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Development of a Conservative Hamiltonian Dynamic System for the Early Detection of Leaks in Pressurized Pipelines

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Abstract

In this study, we propose an innovative approach for real-time leakage detection in pipelines by integrating conservative Hamiltonian equations and experimental Internet of Things (IoT) technologies. The proposed method combines a hybrid model that utilizes sensors and IoT devices to acquire real-time data and solves the coupled system of Hamiltonian equations using the ODE45 numerical integration method. Spectral frequency analysis is an essential part of this method, as it reveals specific patterns in the pressure and flow signals. The findings highlighted 95% accuracy in leak detection, which was validated through a comparison of the theoretical and experimental data. The novelty of this approach lies in its ability to maintain constant total system energy, thereby enabling continuous monitoring for early leak detection. As an improvement, the proper handling of sensor signals is emphasized, underscoring its contribution to the efficient management of water resources in potable water distribution systems.

Keywords: ®Arduino; Conservative Systems; Hamiltonian System; IoT; Leaks; ODE45, Real-Time; Pipelines.

1. Introduction

The primary objective of early leakage detection in pipelines is to reduce the amount of water lost from potable water distribution systems. However, the existing methods for leak localization and quantification have significant limitations in terms of accuracy and detection capability. Leak detection and localization are challenging because of the spatiotemporal dynamics of the process variables [1]. The current instrumentation of potable water distribution networks is marked by deficiencies, contributing to a loss of approximately 40% of the treated water at water treatment plants. For example, Speziali et al. (2021) [2] indicated that water loss rates due to leaks remain exceedingly high for service providers. Additionally, the strategic placement of sensors is crucial for monitoring and preventing failure events in water distribution networks (WDNs).

Verde & Torres (2016) [3] mentioned that each leak in a hydraulic system introduces three unknown fluid-related parameters: leak position, flow loss, and friction factor. These parameters play a crucial role in understanding and characterizing the impact of leaks on a system. An unstable or transient flow is characterized by physical conditions, including liquid compression and the resistance of the pipe material to deformations caused by a wavefront. This phenomenon can result in elastic or inelastic transients. In elastic situations, the method based on characteristics is effective for addressing transient scenarios. Similarly, in inelastic systems, such as those with rigid column pipes for nonstationary flows, the liquid flowing through the pipe is incompressible, and the pipe material does not undergo

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deformation. In this case, the system is affected only by inertia and friction. In addition, because the liquid density and pipe cross-sectional area remain constant, the instantaneous velocity remains the same throughout the control volume. The solution for both systems is based on the continuity and momentum equations, allowing the determination of the velocity and piezometric head along the pipe under a one-dimensional flow configuration. Numerical model validation is carried out by comparison with measurements of real transients and depends on pre-established boundary conditions in terms of algebraic and/or differential equations based on the physical properties of the fluid and pipe material. It is important to highlight that the transient itself is not a problem; the issue lies in the speed of change generated by the closing velocity of the control valves. In other words, the transient state originates between the two steady states. Rodríguez Calderón & Pallares Muñoz (2007) [4] suggested that a transient state can be induced through the instantaneous closure of a downstream valve, generating a wavefront that oscillates through the fluid.

This effect can be reduced or even prevented by gradually activating the flow control devices. Firouzi et al. (2020) [5] asserted that a water hammer can manifest when sudden disturbances occur under stable flow conditions at a specific point in the pipeline owing to instantaneous events. Disturbances originating from leaks in the pipeline can be detected by considering the flow in a pipeline as a conservative Hamiltonian dynamic system. These leaks cause significant changes in the energy within the system. Sánchez-Jiménez et al. (2020) [6] indicated that these changes can be detected through the real-time monitoring of state variables. For instance, the Euler–Lagrange approach has been employed to model pipes conveying water and experiencing leaks under the theory of a rigid column of water. Macias & Kooktae (2022) [7] asserted that it is possible to pinpoint the source of a leak in a gas pipeline using a Hamiltonian approach with the utilization of a backtracking horizon. Similarly, Schneider et al. (2002) [8] studied the transient chaos variations when changing the orientation of a major leak within a chaotic region in closed Hamiltonian systems. On the other hand, Torres and Besancon (2019) [9] described the dynamics of incompressible flow in rigid pipes with leaks and partial obstructions using port-Hamiltonian (pH) models, where interconnected modules are established. Consequently, it is possible to study the behavior of leaks in a hydraulic system based on a Hamiltonian dynamic model derived from the state variables. Perryman et al. (2022) [10] designed controllers for a nonlinear model of a Water WDN, demonstrating that the system led to a pH system while preserving system stability. The energy behavior in the system can be monitored by coupling Hamiltonian differential equations with mathematical functions determined for each leak in terms of flow and pressure. The equations of motion describe the evolution of the generalized coordinates and their conjugate moments over time. On the other hand, the energy conservation equation indicates the total energy in the system over time.

Owing to the occurrence of a leak in a hydraulic system, which generates a pressure differential and mass outflow, it is possible to represent the fluid behavior in a pipeline in terms of energy and determine the changes caused by disturbances inherent to leaks in the system. The Hamiltonian model allows the simulation and analysis of different leak scenarios, enabling the assessment of the impact of these conditions on the dynamic variables of the system. Lopezlena (2014) [11] proposed the development of a computer system for the detection and localization of leaks in pipelines based on a real-time transient model (RTTM), inspired by the pH algorithm. This algorithm is an optimization technique that is used to model distributed pH fluids (DPHFs). Rashad et al. (2021) [12] demonstrated that the pH model corresponds solely to the kinetic energy of a fluid to represent a variety of fluid dynamic systems. The pH algorithm establishes a series of ports to form a physical system to determine the aim of finding optimal control conditions. These port-controlled Hamiltonian models were derived from a theoretical systems approach based on energy modeling. These models are square, indicating that they have an equal number of state variables and control inputs.

Beattie et al. (2018) [13] suggested that these models are passive, suggesting that they do not generate energy internally and can only dissipate or store energy. The configuration of the Hamiltonian system and the flow rate and pressure differential functions yield a set of differential equations that can be solved using the ODE45 algorithm. This algorithm can find a numerical solution for the proposed system of differential equations, thereby allowing for analysis and an understanding of the system's behavior over time. For example, Zi Li et al. (2019) [14] used the built-in ODE45 function in Matlab® to calculate the movement of pipeline inspection gauges conveying petroleum and the motion of the Solghar and Nieckele models. Consequently, the main challenge in establishing a Hamiltonian model for leak detection is determining suitable functions that can accurately represent the flow rate and pressure in the system. Furthermore, the choice and application of appropriate filters for processing the signals emitted by the sensors significantly affect the final model results. It is crucial to select suitable filters to eliminate noise and unwanted disturbances from signals to obtain more accurate measurements for leak detection in a hydraulic system.

In contrast, Bendimerad et al. (2024) [15] studied 1D pH systems with dissipation, such as the Dzekster model for water filtration and viscous nanorods. Equations and implicit representations in Stokes–Lagrange subspaces are derived, addressing pH formulations for Timoshenko and Euler–Bernoulli beams. They explored transformations between explicit and implicit representations. Zhang et al. (2024) [16] observed dynamics with noise in observational data through Hamiltonian mechanics and proposed the Hamiltonian neuron Koopman operator (HNKO), which incorporates mathematical knowledge to automatically discover and maintain conservation laws. Eidnes et al. (2023) [17] proposed a hybrid machine-learning approach using Hamiltonian formulations for mechanical systems, whether conservative or non-conservative. Additionally, they introduced pseudo-Hamiltonian algorithms as a generalization of the Hamiltonian formulation through a pH formulation.

Therefore, the proposed approach to fill this research gap focuses on comprehensively addressing the early detection of leaks in hydraulic systems by combining mathematical models with synchronous flow and pressure signal detection. The proposed model integrates Internet of Things (IoT) technologies and mathematical modeling, structuring a hybrid model based on Hamiltonian equations and signal monitoring to describe the dynamics of the hydraulic system. The experimental model involves the installation of pressure and flow sensors at strategic points in the hydraulic system. Sensors such as USP-G41-1.2 and YF-S201 measure pressure and flow at different locations. The acquired data were filtered to eliminate the inherent sensor signal noise. Once the noise was removed, a mathematical model based on the conservative Hamiltonian equations was incorporated. These equations describe energy conservation in the hydraulic system and allow for the representation of interactions between variables, including disturbances caused by leaks, thereby providing a crucial analytical tool for understanding system dynamics.

To solve the coupled system of Hamiltonian equations and regression functions derived from the sensor data, the numerical integration method ODE45 (Runge–Kutta method) was employed. Finally, different scenarios were generated for the two simultaneous leaks. The Hamiltonian model allows for the analysis of the energy impact on the system generated by the two leaks (energy balance). Different leakage scenarios with temporal variations were considered, which contributed to the applicability of the proposed approach.

