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Macroscopic Traffic Characterization Based on Distance Headway

Amir Iftikhar [1*], Zawar H. Khan [2], T. Aaron Gulliver [1], Khurram S. Khattak [3], Irfan Ahmed [4]

[1] *Department of Electrical and Computer Engineering, University of Victoria, Victoria, BC V8W 2Y2, Canada.*

[2] *College of Computer Science and Engineering, University of Ha'il, Ha'il, 55476, Saudi Arabia.*

[3] *National Center for Big Data & Cloud Computing, University of Engineering and Technology (UET), Peshawar 2023, Pakistan.*

[4] *Department of Electrical Engineering, University of Engineering and Technology Peshawar, Jalozai Campus, Nowshera, Pakistan.*

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Abstract

Accurate traffic characterization is essential for congestion mitigation. In this paper, a traffic model is proposed that incorporates distance headway in the well-known Lighthill, Whitham, and Richards (LWR) model. Velocity is influenced by the headway distance between vehicles. When this distance is small, the velocity is low, and when it is large, the velocity is high. The proposed and LWR models are implemented in MATLAB, and the performance is evaluated for different values of distance headway. The results show that traffic with the proposed model evolves with smaller changes that are more accurate and realistic than with the LWR model.

Keywords: Traffic Congestion; Macroscopic Model; Distance Headway; Explicit Upwind Difference Scheme.

1. Introduction

The economic success of a nation depends on the transportation infrastructure. Therefore, efficient use of this infrastructure is very important. Traffic congestion is a significant issue in urban environments as it results in underutilized infrastructure. Congestion occurs when traffic flow is greater than the road capacity [1]. The 2017 TomTom Traffic Index [2] indicates that congestion levels in Mexico City, Bangkok, Jakarta, Istanbul, and Beijing were 66%, 61%, 58%, 49%, and 46%, respectively. According to the Texas Transportation Institute, the average commuter spends 42 hours each year in traffic [3]. In traffic, drivers adjust their speed based on the distance headway between vehicles. This headway must be maintained so there is sufficient time to avoid accidents.

Traffic can be classified as homogeneous or heterogeneous and equilibrium or nonequilibrium. Heterogeneous traffic has variations in headway, whereas in a homogeneous flow, the headway is approximately constant. Thus, changes in headway in a homogeneous flow are negligible. At equilibrium, traffic changes are density dependent, but this is not the case in non-equilibrium traffic [1]. Stop-and-go traffic occurs more often in heterogeneous conditions as the headway varies and there is excessive acceleration and deceleration [4]. Traffic models can be macroscopic, microscopic, or mesoscopic. Macroscopic models consider average parameters in characterizing traffic flow. These models are considered here due to their low complexity.

The Lighthill-Witham-Richards (LWR) model is a macroscopic model that characterizes small changes in traffic flow [5]. With this model, traffic is always at equilibrium [6]. It is based on the law of conservation and is a system of partial differential equations (PDEs). The assumption is that there are a fixed number of vehicles on a long infinite road

* Corresponding author: amiriftikhar@uvic.ca

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[7]. However, this assumption can produce unrealistic traffic behavior [8]. In Gani et al. [9], a constant term was added to the LWR model to characterize traffic changes, but this does not produce realistic results for different traffic conditions [10]. A model for high-density traffic at a junction on a straight highway was presented by Holden & Risebro [11].

Uniform traffic on a freeway was examined in [5], but changes in density were ignored [12]. The concept of vehicles following one another on a roadway was considered by Jin [13] to develop a model. With this model, vehicles maintain a minimum distance and time between them to align the changes. However, the results are unrealistic for large traffic changes [1, 14, 15]. A model based on real data for freeways in Athens, Greece, was given in Papageorgiou [16]. The drawback of this model is that it does not consider deceleration as traffic approaches congestion [17]. Driver response in heterogeneous traffic was considered by Wong & Wong [18] to improve the LWR model. This was achieved by incorporating the headway distribution based on speed during congestion. However, with this model, faster vehicles overcome slower vehicles in congestion, which is impossible as uniform speed and headway are assumed [19].

An improved LWR model was proposed in [20] to determine vehicle travel time. This model considers traffic changes as a function of minimum and maximum density, speed, and headway, but the travel time distribution is not accurate for all traffic conditions. Moreover, the LWR model is based on the equilibrium flow of vehicles, and real traffic flow is typically not in equilibrium. Further, this model assumes vehicles adjust their speed in zero time at equilibrium [21]. The LWR model is inaccurate for stop-and-go traffic [22] and where large changes occur [8]. Hence, a new model is proposed that considers realistic parameters to overcome the drawbacks of the LWR model.

The proposed model characterizes changes in traffic density based on distance headway. During alignment to forward traffic conditions, the velocity increases with this headway. For a small headway, a driver is more aware of the conditions ahead and reduces speed. Conversely, the LWR model considers a long, ideal road with only small changes in traffic, which can result in unrealistic behavior when these changes are large. To illustrate this, the proposed model is compared with the LWR model for an inactive traffic bottleneck on a 1000 m road.

The rest of this paper is organized as follows. Section II presents the LWR and proposed models. In Section III, the explicit upwind difference scheme is employed to implement these models. A comparison of the LWR and proposed models is presented in Section IV. Finally, some concluding remarks are given in Section V.

