Available online at www.CivileJournal.org

Civil Engineering Journal

Vol. 3, No. 11, November, 2017

Estimation of Origin – Destination Matrix from Traffic Counts Based On Fuzzy Logic

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Received 06 October 2017; Accepted 29 November 2017

Abstract

Determining trip demand matrix is among the basic data in transportation planning. This matrix is derived by surveys, interviews with citizens or questionnaires that required time, money and manpower. Thus, in recent years, demand estimation methods based on network information is taken into consideration. In these methods with the information including: volume, travel time, capacity of the links and initial demand matrix it is possible to estimate the demand matrix. In this paper, we removed the additional parameters in previous studies and used a simple solution to estimate the matrix. This paper proposes a Fuzzy-PFE estimation method that allows to improve the estimation performances of PFE estimator. The objective function presented based on the reduction of travel time and travel time of routs in networks is uncertain. The method is developed by fuzzy sets theory and fuzzy programming that seems to be convenient theoretical framework to represent uncertainty in the available data. The new model is the removal of iterative process of origin - destination matrix estimation using travel time and increase convergence of the model for the large-scale and congested networks by applying little changes in the basic model. In this paper we used TRANSCAD Software to determine the shortest path in the network and optimization of objective function is performed by CPLEX.

Keywords: Origin – Destination Matrix; Traffic Volume; Fuzzy Logic; CPLEX.

1. Introduction

Traditional methods of estimating ODM are through large scale sampled surveys like home interview survey, roadside interview and license plate method conducted once in every 1-2 decades. But in situations of financial constraints these surveys become impossible to conduct. And by the time the survey data are collected and processed, the O-D data obtained become obsolete [1]. Effective and theoretically consist methodologies have been proposed to estimate origin-destination matrices by using aggregate data such as traffic counts and /or demand counts. The use of information derived from traffic counts is attractive because they are cheap, easy and immediate data to collect. Usually, the basic information required by these classical methodologies are an initial estimate of the O_D matrix (i.e. target O-D demand) and a set of traffic counts observed on the links of the considered network [2].

Actually, in addition to the target demand and traffic counts, other sources of information could be available (i.e. expert knowledge about demand flows and link flows, spot data on trip matrix, outdated trip matrices) as well as data can be collected with different methods. This information in general is affected by a level of uncertainty or can be

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http://dx.doi.org/10.28991/cej-030946

incomplete. Uncertain, imprecise or vague data are often used in transportation science and engineering, so the handling of such a problem has become a great challenge for researchers.

Recently, great attention has been given to new paradigms developed in theoretical framework of Fuzzy Set in which fuzzy Logic and Possibility Theory are the mathematical tools most used for solving transportation problem [3]. Few authors have studied the opportunity to consider also this knowledge together with the objective information on O-D matrix estimation. Link volumes information is used by Shafahi and Foturechi that suggest to improve the method described in Spiess (1990) by estimating the link flow by a fuzzy set based assignment method [2].

2. Literature Review

Initially the researchers tried to relate the trip matrix as a function of models (like the gravity models) with related parameters. Some of the researchers like Robillard (1975), Hogberg (1976) used Gravity (GR) model based approaches and some (Tamin and Willumsen, 1989; Tamin et al, 2003) used Gravity-Opportunity (GO) based models for estimating ODM. These techniques require zonal data for calibrating the parameters of the demand models. The main drawback of the gravity model is that it cannot handle with accuracy external-external trips [3].