1.1. Hamiltonian Conservative System

The development of a Hamiltonian conservative system for leak detection in a simple pipeline begins with the kinetic energy associated with the flow, the potential energy represented by pressure, and a function that determines the energy derived from leaks. The kinetic energy is linked to the flow passing through the pipeline. This energy depends on the velocity and mass of the fluid and can be calculated using the kinetic energy equation. Potential energy is related to the fluid pressure in the pipeline. Sultana & Rahman (2013) [18] indicated that these forms of energy involve a function that represents the energy associated with leaks. This function captures the effect of leaks on the system and can vary depending on the location and magnitude of the leaks and is used to model and quantify the hydraulic head loss owing to leaks in the pipeline. The Hamiltonian conservative approach aims to develop a system of differential equations that describes the evolution of these different forms of energy over time. This allows for the detection and quantification of leaks based on changes in flow, pressure, and energy associated with the leaks. To develop this system, the following equations are considered:

$$H = \int \frac{Q^2}{2A} dx + \int \frac{P}{A^2} dx + Perturbation_1(t) + Perturbation_2(t) \tag{1}$$

where the first integral represents the kinetic energy of the system, the second integral represents the potential energy, $Perturbation_1(t)$ and $Perturbation_2(t)$ are functions that model the perturbations associated with leaks over time in terms of pressure. These perturbations were obtained from the proposed experimental system based on a mathematical function generated from the pressure data at each leak location over time. For the data obtained from the USP−G41−1.2 sensor, a statistical analysis was performed, determining the mathematical function that best fits the observed data and obtaining measurements of the perturbation generated by the leak at different moments. The Bernoulli equation is used to develop the system.

$$P + \frac{Q^2}{2A} = Constant + Perturbation_1 + Perturbation_2 \tag{2}$$

where P represents the pressure in the pipeline, A is the cross-sectional area of the pipe, and $Perturbation_1$ and $Perturbation_2$ are the perturbations at each leak in terms of pressure. Subsequently, by applying the mass-conservation equation, we obtain:

$$\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/2)}{\partial x} = Leak_1 + Leak_2 \tag{3}$$

In this context, Q represents the flow rate in the pipeline, $Leak_1$ and $Leak_2$ the leak rates in terms of the flow. From the Hamiltonian, the corresponding equations can be derived using partial derivatives concerning flow rate Q and pressure P variables:

$$\frac{\partial P}{\partial t} = -\frac{\partial H}{\partial Q} \tag{4}$$

where $\frac{\partial P}{\partial t}$ represents the partial derivative of pressure concerning time, $\frac{\partial H}{\partial Q}$ represents the partial derivative of the Hamiltonian function H concerning the flow:

$$\frac{\partial Q}{\partial t} = \frac{\partial H}{\partial P} \tag{5}$$

where $\frac{\partial Q}{\partial t}$ represents the partial derivative of flow concerning time and $\frac{\partial H}{\partial P}$ represents the partial derivative of the Hamiltonian function H concerning pressure P. From this approach, it is possible to describe the temporal evolution of the flow and pressure in the experimental model considering the conservation of energy and the influence of leaks on the system. To determine the flow rate at each leak, the flow data were collected from the YF-S201 sensor at the location of each leak over time. These data allowed us to obtain measurements of the flow rate at different times and find a mathematical function that models the behavior. Thus, it is possible to implement an energy-conservative approach in the system that can contribute to the detection of abnormal changes in flow and pressure caused by simultaneous leaks in the pipes. Consequently, the Hamiltonian model allows the prediction of how the variables of the hydraulic system evolve over time. Disturbances caused by possible leaks in the pipeline were identified by monitoring the changes in pressure and flow in real-time.

This was achieved by comparing the sensor measurements with a theoretical model comprising Hamiltonian differential equations. Therefore, the state variables for this study are defined as follows: $-q(x,t)$ represent the flow rate at position x and time t and $-q(x,t)$ represent the pressure at position x and time t, Thus, the coupling of the Hamiltonian equations is given by:

$$H(q,p,x,t) = \int \left[p(x,t) + \frac{1}{2}q^2(x,t)\right] dx \tag{6}$$

Therefore,

$$\frac{\partial q}{\partial t} = -\frac{\partial H}{\partial p} = -\frac{\partial p}{\partial x} \tag{7}$$

For leaks 1 and 2, we have

$$\frac{\partial p}{\partial t} = -\frac{\partial H}{\partial q} = -\frac{\partial(q^2/2)}{\partial x} + Leak_1 + Leak_2 \tag{8}$$

The energy contributions to the hydraulic system are represented by $(Leak_1, Leak_2)$. In this way, the boundary conditions for the initial flow rate are given by: $-q(x,0) = q_0(x)$. In terms of the initial system pressure, we have: $-p(x,0) = p_0(x)$. These boundary conditions directly depend on the configuration of the hydraulic system. Once a Hamiltonian system is established to detect the two leaks in the system, it is important to process the signals emitted by the sensors. Hence, various filters, statistical analyses, noise-detection tests, and transformations must be implemented. For example, the fast Fourier transform (FFT) allows the decomposition of pressure and flow signals into their frequency components, transitioning from the time domain to the real-time frequency domain. The FFT identifies patterns and establishes unwanted frequencies in the signal, whether high or low. For example, Przystupa et al. (2020) [19] used the standard FFT algorithm to analyze the dynamic responses of a hydraulic pump within a low range of operating frequencies. Consequently, the spectral power behavior of the pressure and flow signals was determined for each leak, displaying the energy distribution at different frequencies. The application of filters, such as low-pass, high-pass, or moving average filters, involves a detailed study of the moment at which the leak occurs. This is crucial because the leak generates a pressure differential that may be confused with noise in the signal. Ji'e et al. (2022) [20] state that the dynamic behaviors linked to various parameters can be identified from the Lyapunov exponent.

2. Materials and Methods

2.1. Experimental Method

The design adopted a hybrid approach that combined conservative Hamiltonian equations and Internet of Things (IoT) technologies for real-time leak detection in pipelines. The execution was carried out by real-time data acquisition using USP-G41-1.2 pressure sensors and YF-S201 flow sensors, positioned at distances of 0.3 and 0.9 m, respectively. These data were incorporated into a mathematical model that solved a coupled system of Hamiltonian equations using the ODE45 numerical integration method. For data analysis, FFT was applied to the signals, allowing for the identification of patterns and subpeaks in the frequency spectrum. The findings, highlighting 95% accuracy in leak detection, were validated by comparing the theoretical and experimental data. This comprehensive approach, from design to data analysis, reveals a robust methodology for addressing the issue of leaks in hydraulic systems by leveraging both mathematical models and advanced real-time monitoring technologies. Figure 1 illustrates the methodological phases that determined the development of the study, including data acquisition using sensors, noise detection in signals, application of filters, spectrogram analysis, coupling of Hamiltonian equations, and prediction of flow and pressure signals over time and along the pipeline.

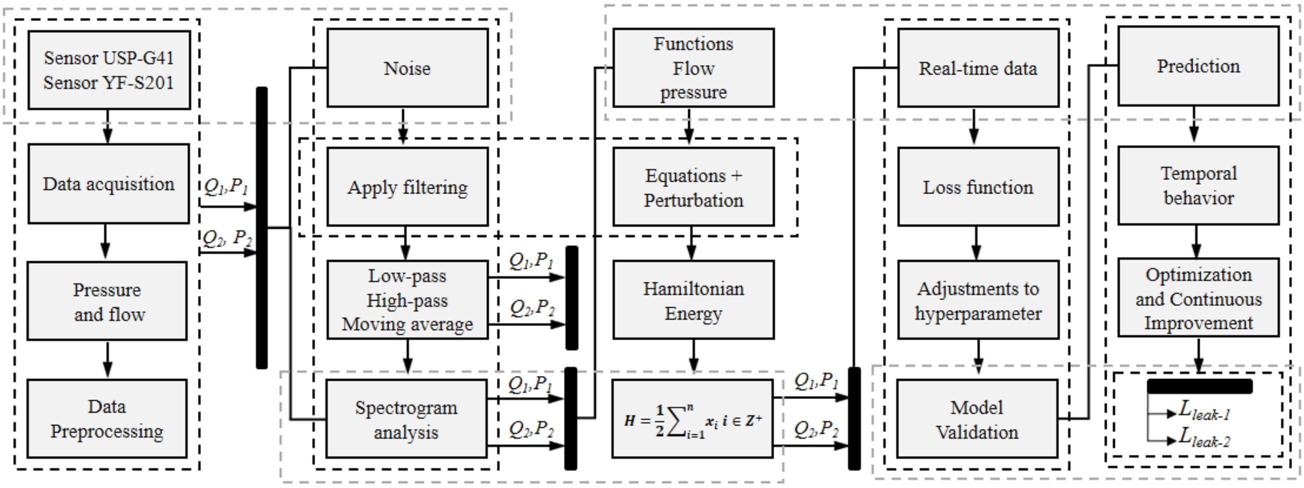


Figure 1. Flow Diagram (Modeling)

Table 1 presents the parameters used to formulate the PDE system of partial differential equations that describe the dynamics of the hydraulic system affected by leaks. The application of the conservative Hamiltonian method in this context aims to conserve the energy in the system as it evolves over time.

Table 1. Table 1. Hydraulic model parameters

Parameter	Value	Unit
Pressure (P)	8-14.5	psi
Flow (Q)	4-12	L/min
Pipe diameter	1/2"	inch
Pipe length	1.5	meters
Location leak_1	0.3	meters
Location leak_2	0.9	meters

Information on the physical constants of the fluid (water) is provided in Table 2.

Table 2. Physical parameters

Parameter	Value	Unit
Temperature	18	°C
Viscosity	0.001002	Pa·s

The Hamiltonian model for leak detection in hydraulic systems is significantly influenced by several key parameters. The pipe length (L) determines the temporal propagation of disturbances, such as leaks, throughout the system. The division of the pipe into intervals (N) and the spatial step (Δx) influence the ability of the model to accurately represent the distribution of these perturbations caused by leaks along the pipe. The time step (Δt) affects the temporal resolution of the model, being essential for capturing instantaneous events in the system. The initial and boundary conditions define the initial state of the system and the conservation of pressure at the ends of the pipe, respectively, thus shaping its temporal evolution. Similarly, the physical constants of a fluid, such as the density and viscosity of water, introduce fundamental properties that determine the response of the system to perturbations. The proper choice and precise configuration of these parameters are essential for constructing a Hamiltonian model. Consequently, sampling datasets of the flow and pressure signals were created for different time points $(t = 5\ s, t = 15\ s,\ t = 10\ s,\ t = 20\ s)$, establishing the dataset necessary to validate and fit the model. Table 3 lists the temporal conditions and initial and boundary conditions that were applied in this analysis.

Table 3. Temporal parameters

$\Delta x = L/N$	N	Δt	Initial condition (Flow)	Initial condition (Pressure)	Temporal iteration	Spatial condition
0.1	15	0.1	$Q(x,0) = 0$	$P(x,0) = 1$	$0, \Delta t, 2t, \ldots, T$	$P(L,t) = 1$

2.2. IoT Experimental Hydraulic Design

Figure 2 represents the experimental design implemented, which is based on a 1.5-m-long pipe, with two regulating valves driven by a centrifugal pump: 0.37 KW, 30 – 80 L/min. Four holes were created in the pipe, located at distances of 0.3 and 0.9 m from the beginning of the pipe, with a hole diameter of 2 mm in each one. Eight sensors were used to monitor the system. Electronics Hobby (2018) [21] proposed a precision flow meter; therefore, a YF-S201 flow meter was used, which was calibrated using a LUXQ flow meter with a two-wire 4–20-mA output. On the other hand, to capture the pressure signals, the USP-G41-1.2 sensor was implemented, which was calibrated using a SCJN-series pressure gauge with a precision of ± 0.5% FS. The sensors were connected to an ESP8266 board, which acted as a microcontroller and captured the signals from the sensors. These signals are transmitted via WiFi to the Matlab® server using ThingSpeak, with a 1 s delay between each transmission. Graphs of the behavior of the pressure and flow upstream, downstream, and on the abscissa of the leaks are shown in real-time through the website.