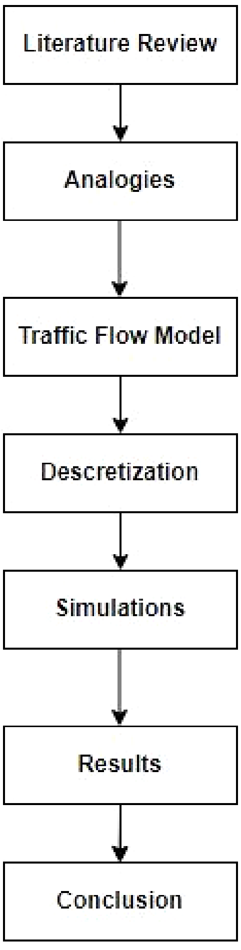

Figure 1. Flowchart of the methodology employed

2. Traffic Flow Models

The LWR model is based on vehicle conservation on a long road so the number of vehicles entering the road is equal to the number leaving. With this model, changes in traffic flow occur when the road density changes. It is assumed that only small changes occur, so traffic aligns in zero time [8]. The LWR model is commonly used because of its low complexity [5] and can be expressed as:

$$\rho_t + (\rho v)_x = 0 \tag{1}$$

where ρ is density, v is velocity, the subscript t denotes partial derivative with respect to time, and the subscript x denotes partial derivative with respect to distance. Flow is the product of density and velocity ρv. The distance headway is covered during alignment to forward conditions. The LWR model assumes this alignment occurs in zero time and thus ignores the distance headway. To avoid an accident, the velocity during alignment should be low when the headway is small [23].

Mixed traffic includes a variety of vehicles such as bicycles, cars, carts, and trucks. In mixed heterogeneous traffic, drivers often do not follow lanes. In this case, a driver adjusts to traffic stimuli based on the lateral D and forward h (distance) headways. Changes in velocity are small when D is large, and large changes and stop-and-go traffic occur when D is small. The desired velocity [24] based on the lateral and forward headways can be expressed as:

$$v_{opt}(h, D) = v_m \left(\frac{h^2}{h_m^2 + D^2}\right) \tag{2}$$

where v_m is the velocity limit on the road and h_m is the maximum distance headway. During congestion and in mixed traffic, D is negligible so that:

$$v_{opt}(h, D) = v_m \left(\frac{h}{h_m}\right)^2 \tag{3}$$

This is the velocity attained when traffic deviates from h_m, and this is achieved via acceleration and deceleration [24]. The goal of drivers is to maintain a safe distance between vehicles. During this process, they adjust the speed to achieve a safe headway [24].

In a homogeneous flow, drivers maintain the maximum headway $h = h_m$ [25], in which case $v(h, D) = v_m$. Substituting Equation 3 in 1 gives:

$$\rho_t + \left(\rho v_m \left(\frac{h}{h_m}\right)^2\right)_x = 0 \tag{4}$$

This indicates that the temporal change in density is based on the change in velocity due to the distance headway. For a small headway, the density is large and alignment to forward stimuli is slow, while for a large headway, the density is small and alignment is fast. When $h = h_m$, Equation 4 becomes

$$\rho_t + (\rho v_m)_x = 0 \tag{5}$$

so the temporal change in density is independent of the headway.

3. Model Simulation

The explicit upwind difference scheme (EUDS) is a discretization technique [26, 27] for simulating dynamic systems. It can provide accurate results and has lower complexity than the Godunov [28] and Force [29] schemes. Therefore, EUDS is employed in this paper to implement the LWR and proposed models. It calculates the backward difference in space and the forward difference in time [30].

Consider a road divided into N equidistant segments. The road length is L with segment length $L/N = \Delta x = x_i - x_{i-1}$. The total time T is divided into M equal time steps $T/M = \Delta t = t_{n+1} - t_n$. The traffic density at the nth time step in the ith road segment is given by:

$$\frac{\partial \rho}{\partial t} = \frac{\rho_i^{n+1} - \rho_i^n}{t_{n+1} - t_n} \tag{6}$$

and the corresponding traffic flow is:

$$\frac{\partial q}{\partial x} = \frac{q_i^n - q_{i-1}^n}{x_i - x_{i-1}} \tag{7}$$

Substituting $q = \rho v$ gives:

$$\frac{\partial q}{\partial x} = \frac{(\rho v)_i^n - (\rho v)_{i-1}^n}{x_i - x_{i-1}} \tag{8}$$

The LWR model discretized using EUDS is then:

$$\frac{\rho_i^{n+1}-\rho_i^n}{t_{n+1}-t_n}+\frac{(\rho v)_i^n-(\rho v)_{i-1}^n}{x_i-x_{i-1}}=0 \tag{9}$$

and substituting $\Delta x = x_i - x_{i-1}$ and $\Delta t = t_{n+1} - t_n$ gives:

$$\rho_i^{n+1}=\rho_i^n-\frac{\Delta t}{\Delta x}\left((\rho v)_{i-1}^n-(\rho v)_i^n\right) \tag{10}$$

which is the density update for the LWR model in segment i for time step $n+1$.

For the proposed model, $q = \rho v_m \frac{h^2}{h_m^2}$ and substituting this in Equation 7 gives:

$$\frac{\partial q}{\partial x}=\frac{(\rho v_m\frac{h^2}{h_m^2})_i^n-(\rho v_m\frac{h^2}{h_m^2})_{i-1}^n}{x_i-x_{i-1}} \tag{11}$$

so the temporal change in density $\frac{\partial \rho}{\partial t}$ for the proposed model is the same as for the LWR model given in Equation 6. The proposed model discretized using EUDS is then:

$$\frac{\rho_i^{n+1}-\rho_i^n}{t_{n+1}-t_n}+\frac{(\rho v_m\frac{h^2}{h_m^2})_i^n-(\rho v_m\frac{h^2}{h_m^2})_{i-1}^n}{x_i-x_{i-1}}=0 \tag{12}$$

and the corresponding density update is:

$$\rho_i^{n+1}=\rho_i^n+\frac{\Delta t}{\Delta x}\left(\left(\rho v_m\frac{h^2}{h_m^2}\right)_{i-1}^n-\left(\rho v_m\frac{h^2}{h_m^2}\right)_i^n\right) \tag{13}$$