Recently, models based on path flow estimator which determines ODM according to the solutions of path flows have been adopted. It is a single level mathematical program in which the interdependency between O-D trip table and route choice proportion (congestion effect) is taken into account. The core component of Path Flow Estimation (PFE) is a Logit based path choice model, which interacts with link cost functions to produce a stochastic user equilibrium traffic pattern. Sherali et al (1994) proposed a linear path flow model employing user equilibrium based solution for reproducing the observed link flows (known for all links) [4]. The procedure utilizes shortest path network flow programming sub-problem and a column generation technique is applied to generate the paths out of alternate paths that will determine the optimal solution to the linear programming model. To avoid the path enumeration required in the model proposed by Sherali et al (1994), Nie and Lee (2002) solved the linear programming model considering an exogenous K-shortest-path for determining the equilibrium path flow pattern. Nie et al (2005) further extended the decoupled path flow estimator by Nie and Lee (2002) considering the generalized least squares framework in aspect of the limitations of the linear programming structure [1]. Sherali et al (2003) enhanced the linear programming model of Sherali et al (1994) for situations where only a partial set of link volumes are available. This introduces nonlinear cost function because of the dependence of the link travel cost on link volumes and a fixed point solution is tried. Further tests using larger and real-size networks are required with these PFE based models for better assessment and efficiency checking of these models [5].

Sherali et al (2003) have developed a successive approximation, sequential linear programming approach for computing a fixed point for a nonlinear model that is designed to estimate an OD trip-table, given incomplete link flow information on the network. The approach utilizes shortest path network flow subproblems in order to implicitly determine a path decomposition of flow that tends to reproduce the observed flows as closely as possible, and that determines missing link volumes as driven by user-equilibrium principles. The approach is also designed to accommodate the case in which it is required to produce a solution that has a tendency to match a specified, prior trip-table, perhaps from among several alternative equilibrium solutions that replicate the observed link volumes [5].

Toledo and Kolechkina (2013) presented linear approximations of the assignment matrix using traffic counts related to specific road segments and other past demand information. For congested networks, Frederix et al. (2014) estimated dynamic O-D matrices using a hierarchical decomposition scheme. The main idea consists in distinguishing the congested subareas for estimating more accurate O-D matrices. Besides, Djukic et al. (2014) introduced a new formulation based on Kalman filter where they demonstrate an effective quality improvement of the O-D matrix estimation. In addition, Perrakis et al. (2012) proved that good estimates of O-D flows can be derived from historical data (e.g. census) by applying Bayesian statistics. Validation tests of their model suggested good predictive results (Perrakis et al., 2015) [6].

The basic elements of the theory of fuzzy sets were introduced by Zadeh (1965) [7] and their application to linear programming in a fuzzy environment has been popularised by Zimmermann (1983) and Rommelfanger (1996), among others. We now discuss some of the research that has been reported in the literature on the application of fuzzy sets and fuzzy logic to O–D matrix estimation. Reddy and Chakroborty (1998) developed a fuzzy bi-level inference based assignment method and applied a maximum entropy model in the upper level [8]. Nanda and Kikuchi (1993) presented a two-stage method for when trip generation and trip attraction counts are approximate. In the first stage the existence of a consistent travel pattern is investigated. If a pattern is found to exist then back-propagation neural networks are used to identify an estimated O–D matrix in the second stage. Biletska et al. (2009) developed a dynamic two-stage method for short-time O–D matrix estimation at just a single signalized intersection for a traffic light cycle using fuzzy-timed high-level Petri nets. Since the data used to estimate the matrix are imprecise, the authors represent them as fuzzy numbers. Jassbi et al. (2011) have presented a three phased variables to O–D matrix entries. Fuzzy rule bases in the model rely upon transportation experts' subjective patterns. Despite the potential of using fuzzy logic for O–D matrix estimation, there has been little attention given to the topic in the literature. Furthermore, as far as we are aware, the method presented in this paper is the first to use the combination of fuzzy linear programming (FLP) and successive linear approximation (SLA) for this challenge with incomplete and imprecise data [9].

Les.R.Foulds et al (2013), have reviewed issues concerned with the estimation of OD matrices in congested urban traffic networks when the input data is incomplete and imprecise. They have presented an iterative linear estimation approach, called FLIPSOD, that utilizes the theory of fuzzy sets in order to deal with the imprecision and incompleteness of the given input estimates. Sometimes a user–equilibrium assignment that reflects the given input data does not exist. In this case FLIPSOD has the useful feature that it provides a range of traffic assignments and their corresponding OD matrix estimates, reflecting the spectrum within the range between insistence on the best estimates within fuzzy limits and a user–equilibrium assignment [9].