Figure 2. Experimental design (Hydraulics Laboratory, Universidad Distrital Francisco José de Caldas)

2.3. Signal Processing

After calibrating the sensors, the signals from the USP-G41-1.2 and YF-S201 sensors were processed. First, the signals from both sensors were captured. Once connected to the ESP8266 microcontroller, the pressure sensor transmits its signal through pin A0 as it emits an analog signal that is proportional to the measured pressure in the form of a current. On the other hand, the sensor communicates through pin D4 using a digital signal. Second, the signals are encoded using the Arduino development environment. A control panel is established in Matlab®, where the data obtained from the time series with a sampling frequency of one second are stored. The analysis was performed using two approaches. The first is a stationary approach in which the flow rate in the system does not vary; that is, it remains constant. In this case, the pressure and flow were monitored and recorded as functions of time to obtain information on the behavior of the system under stable conditions. The second approach is nonstationary or transient, where the flow rate varies as a function of time. Urbanowicz et al. (2020) [22] noted that a new simplified model was applied to simulate viscoelastic transient pipe flows and a method of characteristic (MOC) numerical scheme was implemented. By observing the signal in real-time, the reaction of the system to variations in flow can be examined, as well as how the pressure is affected when both leaks occur simultaneously. Figure 3 shows the electronic layout of the proposed prototype, where the corresponding pins are assigned to each sensor.

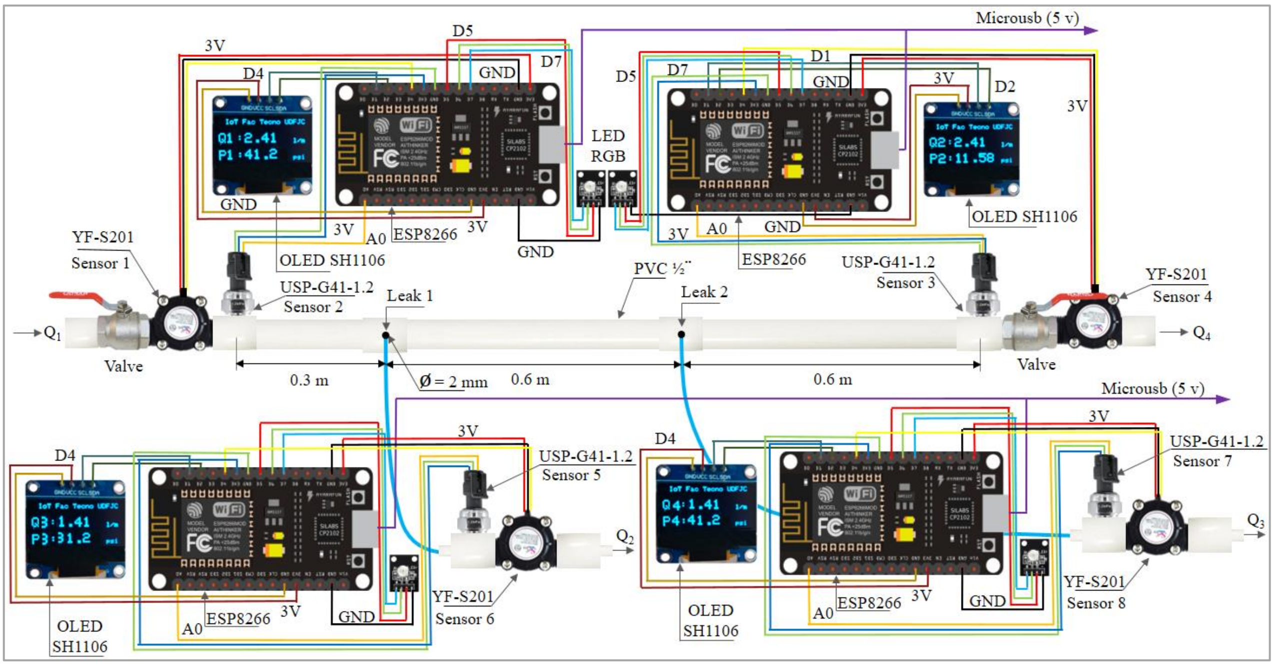


Figure 3. Electronic Prototype

During the analysis of the data from the USP-G41-1.2 and YF-S201 sensors, FFT was applied to the signal to identify patterns, trends, or anomalies in the observed data. When performing this transformation, it was found that the frequency spectrum of the flow sensor (YF-S201) showed the presence of subpeaks at frequencies of 75, 250, and 35 Hz, along with a dominant frequency component. In contrast, the signal from the USP-G41-1.2 sensor, which measures the pressure in the pipeline, exhibited dominant sub-peaks at a frequency of 200 Hz, as well as some minor peaks at 260 and 340 Hz. These findings suggest the existence of specific patterns in the signals measured using both sensors. Figure 4 shows the methodological approach used in this study to develop a conservative Hamiltonian dynamic system for the early detection of pipeline leaks. The development was divided into six phases, ranging from the application of the mass and energy conservation equations to the coupling and solution of the Hamiltonian equations.

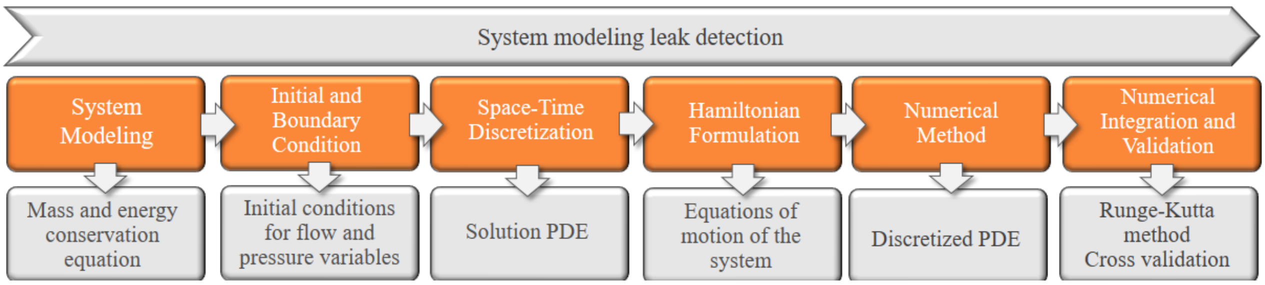


Figure 4. Leak detection modeling

2.4. Filters

The application of filters in a time series implies a reduction in the noise present in the signal, both from the sensor and that generated by environmental conditions. The challenge lies in selecting an appropriate filter that eliminates noise without significantly affecting the key signal information, particularly the detection of peaks and subpeaks that indicate the presence of leaks in piping systems. For example, Kulmány et al. (2023) [23] maintained that calibration could be performed using capacitive sensors. For this reason, the calibration of the sensors was carried out using as a reference a standard average obtained from a pressure manometer of the SCJN series, with a precision of ± 0.5% FS.

In addition, a YF-S201 sensor was used to measure the amount of fluid passing through the system. For this calibration process, a LUXQ series intelligent vortex flow meter with a two-wire 4–20-mA output and high precision was used. The time series of the pressure and flow were affected by the implementation of three filters: low-pass, high-pass, and moving average filters. The statistical behaviors of these filtered signals are shown in Tables 4 and 5.

Table 4. Implemented filter statistics (Pressure)

Filter	Average	Typical error	Median	Standard deviation	Kurtosis	Skewness
Low-pass	45.963	9.013	9.032	52.582	0.053	23.303
High-pass	-0.001	-0.0086	-0.005	-1.27e-06	0.155	0.024
Moving average	50.500	9.002	9.0125	50.500	0.0673	28.961

Table 5. Implemented filter statistics (Flow)

Filter	Average	Typical error	Median	Standard deviation	Kurtosis	Skewness
Low-pass	45.964	5.942	5.965	52.582	0.0407	23.303
High-pass	-0.005	0.002	-0.0008	-1.27e-06	0.144	0.024
Moving average	50.500	5.934	5.9619	50.500	0.049	28.961

In Figure 5, the proposed filters for the pressure and flow signal are shown. It can be observed that, in the case of flow, the high-pass filter eliminates peaks whose amplitudes are in the range of –0.3 to 0.3, which affects the detection of peaks and subpeaks generated by the sudden appearance of leaks in the flow hydraulic system. In statistical terms, it was decided to use the moving average filter for the USP-G41-1.2 and YF-S201 sensors. This filter was programmed in Arduino, which allowed for obtaining a clean signal at the output of the ESP8266 microcontroller, clearly indicating the peaks generated by the two leaks in the system.