For numerical stability, the Courant-Friedrichs-Lewy (CFL) condition should be satisfied [13] so that the traffic flow is accurately approximated during a time step Δt. This condition ensures that the numerical scheme is stable such that small traffic changes are smooth [31, 32]. The CFL condition for the LWR model is:

$$v_m\frac{\Delta t}{\Delta x}\le 1 \tag{14}$$

The maximum velocity is assumed to be $v_m = 17$ m/s for both models, and choosing $\Delta x = 5$ m and $\Delta t = 0.09$ s gives:

$$v_m\frac{\Delta t}{\Delta x}=17\times\frac{0.09}{5}=0.306<1 \tag{15}$$

so, traffic behavior with the LWR model will be stable. For the proposed model, the velocity considered for the CFL condition is:

$$v_m\left(\frac{h}{h_m}\right)^2 \tag{16}$$

which is larger than v_m for $h > h_m$. From (14), the CFL condition for the proposed model is:

$$v_m\left(\frac{h}{h_m}\right)^2\frac{\Delta t}{\Delta x}<1 \tag{17}$$

For $\Delta t = 0.09$ s, $\Delta t = 5$ m, and $\frac{h}{h_m} = 0.833$, this gives:

$$17\times(0.833)^2\times\frac{0.09}{5}=0.212<1 \tag{18}$$

For $\frac{h}{h_m} = 1$, the stability condition is:

$$17\times(1)^2\times\frac{0.09}{5}=0.306<1 \tag{19}$$

and for $\frac{h}{h_m} = 1.25$ it is:

$$17\times(1.25)^2\times\frac{0.09}{5}=0.383<1 \tag{20}$$

Thus, traffic behavior with the proposed model will be stable.

Table 1. Simulation Parameters

Name	Parameter	Value
Road step	Δx	5 m
Time step	Δt	0.09 s
Maximum velocity	v_m	17 m/s
Distance headway	h	10, 12, 15 m
Maximum distance headway	h_m	15 m
Headway	$\frac{h}{h_m}$	0.833, 1, 1.25
Length of road	L	1000 m
CFL conditions	$\Delta t/\Delta x$	0.018
Maximum density	ρ_m	1
Minimum density	ρ	0
Total simulation time	T	100 s

4. Performance Results

In this section, the LWR and proposed models are evaluated over a $L = 1000$ m road for $T = 100$ s. The parameters $\Delta t = 0.09$ s and $\Delta x = 5$ m are employed so the CFL condition is satisfied. The minimum normalized density is 0 which represents no traffic on the road and the maximum density is 1 which indicates 100% of the road is occupied with vehicles. The initial density distribution is given in Figure 2. This shows that the density is 0.1 from 0 m to 200 m, 0.5 from 200 m to 700 m, and 0 from 700 m to 1000 m.

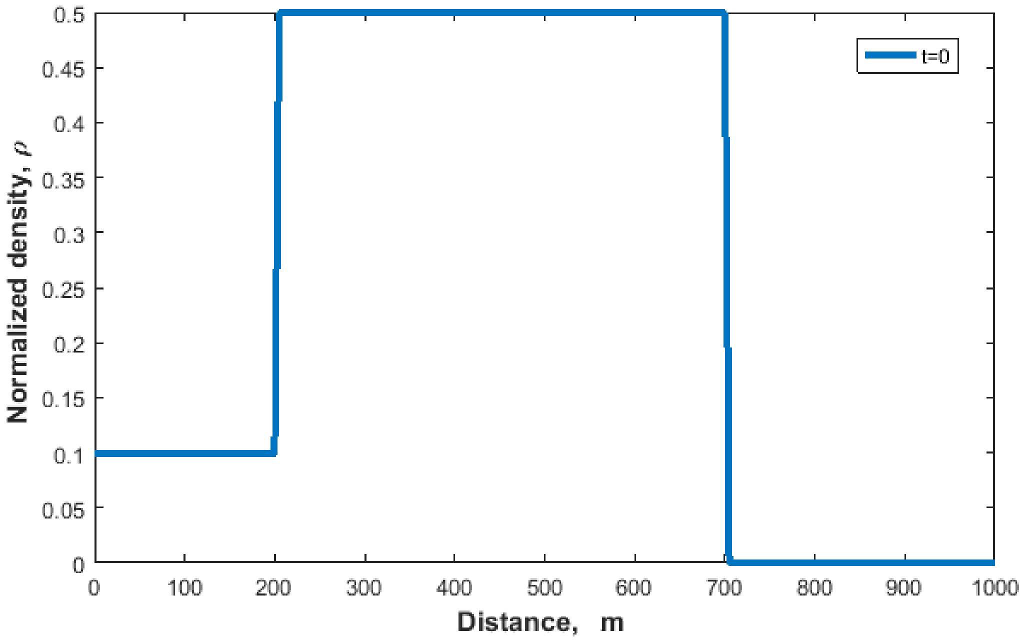

Figure 2. The initial ($t = 0$ s) normalized density distribution on a 1000 m road

Figure 3 presents the normalized traffic density with the LWR model at 10 s. At 0 m the density is 0.1 and decreases to −0.1 at 10 m, which is impossible. It then increases to 10.0 at 11 m and is 8.0 from 50 m to 210 m. At 211 m, the density is 4.0 and increases to 13.0 at 212 m. It is 8.5 from 250 m to 710 m, decreases to 1.9 at 711 m, increases to 13.9 at 712 m, and is approximately 8.0 from 750 m to 1000 m. Figure 4 shows the density with the LWR model at 100 s. It is 0 from 0 m to 960 m and then varies between −2 and 1.5×10^{38}, which is not possible.