Saadi et al (2017) propose a RF based approach for estimating OD matrices using travel surveys. Most of the existing strategies adopt daily traffic counts for estimating O-D pairs, although using traffic counts can present some limitations. They have opted for a travel survey to calibrate the O-D matrix [6].

3. Methodology

3.1. Specifying of New Model Based on Sherali Model

The model that used in Sherali research is displays in Equation 1.

$$minimize \sum_{(i,j)\in OD} \sum_{K=1}^{n_{ij}} C_{ij}^K x_{ij}^K + M \sum_{a\in A} (y_a^+ + y_a^-) + M_\sigma \sum_{(i,j)\epsilon \overline{OD}} (Y_{ij}^+ + Y_{ij}^-)$$

$$subject\ to\ \ \sum_{(i,j)\in OD} \sum_{K=1}^{n_{ij}} (p_{ij}^k)_a x_{ij}^K + y_a^+ - y_a^- = \overline{f_a} \quad \forall a \in A \tag{1}$$

$$\sum_{K=1}^{n_{ij}} x_{ij}^K + Y_{ij}^+ - Y_{ij}^- = Q_{ij} \quad \forall (i,j) \in \overline{OD}$$

$$x \geq 0\,, y^+ \geq 0, y^- \geq 0, Y^+ \geq 0, Y^- \geq 0$$

$$C_{ij}^k = \begin{cases} c.p_{ij}^k \equiv c_{ij}^k & \forall K \in K_{ij} \\ M_1 c.p_{ij}^k \equiv M_1 c_{ij}^k & \forall K \in \overline{K_{\iota J}} \qquad M_1 \geq 0 \end{cases}$$

That:

C_{ij}^K: a travel time impedance or cost on rout K between O-D pair (i, j)

x_{ij}^K: a flow on the path

M and M_σ are fine parameters for dummy variables related to the links and paths

$\overline{f_a}$: observed flow in the links

$\overline{OD}$: origin- destinations the trip exchange matrix of which is determined

$y^- and\ y^+$: are two nonnegative artificial variable vectors that having respective components for each link $a \in A_v$

For each $(i,j) \in \overline{OD}$, the deviation of the OD trip interchange T_{ij} from the target trip-table value Qij is recorded by the difference of two nonnegative (‘‘artificial’’) variables $Y^- and\ Y^+$.

Q_{ij} is the objective trip table matrix

$K_{ij} = \{k \in \{1, \dots, n_{ij}\}: c_{ij}^k = c_{ij}^*$

$c_{ij}^* = \min\{c_{ij}^k, k = 1, \dots, n_{ij}\}$

$\overline{K_{\iota J}} = \{1, \dots, n_{ij}\} - K_{ij}$

The model presented by Sherali et al (2003) (Equation 1), considered a very important role for shortest path between each origin to destination. M_1 Parameter always greater than 1 and means that drivers in network uses shortest path to arriving your destination and don’t use another routs while is not true. In this paper, we assumed that drivers uses of any routs from origin to destination by TRANSCAD.

In this paper the travel time information of the links are transferred to TRANSCAD software. We use a travel time function (BPR) for determining of travel time based on free flow time. Then, with some changes and the use of fuzzy logic in the model that presented by Sherali et al, equation 2 is presented as the final model.