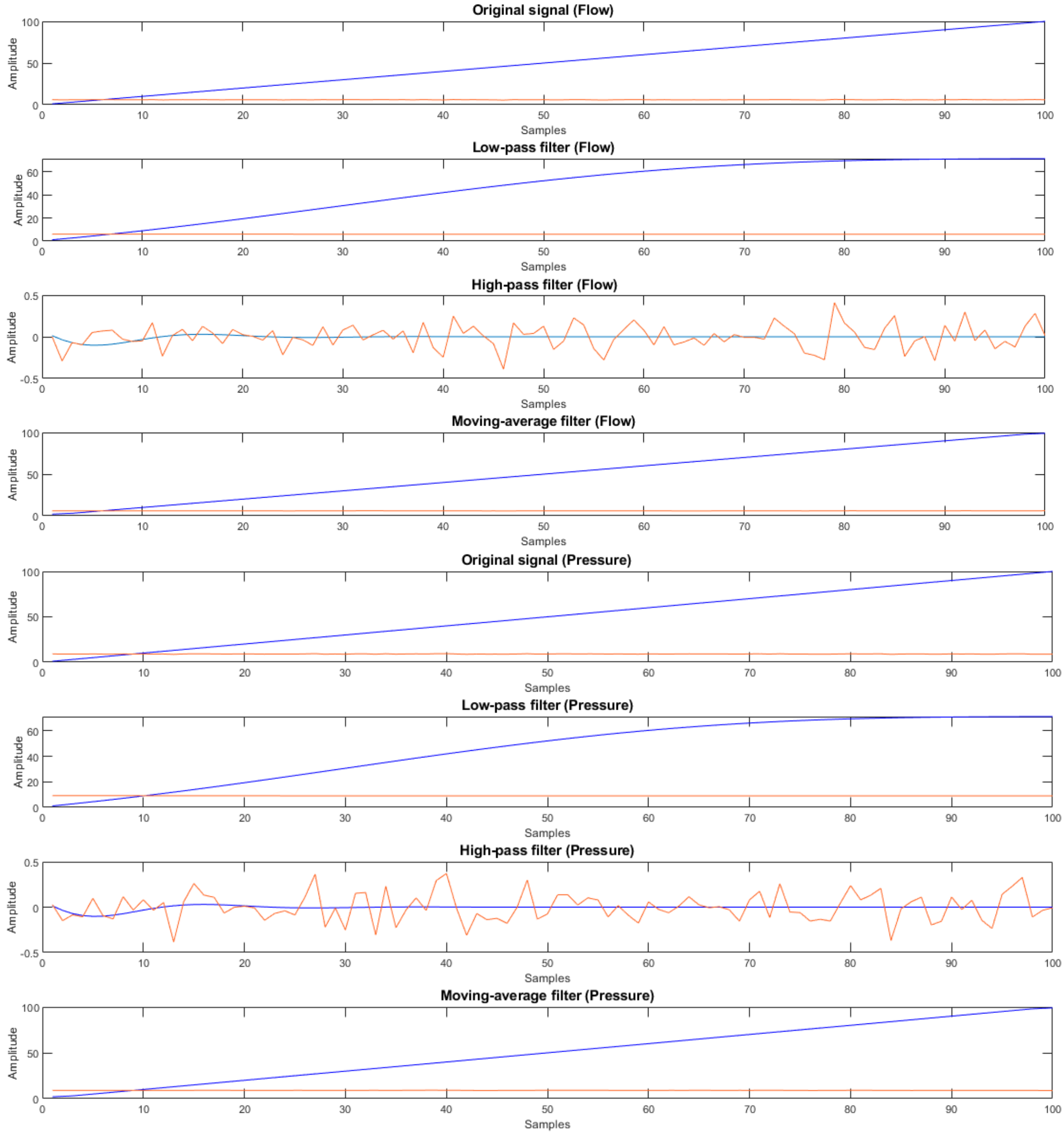


Figure 5. Filters USP-G41-1.2 y YF-S201

During the analysis of the spectral frequency of the flow, significant subpeaks were observed at two specific frequencies: 250 and 35 Hz. (Figure 6). These sub-peaks indicate the existence of prominent frequency components in the flow signal. Regarding the pressure signal, notable sub-peaks were also identified in the frequency spectrum. Specifically, one sub-peak was observed at a frequency of 260 Hz and the other at 340 Hz. These sub-peaks indicate the presence of specific frequency components in the pressure signal. Thus, the analysis of the spectral frequency of both signals provides important information regarding the characteristics and behavior of the studied hydraulic system. This information can be used to identify possible problems or anomalies in the system caused by a leak.

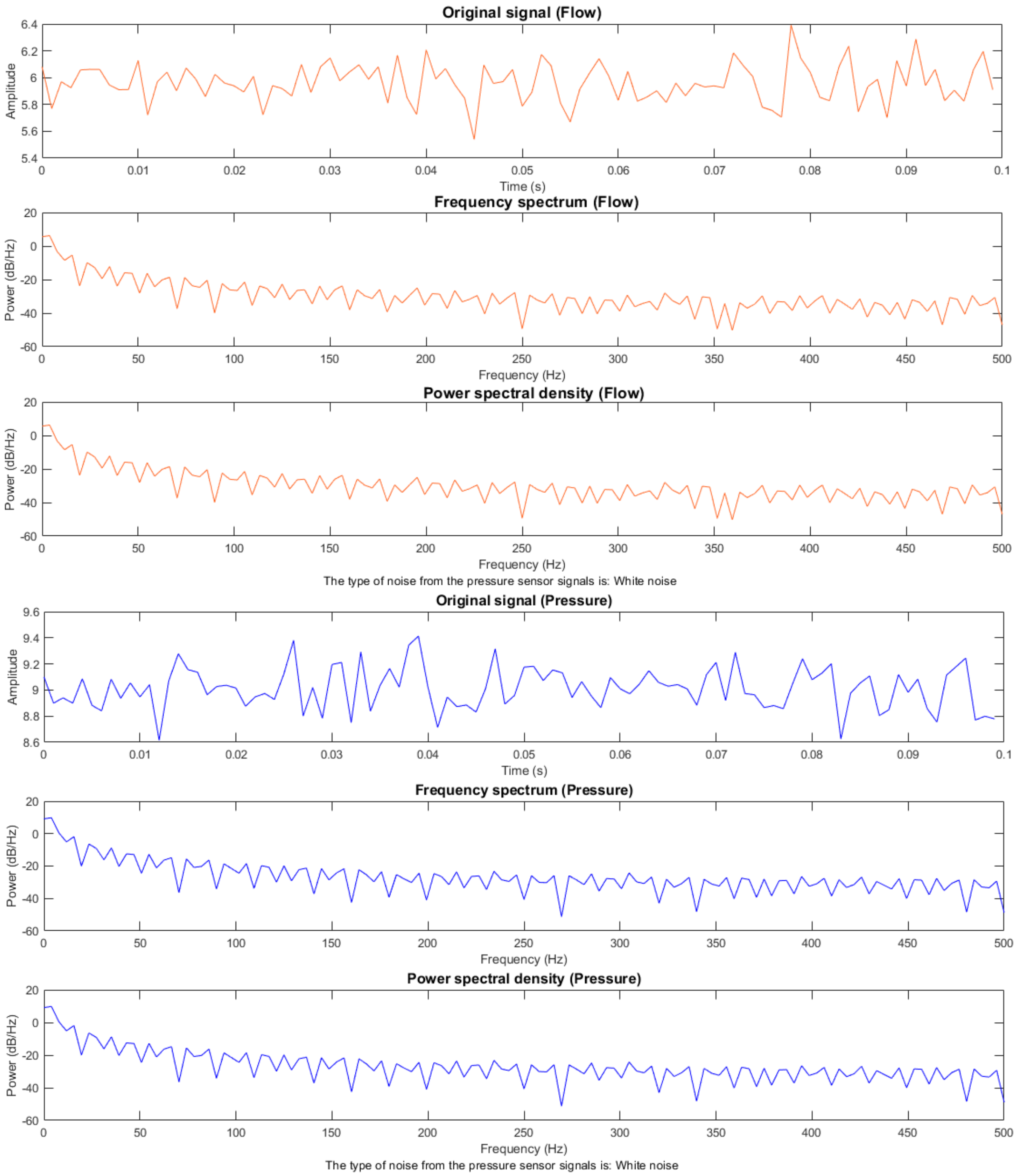


Figure 6. Event detection

The analysis of the spectrogram of the YF-S201 flow sensor allowed for the visualization of the energy distribution as a function of time and frequency. This provides detailed information regarding the variations in flow over time and the frequencies at which they occur.

For example, when observing the spectrogram, recurring patterns were identified for frequencies of 100–200 Hz, evidencing rapid high-energy fluctuations at higher frequencies, whereas at other times, the energy was concentrated at lower frequencies, indicating a more rapid flow stability (Figure 7).

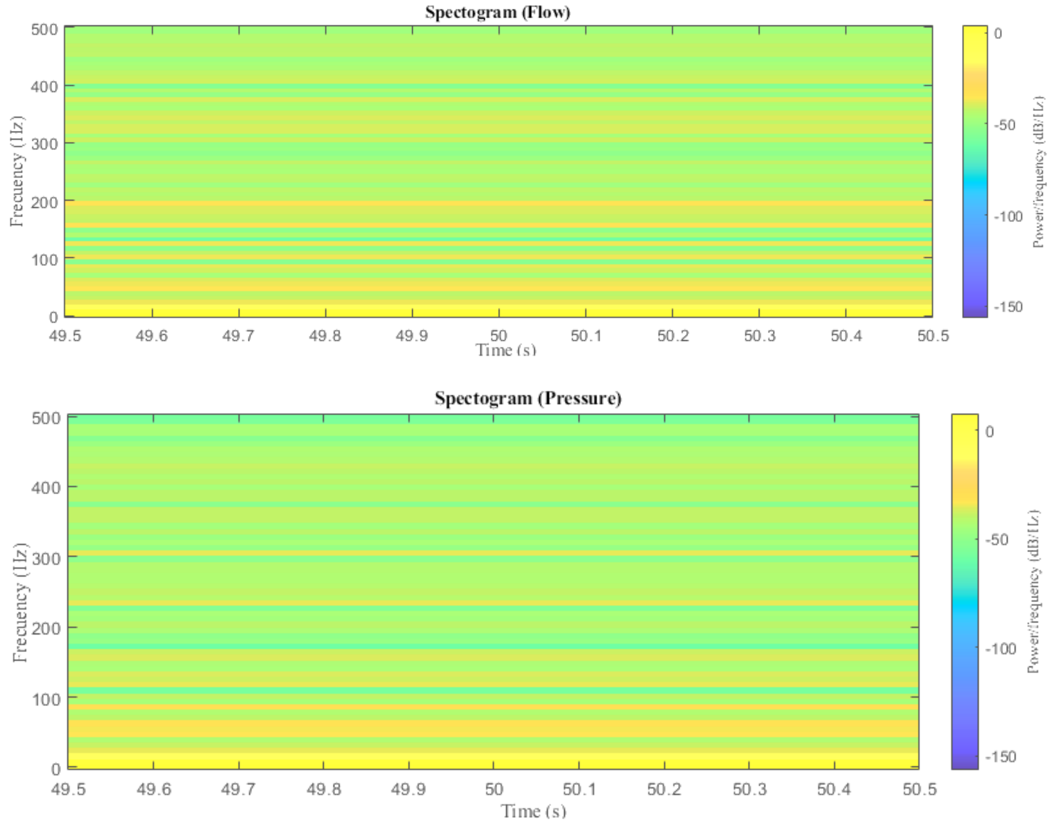


Figure 7. Spectrogram (Flow and pressure)

2.5. Flow Rate and Pressure Functions

To achieve successful coupling of the conservative Hamiltonian method, it is essential to accurately model the pressure and flow behaviors at each leak. This involves carrying out an observation and quantification of the pressure and flow in each leak, to generate a regression that optimally fits the data obtained. These modeled equations are added to the existing Hamiltonian model, thus creating a system of two differential equations that allows for energy contributions to the hydraulic system to be monitored and evaluated. This comprehensive approach provides a more complete and precise understanding of the interactions and energy flows in a system in the event of disturbances (leaks). Figure 8 shows the models for the flow rate and pressure for leak$_1$.