Figure 5 presents the normalized traffic density with the proposed model at 10 s, 50 s, and 100 s for a distance headway of 10 m. At 10 s, the density increases to 0.1 at 15 m and is approximately constant to 205 m. It then increases to 0.5 at 230 m and is approximately constant to 720 m. The density is approximately 0 between 740 m and 1000 m. At 50 s, the density is 0 between 0 m and 40 m, increases to 0.5 at 270 m, and then decreases to 0 at 780 m. The traffic cluster at 100 s is 0 between 0 m to 370 m as it has moved forward. The density is 0.11 at 560 m and from 720 m to 1000 m it is approximately 0.5.

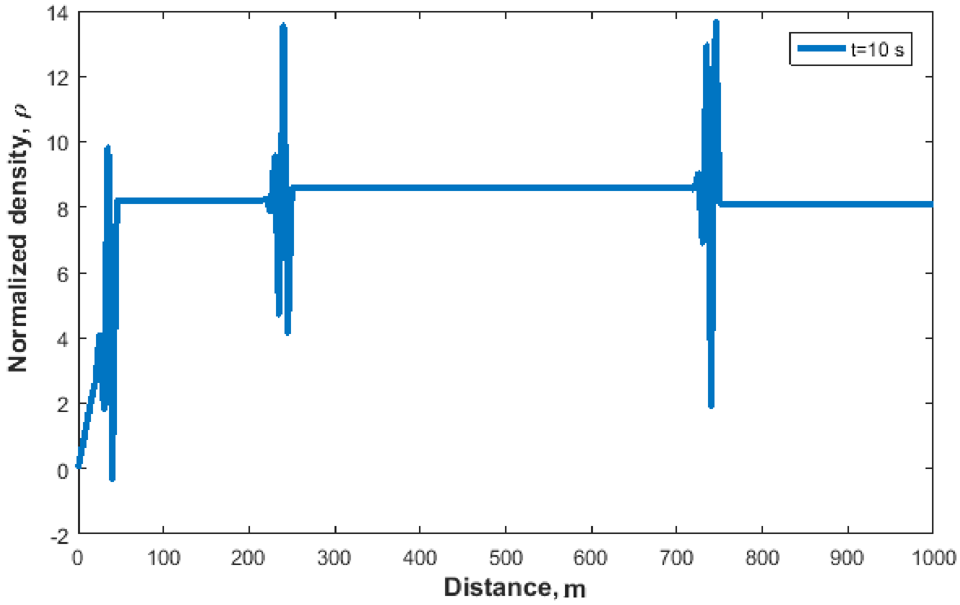

Figure 3. The normalized density with the LWR model at 10 s on a 1000 m road

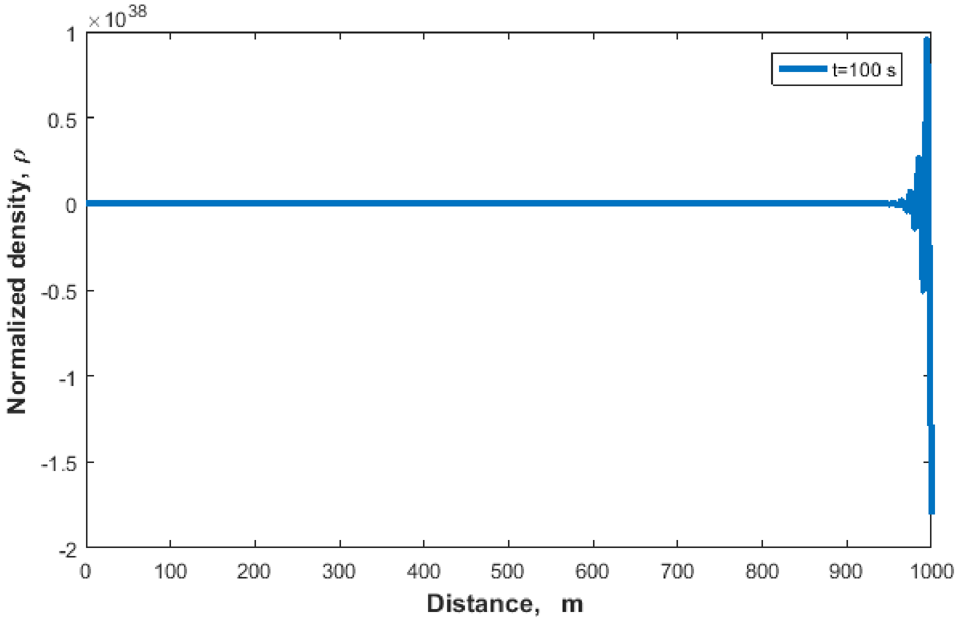

Figure 4. The normalized density with the LWR model at 100 s on a 1000 m road

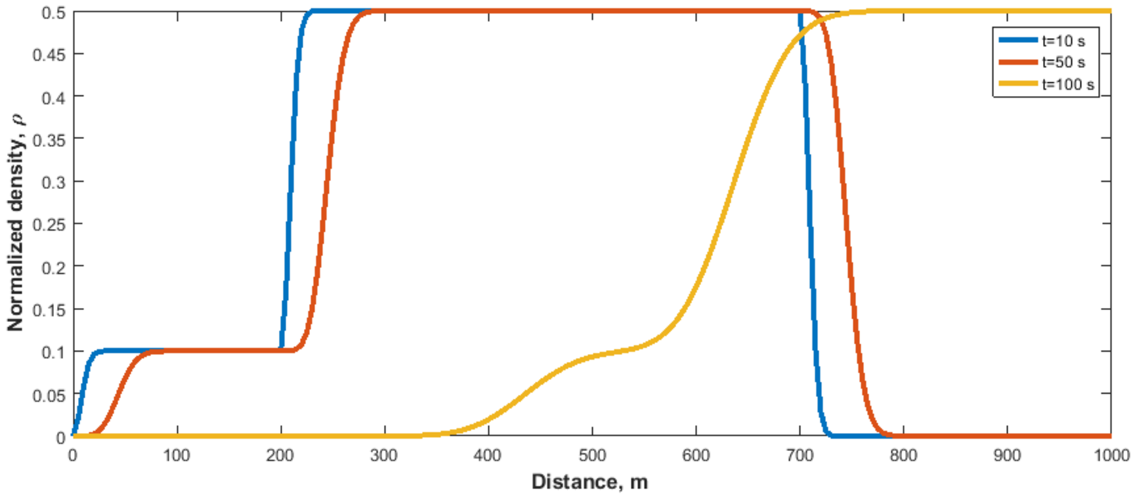

Figure 5. The normalized density with the proposed model for a distance headway of 10 m and $h_m = 10$ m on a 1000 m road at 10 s, 50 s, and 100 s