$$\begin{aligned}
&minimize \sum_{(i,j)\in OD} \sum_{K=1}^{n_{ij}} \widetilde{C_{ij}^{K}} x_{ij}^{K} + M \sum_{a\in A} (y_a^+ + y_a^-) + M_\sigma \sum_{(i,j)\epsilon OD} (Y_{ij}^+ + Y_{ij}^-) \\
&subject\ to \left(\theta \sum_{(i,j)\in OD} \sum_{K=1}^{n_{ij}} \left(p_{ij}^{k}\right)_a x_{ij}^{K} \right) + y_a^+ - y_a^- = \beta\, \overline{f_a} \quad \forall a \in A \\
&\textstyle\sum_{K=1}^{n_{ij}} x_{ij}^{K} + Y_{ij}^+ - Y_{ij}^- = Q_{ij} \quad \forall (i,j) \in OD \\
&x \geq 0\,, y^+ \geq 0, y^- \geq 0, Y^+ \geq 0, Y^- \geq 0 \\
&C_{ij}^{k} = c.p_{ij}^{k} \ \equiv \ c_{ij}^{k} \qquad\qquad \forall K \in K_{ij}
\end{aligned} \tag{2}$$

In this model we used parameter θ that presented as 0 or 1. This parameter is 1 for the links that volume of these are observed and it is 0 for the other links. In addition, the travel time is considered a fuzzy parameter that is transferred into a crisp number by ranking function and included in the new model. However, in this model problem is solved by defining the parameter θ and it is no need for change in the objective function. In addition the optimal condition for the model would be when the dummy variables are zero that this solution can be used to determine the computational volume in CPLEX optimization software.

3.2. Ranking Function

The ranking function is approach of ordering fuzzy numbers which is an efficient. Various types of ranking function have been introduced which are used for solving linear programming problems with fuzzy parameters. When using ranking function for comparison of fuzzy linear programming problem. Usually define a crisp model which is equivalent to the fuzzy linear programming problem, then using the optimal solution of this model as the optimal solution for fuzzy linear programming problem. There are various types of fuzzy numbers, but the triangular and trapezoidal are the most important fuzzy memberships. In this research we use the trapezoidal fuzzy numbers .In fact, the fuzzy number is defined by its corresponding membership function. A trapezoidal fuzzy number can be shown by $\tilde{a} = (a^l, a^u, \alpha, \beta)$ and the Yager ranking function is:

$$R(\tilde{a}) = \frac{1}{2}\left(a^l + a^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right) \tag{3}$$

Figure 1. shown the trapezoidal membership function for fuzzy number which is as follows:

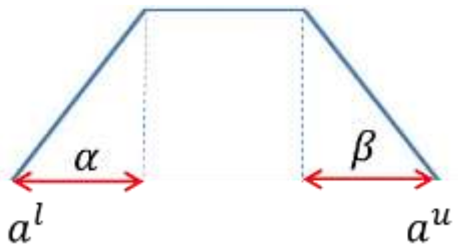

Figure 1. Trapezoidal membership function

In this paper we build the network in TRANSCAD software. Firstly the existing nodes should be plotted and the links should be designed to transfer the nodes to the network. Each link is obtained by connecting of two nodes. For travel time function, we need input parameters such as α and β. These two parameters are used to trip time function of the links Burea Public Roads. Their numerical value is based on the regulation $\alpha = 0.15$ and β = 4. After the input data the data set should be integrated that is done by network creation.

By determining the number of possible paths between any pair of origin- destination and determining the travel time for each path, by taking the output in the format Shape of the TRANSCAD software, the rest of activity should be performed in ARCGIS software. Path and arc information are combined in ARCGIS application and it is determined from which links the path between a pair of origin – destination. It should be noted that the number of selected paths for each network varies based on the network dimensions. For Corridor network which is a small network all selected paths for each origin - destination pair are determined and the information of all paths are transmitted to the CPLEX software to determine the final matrix. But for the Golpayegan network for each origin - destination pair 100 separate paths are chosen and the number of paths along the network information will be transferred to the respective application. To determine the links through which the paths pass, the combined information are transferred to Excel and by coding in Macro Excel the new data are presented as a 0 and 1 matrix.

After determining the basic data for the model, now the model is optimized which is done by CPLEX software. In order to use the basic information usable in the CPLEX optimization software it is required to link the data to the relevant application.