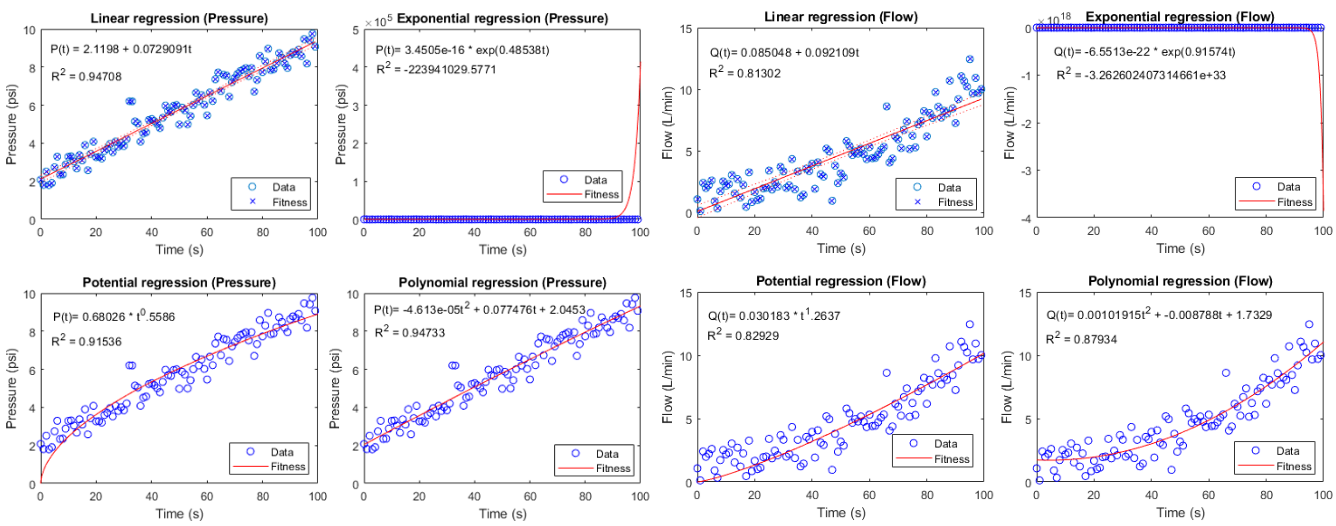


Figure 8. Flow rate and pressure models for initial leak

For leak$_2$, located 0.9 m from the upstream flow sensor, a linear regression function is established for pressure with a coefficient of determination equal to 0.948. Likewise, for the behavior of the flow rate of leak$_2$, a polynomial was obtained with a coefficient of determination equal to 0.849 (Figure 9).

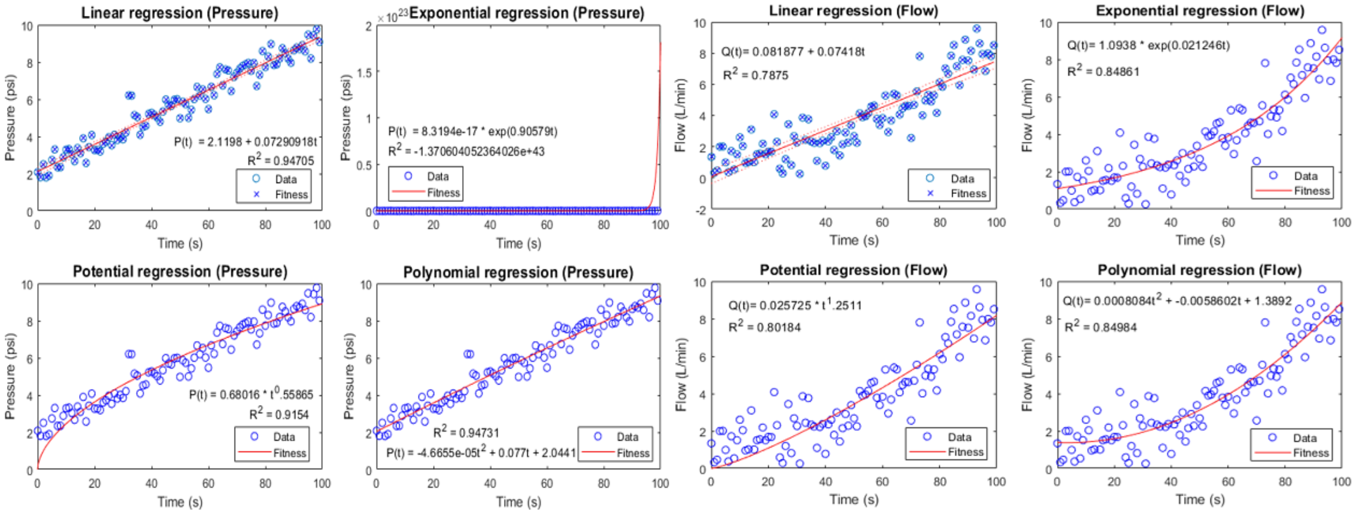


Figure 9. Flow rate and pressure models for the final leak

Table 6 presents a list of functions relating to the flow rate and pressure in the context of leaks.

Table 6. Flow rate and pressure models

Leak	Location (m)	Flow function	Pressure function
$Leak_1$	0.3	$Q_1(t) = 0.00101915t^2$-0.008788t+1.7329	$P_1(t) = 2.1198 + 0.0729091t$
$Leak_2$	0.9	$Q_2(t) = 0.00080849t^2$-0.0058602t+1.3892	$P_2(t) = 2.1195 + 0.072918t$

2.6. Hamiltonian System for Leak Detection

According to Bedford (2021) [24] the application of Hamilton's principle in classical fluid mechanics involves the use of this principle to describe and analyze the behavior of moving fluids in energetic terms. The implementation of the Hamiltonian model in the study of leaks requires mathematical models that describe the disturbances and flow rates of each leak. In this study, the real-time monitoring of pressure and flow at four strategic points along a pipeline was proposed. Four YF-S201 sensors were used to measure the flow, which provided information regarding the flow of water in the system along the pipe and at predefined leak points. These sensors were placed at the beginning of the system, at the end of the pipe, and each of the induced leaks. The mass change rate owing to leakage was modeled as a function of the flow rate concerning time. In addition, four USP-G41-1.2 sensors were installed to capture pressure records at the points mentioned above.

This allowed disturbances associated with the pressure in the system to be detected. Thus, it is essential to consider the temporality of events when analyzing dynamic systems. At the onset of a leak, a transient event occurs in which the flow is unstable and fluctuating. As time progressed, the flow tended to stabilize and approached a steady state where it remained constant. Therefore, when modeling a leak, it is necessary to consider its temporal evolution. This involves considering how the flow rate changes over time and approaches a steady state. By considering this temporal evolution in a hydraulic system in the presence of two simultaneous leaks, the incidence of leaks can be evaluated in terms of flow and pressure. First, the positions of the leaks in the pipe are identified by establishing the corresponding abscissa. Subsequently, the physical characteristics of the leak, such as the size, shape, and duration, were established. The size of the hole significantly affects the behavior of the leak over time; in this study, the hole size for each leak was equal to 2 mm. Once this was completed, the flow rate was modeled for the flow rate and disturbances in terms of pressure, determining four functions that represented the temporal evolution of the two leaks induced in the system. Subsequently, these functions were added to the mass-conservation and Bernoulli equations to configure the Hamiltonian dynamic models. Figure 10 shows the methodology proposed in this study.

Subsequently, a simulation of the model with different leakage scenarios was performed to evaluate the detection and quantification capacities of the mass generated by the two leaks. These simulations made it possible to adjust the model parameters and optimize their performance. Various tests were conducted to validate the Hamiltonian model based on the experimental model. Controlled leaks were induced in the hydraulic system and compared with the measurements obtained from model predictions. The advantage of using a Hamiltonian dynamic system is that it allows the behavior of state variables, such as the flow and pressure in the pipe, to be precisely modeled and analyzed based on the conservative properties of the system.