Figure 6 presents the normalized traffic density with the proposed model at 10 s, 50 s, and 100 s for a distance headway of 12 m. At 10 s, the density is 0 between 0 m and 2 m, 0.1 from 20 m to 200 m, 0.5 from 220 m to 700 m, and 0 at 720 m. At 50 s, the density is 0 between 0 m and 80 m, and approximately 0.1 from 150 m to 290 m. It is 0.5 from 380 m to 780 m and 0 at 880 m. At 100 s, the density is 0 between 0 m and 530 m, and from 960 m to 1000 m it is approximately 0.5. Figure 7 presents the normalized traffic density with the proposed model at 10 s, 50 s, and 100 s for a distance headway of 15 m. At 10 s, the density is 0 between 0 m and 80 m, 0.5 at 350 m, and then decreases to 0 at 850 m. At 50 s, it is 0 between 0 m and 160 m, 0.5 at 450 m, and then decreases to 0 at 960 m. At 100 s, the density is 0 between 0 m and 870 m and increases to 0.06 at 1000 m.

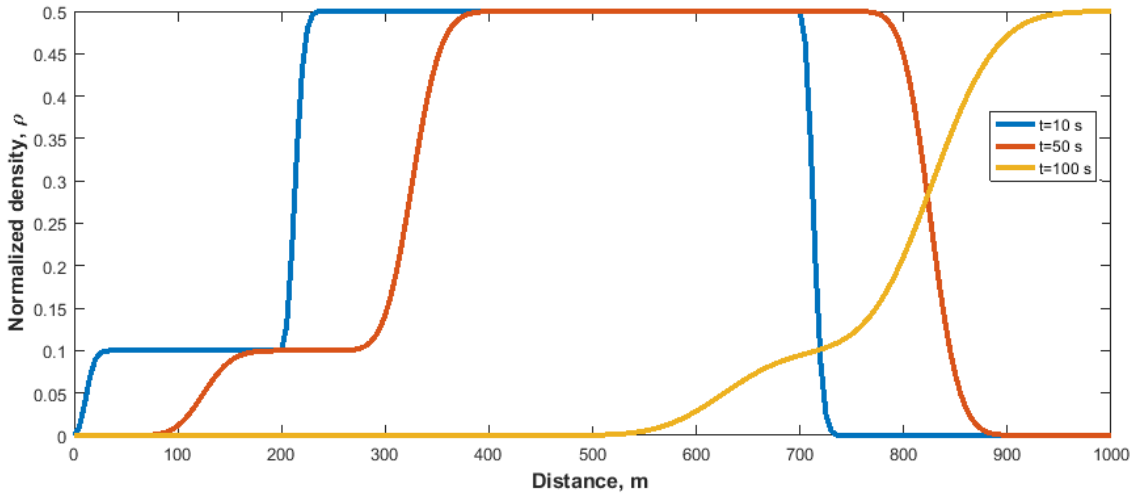

Figure 6. The normalized density with the proposed model for a distance headway of 12 m and $h_m = 10$ m on a 1000 m road at 10 s, 50 s, and 100 s

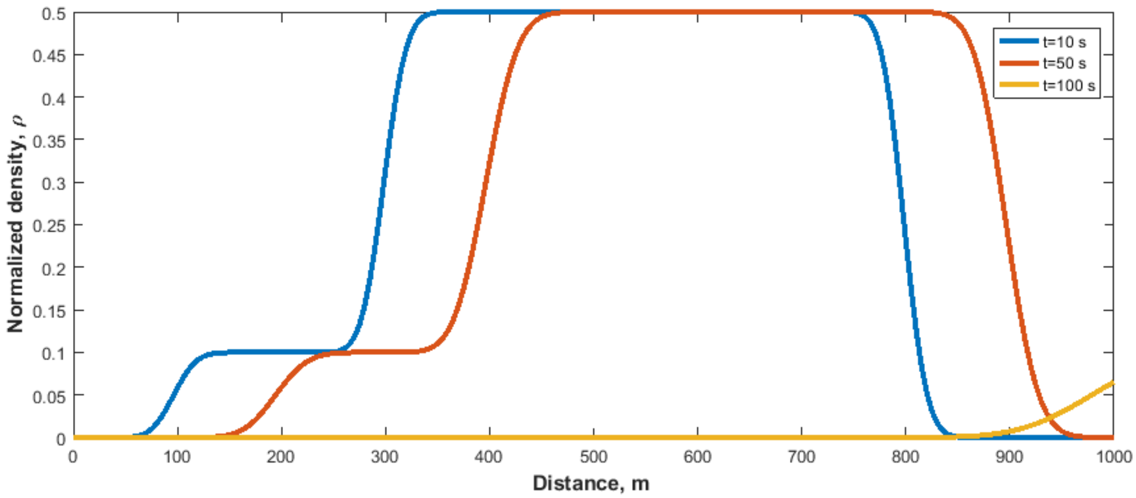

Figure 7. The normalized density with the proposed model for a distance headway of 15 m and $h_m = 10$ m on a 1000 m road at 10 s, 50 s, and 100 s