The information on each path including travel time of the existing paths between each Origin – destination pair is stored in Excel. After finishing this process and coding in CPLEX software the Run of software is used and the Origin – destination matrix is determined. In addition to the traffic allocation performed by TRANSCAD software, the CPLEX software presents a different computational volume by determining the parameters that determine the positive and negative deviation from the volume and origin – destination matrix that this computational volume is compared by the computational volume determined by TRANSCAD software and the existing difference is determined.

4. Computational Results

In this section we present some preliminary computational experience with new model on two sample networks.

4.1. Corridor Network

In this section, we report results comparing new model in CPLEX with User Equilibrium assignment and Stochastic User Equilibrium assignment in TRANSCAD for link volume.

Table 1. Corridor Network characteristics

Link Index	Node		Observed link volume	Capacity	Free Flow cost
	From	To			
1	4	9	2400	2526	8.9
2	10	5	1600	2105	8.9
3	6	5	100	105	35.65
4	6	7	5000	5263	8.9
5	6	8	500	526	8.9
6	7	1	500	526	8.9
7	7	9	4500	4736	17.82
8	8	10	500	526	17.82
9	9	4	2000	2105	8.9
10	9	10	1500	1579	8.9
11	9	11	4900	5157	17.82
12	5	10	2000	1684	8.9
13	10	9	1500	1579	8.9
14	10	12	900	947	17.82
15	11	2	4800	5052	17.82
16	11	12	300	316	8.9
17	12	3	1000	1053	17.82
18	12	11	200	211	8.9

The pertinent characteristics of this network consists of 6 zones, 6 intersection nodes and 18 links are displayed in Table 1. Figure 2 presented the links, nodes and origin – destination pair for Corridor network.

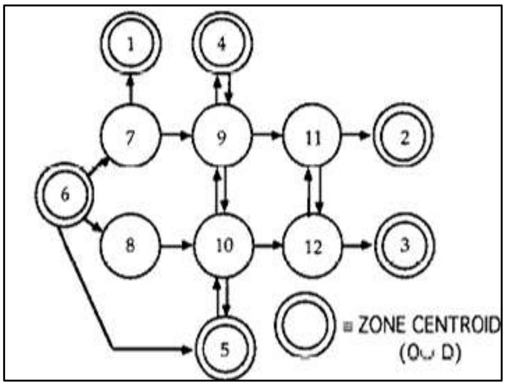

Figure 2: Links and nodes of Corridor network [4]

Two important factors that affect the OD estimation process are the extent and quality of link volume availability and the quality of the prior trip-table that is used to guide the solution. In order to compare test results, two measures of closeness were used. The first is based on the replication of observed volumes for the link $a \in A_v$, and the second is the closeness of the estimated trip-table to the correct table. These measures are defined as follows:

$$MAE\ (Mean\ Absolute\ Error) = \frac{\sum_{a\in A}(|f_a^* - \overline{f_a}|)}{\sum_{a\in A}\overline{f_a}} \tag{4}$$

$$MAE\ (TT) = \frac{\sum\left|T_{ij} - T_{ij}^*\right|}{\sum T_{ij}^*} \tag{5}$$

Table 2. Conclusion of Corridor Network

Link Index	Observed link volume	Result of UE assignment	Result of SUE assignment	CPLEX Volume	Difference between UE & Observed Vol.	Difference between SUE & Observed Vol.	Difference between CPLEX & Observed Vol.
1	2400	2400	2400	2400	0	0	0
2	1600	1600	1529	1600	0	71	0
3	100	100	171	100	0	71	0
4	5000	5400	5000	5000	400	0	0
5	500	500	429	500	0	71	0
6	500	500	500	500	0	0	0
7	4500	4900	4500	4500	400	0	0
8	500	500	429	500	0	71	0
9	2000	2300	2000	2000	300	0	0
10	1500	1450	1500	1500	50	0	0
11	4900	5267	4857	4900	367	43	0
12	2000	2300	2000	2000	300	0	0
13	1500	1717	1457	1500	217	43	0
14	900	933	943	900	33	43	0
15	4800	5200	4800	4800	400	0	0
16	300	350	300	300	50	0	0
17	1000	1000	1000	1000	0	0	0
18	200	283	243	200	83	43	0