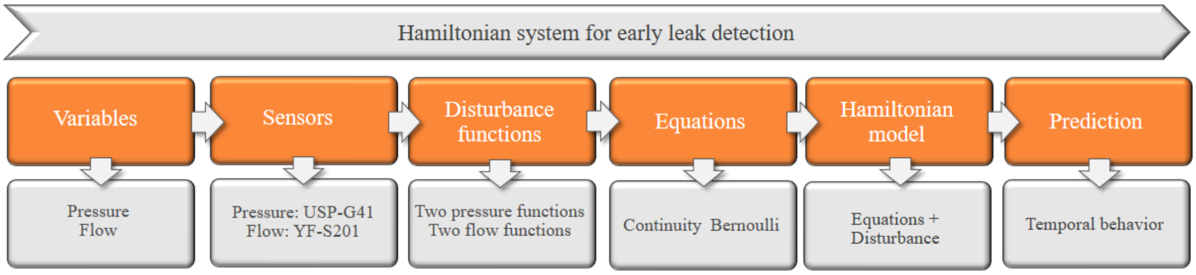


Figure 10. Hamiltonian method for leak detection

3. Results and Discussion

If we consider a pipe with two leaks and the functions for the flow and pressure in each leak, it is possible to propose a system of Hamiltonian equations that considers both the conservation of mass and energy in the pipe, as well as

$$\frac{\partial Q}{\partial t} = -\frac{\partial H}{\partial P} = -\frac{\partial (P^2/2A)}{\partial x} + Leak\big(Q_1(t)\big) + Leak\big(Q_2(t)\big) \tag{9}$$

where *Q(t)* is the flow, *P(t)* is the pressure, A is the cross-sectional area of the pipe, $Leak_1(Q_1(t))$ is leak rate$_1$ and $Leak_2(Q_2(t))$ leak rate$_2$. For the pressure in the system, we have

$$\frac{\partial P}{\partial t} = -\frac{\partial H}{\partial Q} = -\frac{\partial (Q^2/2A)}{\partial x} + Perturbation_1(P_1(t)) + Perturbation_2(P_2(t)) \tag{10}$$

where $Perturbation_1(P_1(t))$ is $Pressure_1$ in leak$_1$ and $Perturbation_2(P_2(t))$ is $Pressure_2$ in $Leak_2$. In this study, it was found that the first leak ($Leak_1$) was located at a distance of 0.3 m from the initial valve, while the second leak ($Leak_2$) was located at a distance of 0.9 m from the initial valve. Using the data collected by YF-S201 for flow and USP-G41-1.2 for pressure, four mathematical models were developed to describe the observed variables concerning time. Therefore, the system of Hamiltonian equations is represented by

$$\frac{\partial Q}{\partial t} = -\frac{\partial H}{\partial P} = -\frac{\partial (P^2/2A)}{\partial x} + 0.00101915t^2 - 0.008788\text{t} + 1.7329 + 0.00080849t^2 - 0.0058602\text{t} + 1.3892 \tag{11}$$

$$\frac{\partial Q}{\partial t} = -\frac{\partial (P^2/2A)}{\partial x} + 0.00182764t^2 - 0.0146482\text{t} + 3.1221 \tag{12}$$

For pressure, we have

$$\frac{\partial P}{\partial t} = -\frac{\partial (Q^2/2A)}{\partial x} + 2.1198 + 0.0729091t + 2.1195 + 0.072918t \tag{13}$$

$$\frac{\partial P}{\partial t} = -\frac{\partial (Q^2/2A)}{\partial x} + 4.2393 + 0.1458271t \tag{14}$$

The Hamiltonian system proposed in Equations 12 and 14 was solved using the Runge–Kutta method. Considering the length of the pipe, $L = 1.5\ m$, it was divided into 15 intervals of length (N), where $\Delta x = L/N = 1.5/15 = 0.1$, and time will move forward with one step $\Delta t = 0.1$. The spatial step (Δx) is equal to 15. Subsequently, the variables and initial conditions are initialized, for which vectors Q and P of size $N + 1$ represent the values of Q and P on each abscissa of the pipe. The initial conditions $Q(x, 0) = 0$ and $P(x, 0) = 1$ were established for all x. Then, it iterated in time for each time step $t = 0, \Delta t, 2\Delta t, \dots, T$ (where T is the desired final time) and for each point in the pipeline $i = 1, 2, \dots, N - 1$. Given the above, the spatial derivatives were calculated using centered finite differences (Figure 11).

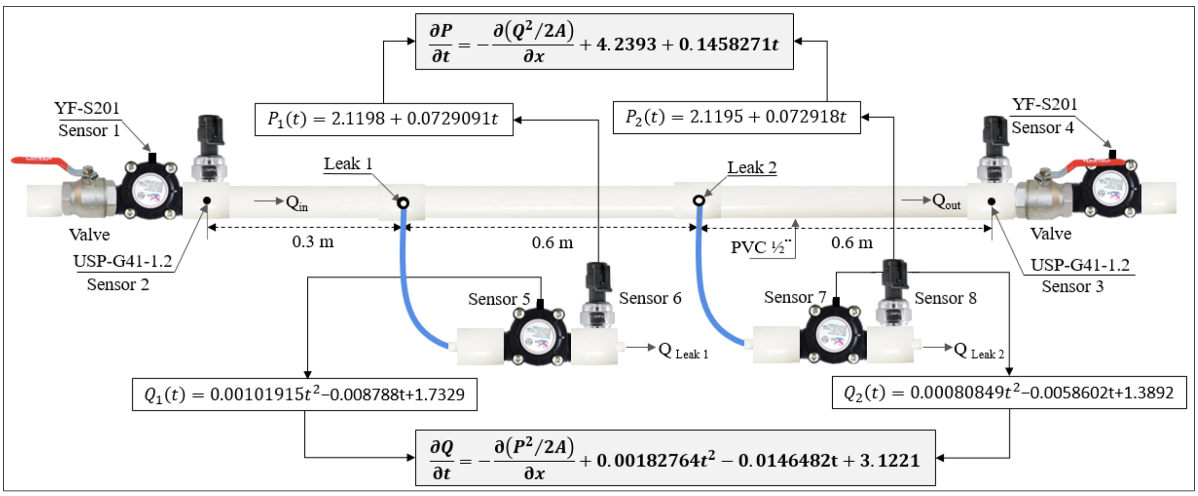


Figure 11. Assembly system of differential equations

Applying the fourth-order Runge–Kutta method to solve the Hamiltonian ensemble, we obtain

$$k_1 = h \cdot f(t_n, P_n) \tag{15}$$

$$k_2 = h \cdot f\left(t_n + \frac{h}{2}, P_n + \frac{k_1}{2}\right) \tag{16}$$

$$k_3 = h \cdot f\left(t_n + \frac{h}{2}, P_n + \frac{k_2}{2}\right) \tag{17}$$

$$k_4 = h \cdot f(t_n + h, P_n + k_3) \tag{18}$$

where h is the time step, t_n is the time at step n, P_n is the value of P at step n, and $f(t, P)$. t is the function on the right side of the difference equation; thus, the update is given by

$$P_{n+1} = P_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{19}$$

In terms of pressure, we have;

$$\frac{\partial(P^2/2A)}{\partial x} \tag{20}$$

For the flow, we have

$$\frac{\partial(Q^2/2A)}{\partial x} \tag{21}$$

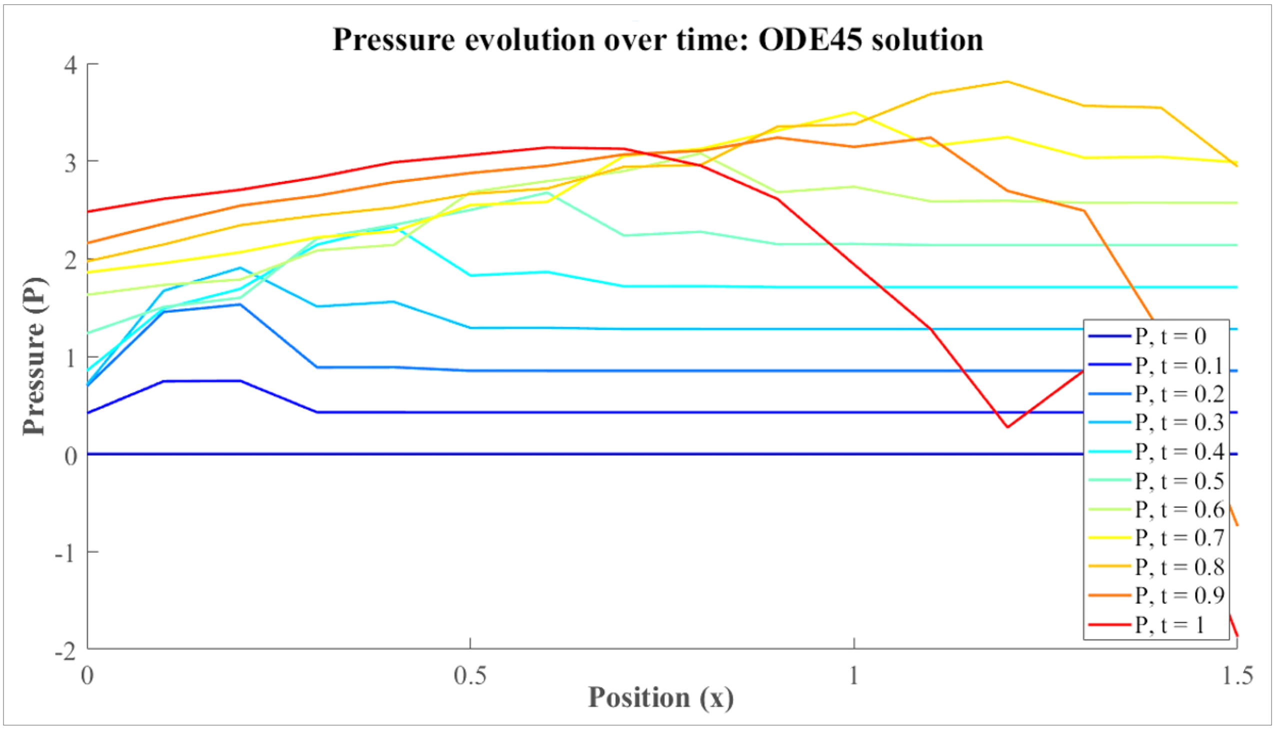
Then, the time derivatives are calculated as $\frac{\partial Q}{\partial t}$ and $\frac{\partial p}{\partial t}$ using the equations of the Hamiltonian system. Next, the fourth-order Runge-Kutta method was applied to update the values of Q and P. From the calculated derivatives, we obtained $k_1Q, k_1P, k_2Q, k_2P, k_3Q, k_3P, k_4Q, k_4P$, where the values of Q and P were updated.

$$Q[i] = Q[i] + (\Delta t/6)(k_1Q + 2k_2Q + 2k_3Q + k_4Q) \tag{22}$$

For the pressure, we have

$$P[i] = P[i] + (\Delta t/6)(k_1P + 2k_2P + 2k_3P + k_4P) \tag{23}$$

Finally, boundary conditions were applied as $P(0, t) = P(L, t) = 1$, to update the values of $P[0]$ and $P[N]$. Figure 12 presents the solution system of differential equations using the fourth-order Runge–Kutta method (ODE45), where the evolution of the state variables is graphed as Q and P at that time. After the ODE45 algorithm was used to solve the system of differential equations that modeled the pressure in a pipe, it was found that there was a minimum pressure equal to 0.3 psi for one second at a distance of 1 m from the pipe. This result indicates that at a specific point in the pipe, there was a significant reduction in the pressure of the fluid caused by the appearance of leaks 1 and 2. The solution obtained using the ODE45 method allowed us to analyze the evolution of the pressure over time and position in the pipeline.