The results in Figures 5-7 indicate that the density with the proposed model is realistic. This shows that traffic moves faster with a larger distance headway. Conversely, the density with the LWR model goes as high as 1.5×10^{38} as shown in Figure 4. This is well above the maximum of 1 so the results are not realistic. At transitions, the density goes above the maximum and fluctuates rapidly over short distances, which is impossible. Further, the density goes below 0 to -0.1 which is nonsense. Thus, the LWR model cannot adequately characterize traffic behavior. The results obtained with the proposed model show that it produces realistic traffic behavior as the density is between 0 and 1. Further, the density becomes smooth over time at abrupt changes in traffic as expected.

The normalized traffic density over on a 1000 m road for 1000 s with a distance headway of 10 m is given in Figure 8. At 100 s, the density is 0.12 at 0 m and increases to 0.35 at 280 m and 0.4 at 500 m. It then decreases to 0 at 800 m and is approximately constant. At 500 s, the density is 0 at 0 m and increases to 0.45 at 780 m. It then decreases to 0 at 800 m and is approximately constant. At 1000 s, the density is 0 at 0 m and increases to 0.5 at 810 m. It then decreases to 0 at 950 m and is approximately constant. These results show that the normalized density stays within the range 0 to 1. Figure 9 gives the corresponding density with the proposed model for a distance headway of 12 m. The results are realistic as the traffic becomes smooth over time. The normalized traffic density with the proposed model over a distance of 1000 m for 1000 s and a distance headway of 15 m is presented in Figure 10. At 50 s, the density is

0.09 at 20 m and increases to 0.15 at 180 m. At 100 s, it is 0.12 and increases to 0.3 at 380 m, while at 500 s it is 0.35 at 650 m and decreases to 0 at 850 m. At 1000 s, the density is 0 between 0 m and 800 m. It increases to 0.5 at 980 m and then decreases to 0 at 1000 m.

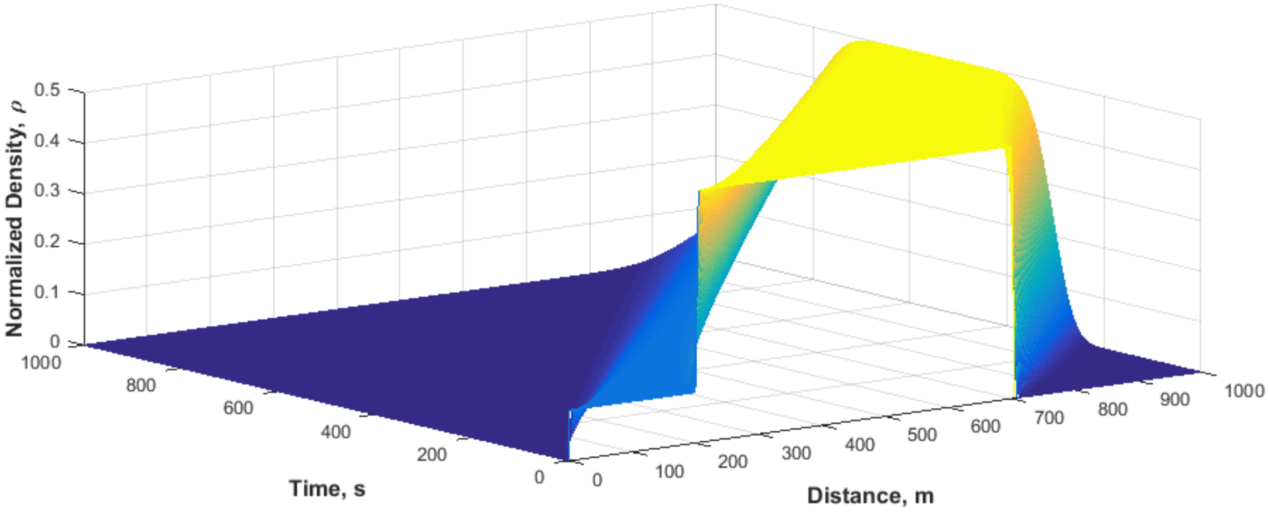

Figure 8. The normalized traffic density with the proposed model on a 1000 m road for a distance headway of 10 m