Based on Table 2, the difference between observed volume and calculated volumes for both User Equilibrium and Stochastic User Equilibrium is equal to 7.6% and 1.3% which is acceptable but the level of difference in observational and computational volume difference of CPLEX software is zero, which presents the high accuracy of this method in CPLEX software.

Table 3. Correct Trip Table

From/To	1	2	3	4	5
4	0	600	700	0	1100
5	0	1700	300	0	0
6	500	2500	0	2000	600

Table 4. CPLEX trip table

From/To	1	2	3	4	5
4	0	600	700	0	1100
5	0	1700	300	300	0
6	500	2900	0	2000	600

Table 3 displays the correct trip table in Corridor Network and Table 4 displays the trip table that obtained from CPLEX software. In Table 5, has been shown the difference between correct trip table and CPLEX trip table.

Table 5. Difference between Correct trip table and CPLEX trip table

From/To	1	2	3	4	5
4	0	0	0	0	0
5	0	0	0	300	0
6	0	400	0	0	0

According to table 5, for two pairs of Origin – Destination, differences between correct trip table and the trip table that obtained from CPLEX software more than zero and this differences for others is zero. *MAE (TT)* for corridor network is 7%.

4.2. Golpayegan Network

Golpayegan Network consist of 6 traffic zones, 170 links that in 59 links of them the volume is counted. Figure 3 displays a zones in Golpayegan and figure 4 display the links of Golpayegan network.

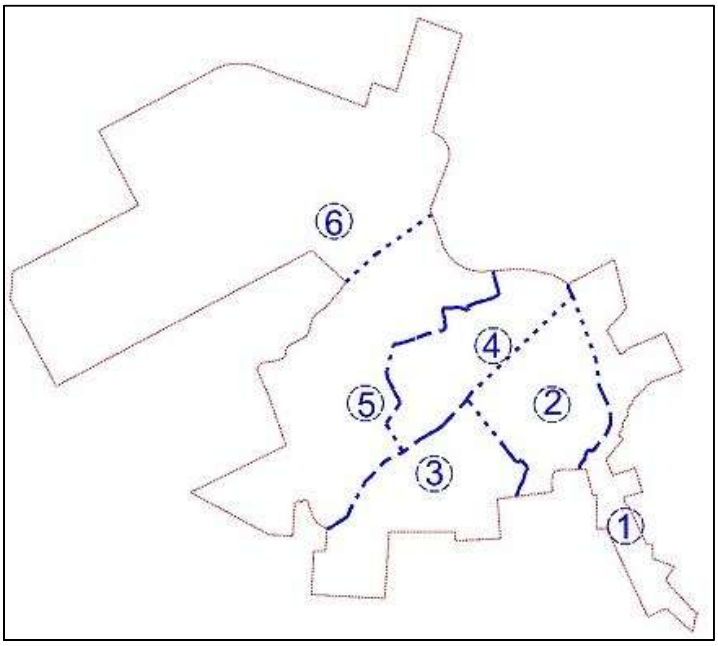

Figure 3: Zones of Golpayegan network

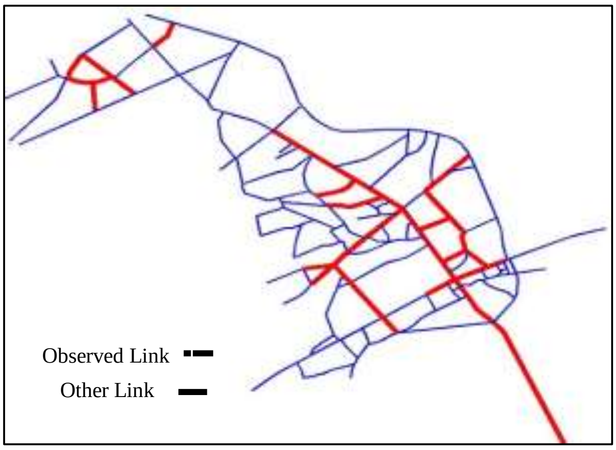

Figure 4: Links of Golpayegan network

Specifications of some of the links in this network displays in Table 6.