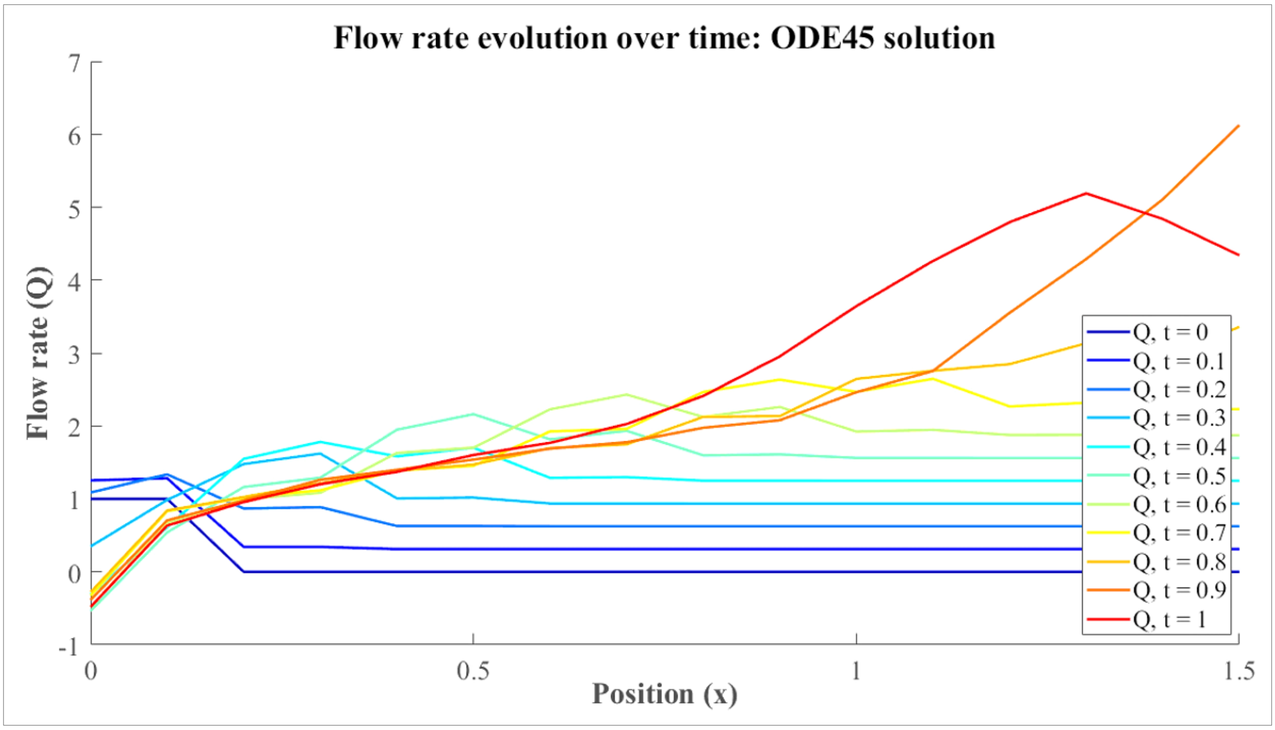
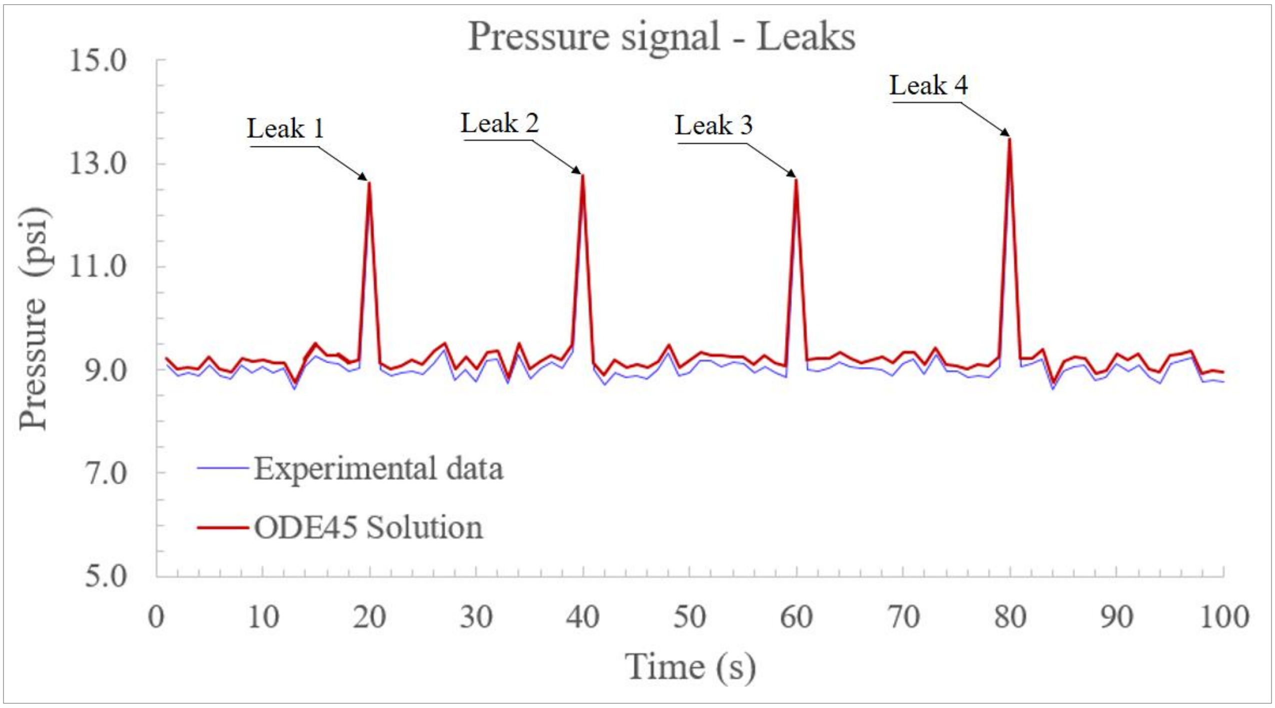


Figure 12. ODE45 Solution (Pipe Length)

Figure 13 shows a comparison of the results obtained using the Hamiltonian model and the data observed in the experimental model. The precision of the pressure and flow quantification for each leak was evaluated, and an accuracy level of 95% was obtained. This demonstrates the ability of the numerical model to accurately estimate the mass and position of each leak.



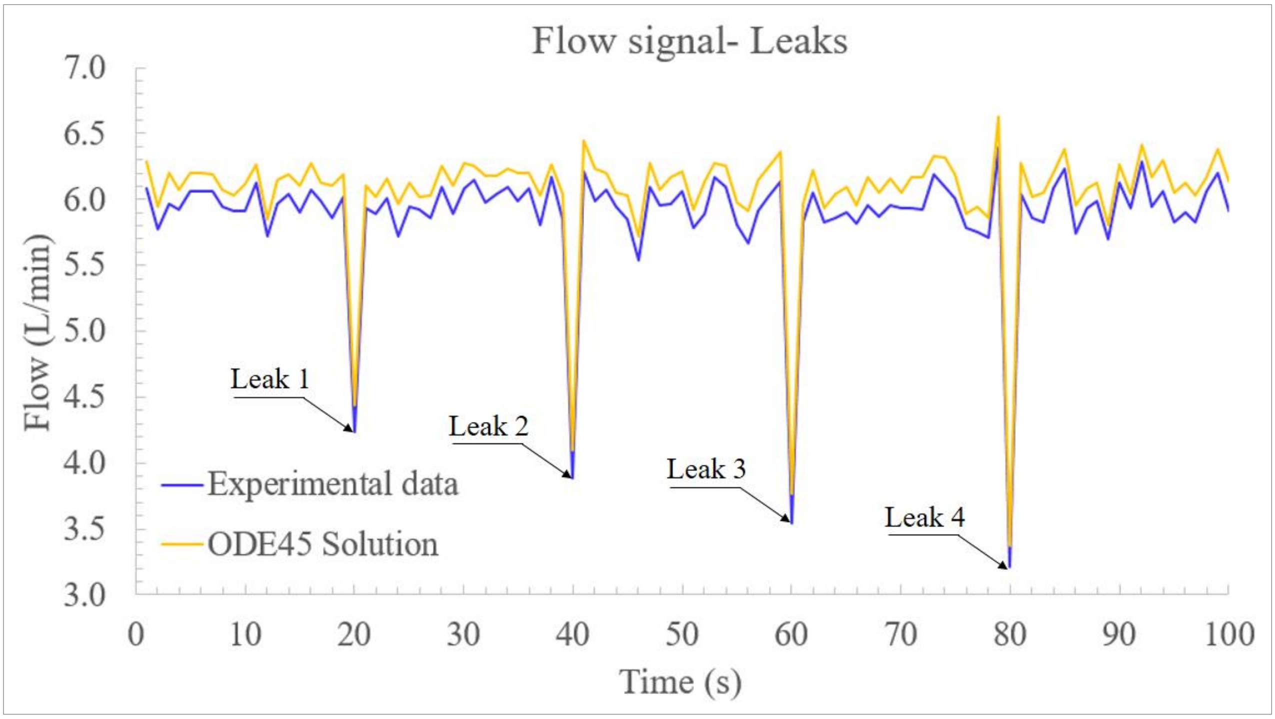


Figure 13. ODE45 Solution (Time)

4. Conclusion

The development of a conservative Hamiltonian dynamic system for the early detection of leaks in pressure pipelines represents a significant innovation in applied research and contributes significantly to the field of applied mathematical modeling. This study demonstrates how this approach can be used to understand and predict the behavior of dynamic systems, such as hydraulic networks. The fusion of conservative Hamiltonian equations with IoT technologies and the implementation of a hybrid model proved to be highly effective, achieving an accuracy of 95% for leak detection. This level of precision validated the ability of the proposed model to identify anomalies in real-time, thus contributing to improvements in the management of drinking water distribution systems.