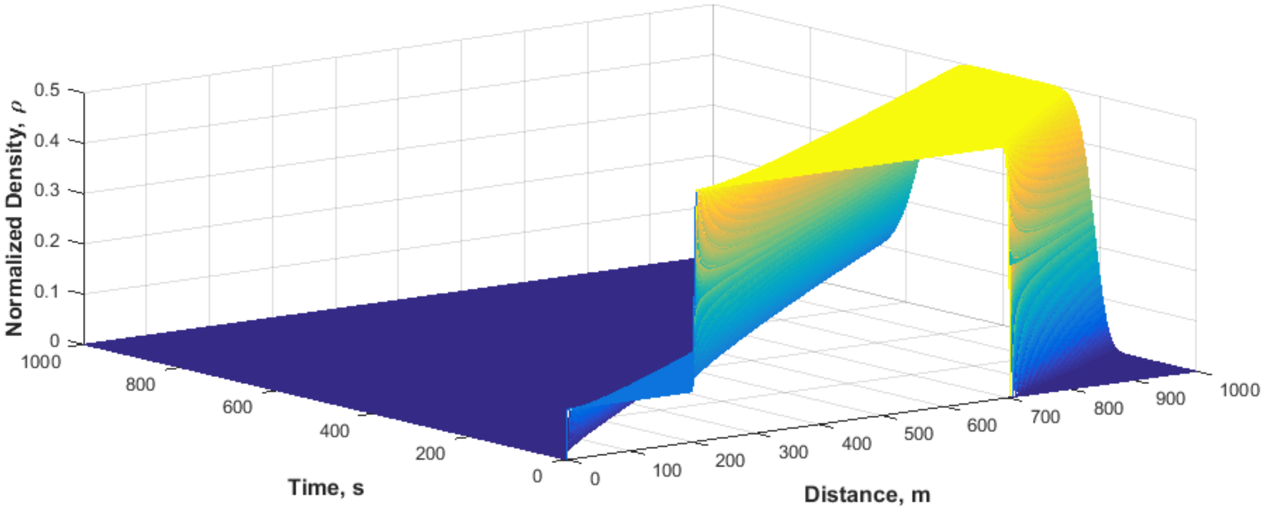

Figure 9. The normalized traffic density with the proposed model on a 1000 m road for a distance headway of 12 m

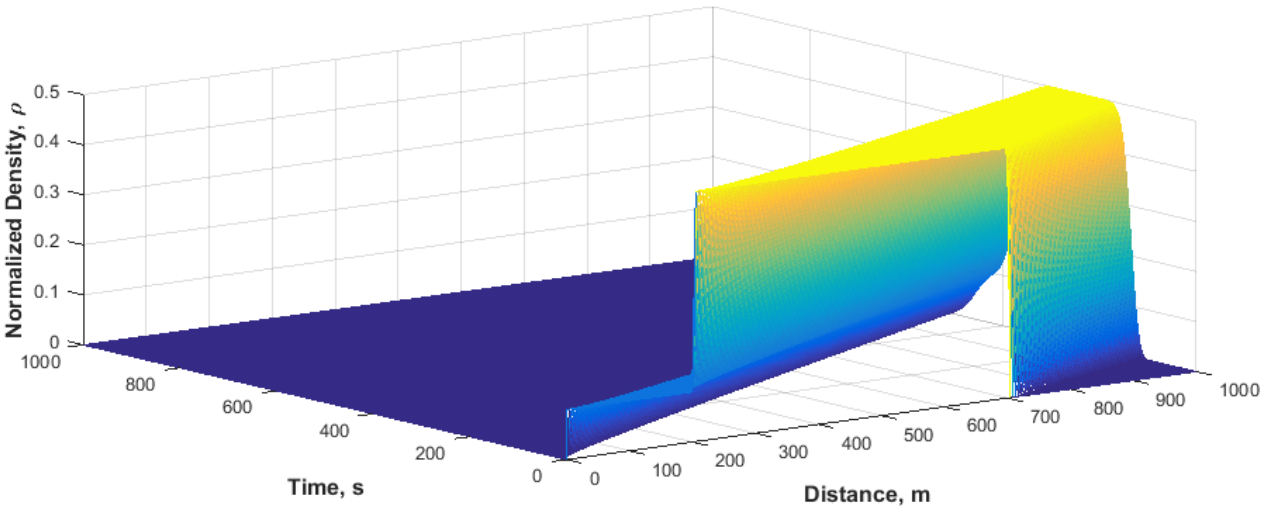

Figure 10. The density behavior with the proposed model on a 1000 m road for a distance headway of 15 m

Figures 8 to 10 give the normalized density for the proposed model with distance headways 10 m, 12 m, and 15 m, respectively. These results show that the traffic behavior is realistic for all headway values. Further, the traffic is faster with a larger distance headway. Conversely, traffic behavior with the LWR model is not realistic. This is because it does not consider the distance headway and traffic changes in zero distance.

4.1. Riemann Problem

The Riemann problem [33] is now solved using the proposed and LWR methods. The length of the road is $L = 100$ m and $T = 100$ s. The distance headway is 15 m for both models. The initial traffic distribution is:

$$\rho = \sin[(x-2)\pi] \tag{21}$$

The normalized traffic density with the proposed model for a distance headway of 15 m is given in Figure 11. This shows that the density stays in the range 0 to 1. It decreases spatially and temporally and becomes smooth. At 0 s, the density is 0 at 0 m, increases to 0.59 at 10 m, and then decreases at 20 m to 0.15. It increases again and reaches 0.56 at 40 m and then decreases at 50 m to 0.12. The density is 0.01 at 100 m. At 75 s, the density is 0 at 0 m and increases to 0.04 at 10 m, 0.07 at 20 m, 0.14 at 40 m, 0.20 at 50 m, and 0.3 at 100 m. At 100 s, the density is 0 at 0 m and increases to 0.01 at 10 m, 0.02 at 20 m, 0.12 at 40 m, 0.18 at 50 m, and 0.3 at 100 m.