Table 6. Specification of some of Golpayegan Links

Link Index	Observed link volume	Capacity	Free Flow cost
10	563	1000	20
11	586	1500	13
14	273	800	15
15	590	800	11
19	386	800	26
20	480	750	23
21	434	800	27
22	650	2100	24
26	656	1000	22
27	563	900	12
28	730	1000	25
41	485	600	31
42	301	500	10
43	14	500	15
65	379	700	26
72	201	400	32
73	659	700	25
74	964	1200	25
75	1315	1400	15
80	853	1500	21
81	985	1200	29
84	290	500	12
108	600	800	10
110	370	500	12
111	742	1000	18

Table 7. Conclusion of Golpayegan Network

Link Index	Observed link volume	Result of UE assignment	Result of SUE assignment	CPLEX Volume	Difference between UE & Observed Vol.	Difference between SUE & Observed Vol.	Difference between CPLEX & Observed Vol.
10	563	0	0	558	563	563	5
11	586	0	0	586	586	586	0
14	273	0	0	273	273	273	0
15	590	383	383	590	207	207	0
19	386	0	0	386	386	386	0
20	480	293	282	480	187	198	0
21	434	0	0	45	434	434	389
22	650	0	0	650	650	650	0
26	656	357	357	656	299	299	0
27	563	357	357	563	206	206	0
28	730	103	118	730	627	612	0
41	485	0	0	0	485	485	485
42	301	0	0	301	301	301	0
43	14	0	0	14	14	14	0
65	379	0	0	288	379	379	91
72	201	0	0	220	201	201	19
73	659	487	487	659	172	172	0
74	964	194	179	983	770	785	19
75	1315	277	273	1315	1038	1042	0
80	853	383	383	590	470	470	263
81	985	157	168	985	828	817	0
84	290	0	0	290	290	290	0
108	600	230	241	600	370	359	0
110	370	0	0	370	370	370	0
111	742	0	0	734	742	742	8

Based on Table 7, the difference between observed volume and calculated volumes for both User Equilibrium and Stochastic User Equilibrium is equal to 73.6% and 73.3% which is high error but the level of difference in observational and computational volume difference of CPLEX software is 6.1%, which presents the high accuracy of this model in CPLEX software.

Table 8. Correct Trip Table

From/To	1	2	3	4	5	6
1	301	427	85	156	22	9
2	427	1796	402	459	230	383
3	86	402	311	147	60	64
4	157	459	147	708	105	340
5	22	230	60	105	128	86
6	79	383	64	340	86	384

Table 9. CPLEX trip table

From/To	1	2	3	4	5	6
1	301	427	695	156	22	9
2	427	1796	402	459	230	383
3	86	402	311	147	60	64
4	157	459	147	708	105	340
5	22	230	60	105	128	86
6	79	383	64	340	86	384

Table 8 displays the correct trip table in Golpayegan Network and Table 9 displays the trip table that obtained from CPLEX software. In Table 10, has been shown the difference between correct trip table and CPLEX trip table.

Table 10. Difference between Correct trip table and CPLEX trip table

From/To	1	2	3	4	5	6
1	0	0	610	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0

According to table 10, for one pairs of Origin – Destination, differences between correct trip table and the trip table that obtained from CPLEX software more than zero and this differences for others is zero. MAE (TT) for corridor network is 6.1%.