The fusion of mathematical models and IoT technologies offers a comprehensive methodology for addressing the challenge of detecting and quantifying leaks in hydraulic systems in real-time. The application of filters to sensor signals, identification of patterns in the spectral frequency, and successful coupling of the flow rate and pressure functions demonstrate the effectiveness of integrating mathematical models and advanced technologies in leak detection and monitoring. The application of the conservative Hamiltonian model in the early detection of leaks can contribute to the efficient management of water resources because of the model's ability to maintain the total energy of the system constant, monitor it in real-time, and generate synchronous responses for decision-making when an event occurs. Finally, the ability to model and understand the interactions between variables in a multi-perturbed system demonstrates the applicability of the conservative Hamiltonian approach to real-world situations. This contribution is relevant to researchers interested in the analysis and modeling of complex systems in various disciplines.

5. Declarations

5.1. Author Contributions

Conceptualization, C.A.G.U. and E.Z.H.; methodology, C.A.G.U., E.Z.H., and E.O.L.M.; software, E.O.L.M.; validation, E.O.L.M.; formal analysis, C.A.G.U., E.Z.H., and E.O.L.M.; investigation, E.O.L.M.; data curation, C.A.G.U. and E.Z.H.; writing—original draft preparation, C.A.G.U., E.Z.H., and E.O.L.M.; writing—review and editing, C.A.G.U., E.Z.H., and E.O.L.M.; visualization, E.O.L.M.; supervision, E.O.L.M. All authors have read and agreed to the published version of the manuscript.

5.2. Data Availability Statement

Data supporting the findings of this study are available upon request from the corresponding authors.

5.3. Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

5.4. Acknowledgements

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5.5. Conflicts of Interest

The authors declare no conflict of interest.

6. References

[1] Xie, J., Huang, B., & Dubljevic, S. (2024). Moving Horizon Estimation for Pipeline Leak Detection, Localization, and Constrained Size Estimation. doi:10.2139/ssrn.4730270.

[2] Speziali, S., Bianchi, F., Marini, A., Menculini, L., Proietti, M., Termite, L. F., Garinei, A., Marconi, M., & Delogu, A. (2021). Solving Sensor Placement Problems in Real Water Distribution Networks Using Adiabatic Quantum Computation. IEEE International Conference on Quantum Computing and Engineering (QCE), Broomfield, Colorado, United States. doi:10.1109/qce52317.2021.00079.

[3] Verde, C., & Torres, L. (2017). Modeling and monitoring of pipelines and networks. Springer, Cham, Switzerland. doi:10.1007/978-3-319-55944-5.

[4] Rodríguez Calderón, W., & Pallares Muñoz, M. R. (2007). A numerical water-hammer model using Scilab. Ingeniería e Investigación, 27(3), 98–105. doi:10.15446/ing.investig.v27n3.14850. (In Spanish).

[5] Firouzi, A., Yang, W., Shi, W., & Li, C.-Q. (2021). Failure of corrosion affected buried cast iron pipes subject to water hammer. Engineering Failure Analysis, 120, 104993. doi:10.1016/j.engfailanal.2020.104993.

[6] Sánchez-Jiménez, A. D., Torres, L., & López-Estrada, F. R. (2020). Euler-Lagrange approach for modeling water pipelines with leaks. Memorias del Congreso Nacional de Control Automático, 1-7.

[7] Macias, G., & Lee, K. (2022). Optimal Gas Leak Localization and Detection using an Autonomous Mobile Robot. Proceedings of the 9th International Conference of Control, Dynamic Systems, and Robotics (CDSR'22), Niagara Falls, Canada. doi:10.11159/cdsr22.122.

[8] Schneider, J., Tél, T., & Neufeld, Z. (2002). Dynamics of "leaking" Hamiltonian systems. Physical Review E - Statistical Physics, Plasmas, Fluids, and Related Interdisciplinary Topics, 66(6), 6. doi:10.1103/PhysRevE.66.066218.

[9] Torres, L., & Besancon, G. (2019). Port-Hamiltonian Models for Flow of Incompressible Fluids in Rigid Pipelines with Faults. 2019 IEEE 58th Conference on Decision and Control (CDC), Nice, France. doi:10.1109/cdc40024.2019.9029170.

[10] Perryman, R., Taylor, J. A., & Karney, B. (2022). Port-Hamiltonian based control of water distribution networks. Systems & Control Letters, 170. doi:10.1016/j.sysconle.2022.105402.

[11] Lopezlena, R. (2014). Computer implementation of a boundary feedback leak detector and estimator for pipelines II: Leak estimation. Memorias del XVI Congreso Latinoamericano, 14-17 October, 2014, Cancún, Mexico.

[12] Rashad, R., Califano, F., Schuller, F. P., & Stramigioli, S. (2021). Port-Hamiltonian modeling of ideal fluid flow: Part II. Compressible and incompressible flow. Journal of Geometry and Physics, 164. doi:10.1016/j.geomphys.2021.104199.

[13] Beattie, C., Mehrmann, V., Xu, H., & Zwart, H. (2018). Linear port-Hamiltonian descriptor systems. Mathematics of Control, Signals, and Systems, 30(4). doi:10.1007/s00498-018-0223-3.

[14] Zi Li, L., Rosli, M. I., & Panuh, D. (2019). Velocity Modelling for Pipeline Inspection Gauge. Jurnal Kejuruteraan, 31(2), 275–280. doi:10.17576/jkukm-2019-31(2)-11.

[15] Bendimerad-Hohl, A., Matignon, D., Haine, G., & Lefèvre, L. (2024). On implicit and explicit representations for 1D distributed port-Hamiltonian systems. arXiv Preprint, arXiv:2402.07628. doi:10.48550/arXiv.2402.07628.

[16] Zhang, J., Zhu, Q., & Lin, W. (2024). Learning Hamiltonian neural Koopman operator and simultaneously sustaining and discovering conservation laws. Physical Review Research, 6(1), 12031. doi:10.1103/PhysRevResearch.6.L012031.

[17] Eidnes, S., Stasik, A. J., Sterud, C., Bøhn, E., & Riemer-Sørensen, S. (2023). Pseudo-Hamiltonian neural networks with state-dependent external forces. Physica D: Nonlinear Phenomena, 446, 446. doi:10.1016/j.physd.2023.133673.

[18] Sultana, S., & Rahman, Z. (2013). Hamiltonian Formulation for Water Wave Equation. Open Journal of Fluid Dynamics, 3(2), 75–81. doi:10.4236/ojfd.2013.32010.

[19] Przystupa, K., Ambrożkiewicz, B., & Litak, G. (2020). Diagnostics of Transient States in Hydraulic Pump System with Short Time Fourier Transform. Advances in Science and Technology Research Journal, 14(1), 178–183. doi:10.12913/22998624/116971.

[20] Ji'e, M., Yan, D., Sun, S., Zhang, F., Duan, S., & Wang, L. (2022). A Simple Method for Constructing a Family of Hamiltonian Conservative Chaotic Systems. IEEE Transactions on Circuits and Systems I: Regular Papers, 69(8), 3328–3338. doi:10.1109/TCSI.2022.3172313.

[21] Phipps Electronics. (2024). YF-S201 Hall Effect Water Flow Meter / Sensor. Phipps Electronics, North Revesby, Australia. Available online: https://www.phippselectronics.com/product/yf-s201-hall-effect-water-flow-meter-sensor/ (accessed on March 2024).

[22] Urbanowicz, K., Duan, H. F., & Bergant, A. (2020). Transient liquid flow in plastic pipes. Strojniski Vestnik/Journal of Mechanical Engineering, 66(2), 77–90. doi:10.5545/sv-jme.2019.6324.

[23] Kulmány, I. M., Bede-Fazekas, Á., Beslin, A., Giczi, Z., Milics, G., Kovács, B., Kovács, M., Ambrus, B., Bede, L., & Vona, V. (2022). Calibration of an Arduino-based low-cost capacitive soil moisture sensor for smart agriculture. Journal of Hydrology and Hydromechanics, 70(3), 330–340. doi:10.2478/johh-2022-0014.

[24] Bedford, A. (2021). Hamilton's Principle in Continuum Mechanics. Springe, Cham. Switzerland. doi:10.1007/978-3-030-90306-0.

Appendix I: Matlab Encoding ®: ODE45

```
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Microcontroller: Esp8266
www.edgarladino.com
-------------------------------------------//

------------Development of a conservative Hamiltonian dynamic system for the early detection of leaks
in pressure pipes-----------//

function main()
    L = 1.5; % Pipe length
    N = 15; % Number of intervals
    dx = L/N; % space step
    SD = 0.1; % Time step
    T = 1; % Final time
% Define initial conditions
    x = 0:dx:L;
    Q0 = zeros(size(x));
    P0 = zeros(size(x));
    Q0(1) = 1; % Initial condition for Q at x = 0
    P0(1) = 1; % Initial condition for P at x = 0
% Solve the system of partial differential equations
    [t, y] = ode45(@rhs, 0:dt:T, [Q0; P0]);
% Plott Q
    figure;
    hold on;
    colors = parula(length(t));
    for i = 1:length(t)
        plot(x, y(i, 1:N+1), 'Color', colors(i,:), 'DisplayName', ['Q, t = ', num2str(t(i))]);
    End
    xlabel('x');
    ylabel('Q');
    legend('show');
    hold off;
% P Chart
    figure;
    hold on;
    for i = 1:length(t)
        plot(x, y(i, N+2:end), 'Color', colors(i,:), 'DisplayName', ['P, t = ', num2str(t(i))]);
    End
    xlabel('x');
    ylabel('P');
    legend('show');
    hold off;
    function dydt = rhs(t, y)
        A = 1; % Pipe Area
        Q = y(1:N+1);
        P = y(N+2:end);

        dQdt = -gradient(P.^2/(2*A), dx) + 0.00182764*t^2 - 0.00146482*t +3.1221;
        dPdt = -gradient(Q.^2/(2*A), dx) + 4.2393 + 0.1458271*t;

        dydt = [dQdt; dPdt];
    End
End
```