed method are experimentally contrasted using two validation tests: (a) a laboratory test that analyses the divergence between the experimental structural response of a small-scale bridge model for different load cases; (b) full-scale girder bridge load test which analyses the divergence between the experimental structural response and that obtained by applying the proposed method during the static load test of the bridge over the Barbate River. The variables analysed in the laboratory test are the deflection and the deformation experienced by the lower fibre in the mid span section of each girder. The results of the laboratory test reflect an optimal adjustment of the proposed method (a maximum deviation less than 10%) compared with traditional ones (consistently between 10- 20%) (Figures 9 and 10). In the full-scale girder bridge load test, a theoretical/empirical comparative analysis of the deflections experienced by the span centres of all the longitudinal girders that make up the deck span under study is carried out. The divergence between the deflection of the most stressed girder obtained experimentally and that obtained theoretically applying the method proposed by the authors is less than 10 percent (Figure 14).

The proposed method for the analysis of the cross-sectional distribution of live loads on girder bridge decks allows the cross-sectional distribution to be determined in different configurations of girder bridge decks without the need to resort to complex calculation models that involve significant computing power and excessive analysis time. The proposed method can also apply to modern synthetic materials such as plastic composites and those with self-repair properties [6]. The simplicity of the method means it could be easily integrated into optimal bridge design strategies [29] or more heuristic approaches [30-32] to challenge today's competitive world in an intelligent way. In addition, it opens the way to future lines of research, such as the application of the method proposed by the authors to the analysis of the transverse distribution of dynamic loads in other types of bridge decks, such as slab-type bridge decks or box section bridge decks. Another future line of research would be the application of the method proposed by the authors to the modal analysis of different types of bridge decks.

## 7. Declarations

### 7.1. Author Contributions

Conceptualization, A.G-A., D.G-S., I.C-U-A., and C.L.C.; methodology A.G-A. and D.G-S.; validation A.G-A., and D.G-S; formal analysis, A.G-A., and D.G-S.; investigation, A.G-A., and D.G-S.; resources, D.G-S., and I.C-U-A.; writing—original draft preparation, A.G-A., and D.G-S.; writing—review and editing, I.C-U-A., and C.L.C.; supervision, A.G-A., D.G-S., I.C-U-A., and C.L.C.; project administration, A.G-A., and D.G-S.; funding acquisition, D.G-S., and I.C-U-A. All authors have read and agreed to the published version of the manuscript.

### 7.2. Data Availability Statement

The data presented in this study are available on request from the corresponding author.

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### 7.5. Conflicts of Interest

The authors declare no conflict of interest.

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