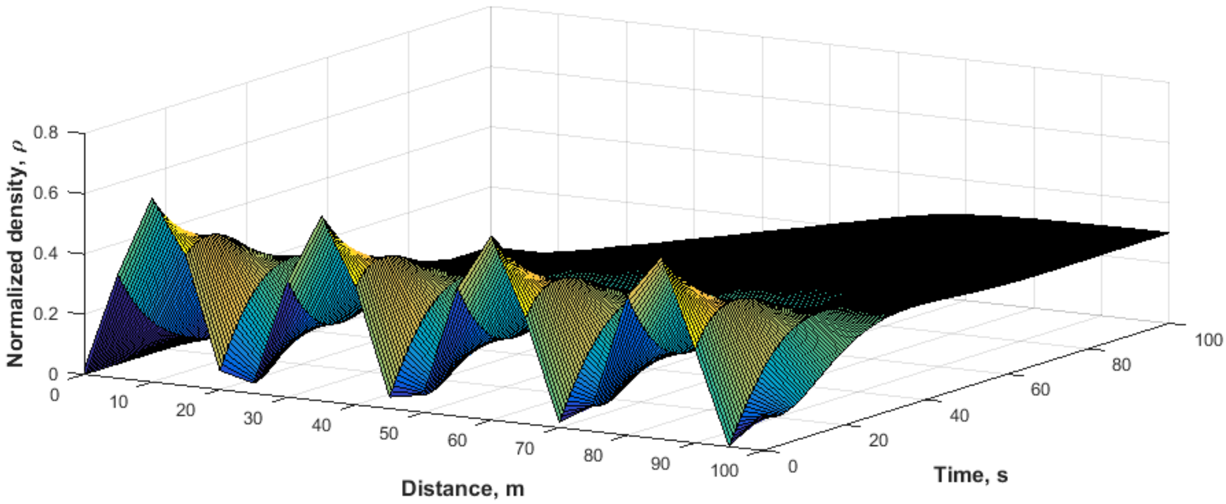

Figure 11. The normalized traffic density with the proposed model for a distance headway of 15 m

Figure 12 presents the normalized traffic density with the LWR model for a distance headway of 15 m. This shows that the model does not provide realistic results when there is a sudden change in density. In particular, oscillatory behavior is observed with a density of 3×10^4 at 70 m, which is impossible. Further, it is -2×10^4 from 90 m to 100 m. These results indicate that the results for traffic changes with the proposed model are more realistic than with the LWR model. This is because the proposed model considers distance headway while the LWR model does not.

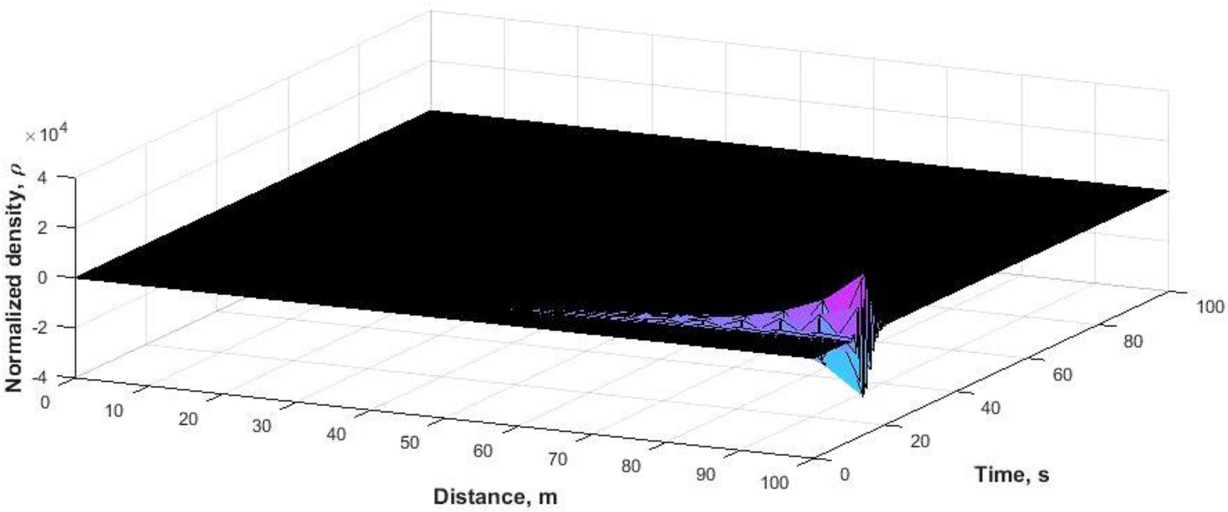

Figure 12. The normalized traffic density with the LWR model for a distance headway of 15 m

5. Conclusion

A new traffic model was presented that incorporates distance headway for realistic traffic characterization. The performance of this model was compared with the well-known LWR model. The results obtained show that traffic with a large distance headway is faster than with a small distance headway, as expected. Further, the density becomes smooth over time and remains within the maximum and minimum values. Conversely, traffic with the LWR model exceeds the maximum and minimum densities, and there are very large variations in density over short distances. This is because the LWR model cannot characterize traffic for a variety of conditions, whereas the proposed model considers the distance headway between vehicles. It was shown that the LWR model cannot characterize large changes in traffic appropriately, whereas the results with the proposed model are smooth. Thus, the proposed model can be used in traffic prediction for planning and intelligent transportation systems (ITSs).

6. Declarations

6.1. Author Contributions

Conceptualization, A.I. and Z.H.K.; methodology, A.I. and Z.H.K.; software, I.A.; validation, A.I., Z.H.K., and T.A.G.; formal analysis, A.I., Z.H.K., and K.K.; investigation, A.I., T.A.G., Z.H.K., K.K., and I.A.; resources, A.I.; data curation, A.I. and I.A.; writing—original draft preparation, A.I.; writing—review and editing, A.I., Z.H.K., T.A.G., and K.K.; visualization, A.I., Z.H.K., I.A., and. K.K.; supervision, T.A.G.; project administration, A.I., Z.H.K., and T.A.G.; funding acquisition, T.A.G. All authors have read and agreed to the published version of the manuscript.

6.2. Data Availability Statement

The data presented in this study are available on request from the corresponding author.

6.3. Funding

University of Victoria, Victoria, BC V8W 2Y2, Canada.

6.4. Conflicts of Interest

The authors declare no conflict of interest.

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