5. Conclusion

We have reviewed issues concerned with the estimation of O–D matrices in urban traffic networks when the input data is incomplete and imprecise. Sherali et al. (2003) mentioned a non-linear formulation for O–D matrix estimation and described a successive linear approximation method for computing a heuristic solution to this formulation. In this paper, we removed the additional parameters in previous studies and used a simple solution to estimate the matrix and improve a model that sherali used it in 2003.

In this model, for cases where the obtained volume by CPLEX software is compared with the observed volume, the result is shown that the MAE (TT) is zero for Corridor network and it is 6% for Golpayegan network. For congested network (Golpayegan), MAE (volume) is 7% and this low error for congested network is due to the type of objective function and constraints that used. The optimal solution contains the case in which the level of artificial variables defined in the objective function are zero and the error rate for determining the computational volume for Corridor network, in SUE is less than UE, due to the fact that the SUE mode is closer to the UE mode.

6. References

[1] Bera, Sharminda, k.v.krishna Rao. "Estimation of origin-destination matrix from traffic counts: the state of the art." European Transport, n.49 (2011): 3-23.

[2] Caggiani, Leonardo, Michele Ottomanelli, and Domenico Sassanelli. "A Fixed Point Approach to Origin–Destination Matrices Estimation Using Uncertain Data and Fuzzy Programming on Congested Networks." Transportation Research Part C: Emerging Technologies 28, no. Supplement C (2013/03/01/ 2013): 130-41. doi: https://doi.org/10.1016/j.trc.2010.12.005.

[3]. Lo, H. P., N. Zhang, and W. H. K. Lam. "Decomposition Algorithm for Statistical Estimation of Od Matrix with Random Link

Choice Proportions from Traffic Counts." Transportation Research Part B: Methodological 33, no. 5 (1999/06/01/ 1999): 369-85. doi:https://doi.org/10.1016/S0191-2615(98)00042-3.

[4]. Sherali, Hanif D., R. Sivanandan, and Antoine G. Hobeika. "A Linear Programming Approach for Synthesizing Origin-Destination Trip Tables from Link Traffic Volumes." Transportation Research Part B: Methodological 28, no. 3 (1994/06/01/ 1994): 213-33. doi:https://doi.org/10.1016/0191-2615(94)90008-6.

[5]. Sherali, Hanif D., Arvind Narayanan, and R. Sivanandan. "Estimation of Origin–Destination Trip-Tables Based on a Partial Set of Traffic Link Volumes." Transportation Research Part B: Methodological 37, no. 9 (2003/11/01/ 2003): 815-36. doi:https://doi.org/10.1016/S0191-2615(02)00073-5.

[6]. Saadi, Ismail, Ahmed Mostafa, Jacques Teller, and Mario Cools. " A bi-level Random Forest Based approach for estimating O-D matrices" Preliminary results from the Belgium National Household Travel Survey". Transportation Research Procedia, n. 25 (2017): 2566-2573. doi:https://doi.org/10.1016/j.trpro.2017.05.301.

[7]. Zadeh, L. A. "Fuzzy Sets." Information and Control 8, no. 3 (1965/06/01/ 1965): 338-53. doi:https://doi.org/10.1016/S0019-9958(65)90241-X.

[8]. Harikishan Reddy, K., and Partha Chakroborty. "A Fuzzy Inference Based Assignment Algorithm to Estimate O-D Matrix from Link Volume Counts." Computers, Environment and Urban Systems 22, no. 5 (1998/09/01/ 1998): 409-23. doi:https://doi.org/10.1016/S0198-9715(98)00037-4.

[9]. Foulds, Les R., Hugo A. D. do Nascimento, Iacer C. A. C. Calixto, Bryon R. Hall, and Humberto Longo. "A Fuzzy Set-Based Approach to Origin–Destination Matrix Estimation in Urban Traffic Networks with Imprecise Data." European Journal of Operational Research 231, no. 1 (2013/11/16/ 2013): 190-201. doi:https://doi.org/10.1016/j.ejor.2013.05.012.