

Civil Engineering Journal

Vol. 3, No. 11, November, 2017



Three-Dimensional Simulation of Flow Field in Morning Glory Spillway to Determine Flow Regimes (Case Study: Haraz Dam)

Amir Reza Razavi ^{a*}, Hassan Ahmadi ^a

^a Department of civil engineering, Roudehen Branch, Islamic Azad University, Roudehen, Iran.

Received 15 October 2017; Accepted 30 November 2017

Abstract

Morning-glory spillways are usually used in dams constructed in narrow valleys or those on steeply sloped supports. Furthermore, one can adopt this type of spillway in cases where guiding and diversion tunnels of adequate diameter are available. One of positive characteristics of these spillways is that, their maximum capacity can be approached at relatively low head. This characteristic can be seen as an advantage in cases wherein maximum outflow from the spillway shall be limited. On the other hand, should water head on top of the spillway exceeds the project baseline head, changes in output discharge will be negligible. Morning-glory spillways are commonly used in large dam construction projects across Iran (e.g. Sefid-Rood Dam, Alborz Dam, Haraz Dam, etc.). Given that spillway is one of the most important axillary structures for dams, accurate and realistic characterization of the hydraulic conditions affecting them seems to be necessary. On this basis, the present research is aimed at accurate determination of flow behavior and discharge coefficient of morning-glory spillways from the flow inlet down to horizontal tunnel of the morning-glory spillway of Haraz Dam. For this purpose, the most significant hydraulic parameters (including flow depth, flow velocity, flow pressure at different sections of the spillway, and rate of outflow at spillway) will be determined. In this study, an effort was made to use the numerical model of Flow3D to numerically model three-dimensional flow based on physical model and actual data from one of the largest and most important morning-glory spillways for calibration and verification purposes, and determine accuracy of the numerical modeling and associated error with simulating the numerical model. Results of this study show that, the flow at morning-glory spillways is controlled in either of three modes: flow control at crest, orifice control, and pipe control.

Keywords: Morning-Glory Spillway; Numerical Modeling; Flow3D; Flow Hydraulics.

1. Introduction

Morning-glory spillway is a vertical and horizontal tunnel conduit which transmits flood flow from a higher level to a lower level at high rate. This type of spillway resembles siphon spillways, except that its function is different from that of siphon spillways. This type of spillway is designed and constructed in narrow valleys wherein supports on either sides of the valley are steeply sloped, or in diversion tunnels. Another advantage of this type of spillway is that, one can achieve a capacity close to their maximum capacity at relatively low heads. This characteristic ends up with ideal performance of morning-glory spillways in cases where maximum outflow at spillway is limited. Accordingly, these spillways suit the dams with adequately large reservoirs. Moreover, in surface wastewater drainage systems or water transmission systems, the morning-glory spillways serve as a conduit for transmitting the flow from upstream to downstream (from catchment area to tunnel discharge system in mountainous watersheds). In such cases, morning-glory spillways are used with particular types of catchments generally known as vortex flow catchments and provide the flow with an angular velocity which develops a rotation flow at morning-glory. As of current, various studies have been performed on morning-glory spillways in the form of experimental studies, numerical modeling and simulation studies,

^{*} Corresponding author: amirrezarazavi@gmail.com

doi http://dx.doi.org/10.28991/cej-030943

> This is an open access article under the CC-BY license (https://creativecommons.org/licenses/by/4.0/).

[©] Authors retain all copyrights.

Civil Engineering Journal

analytic modeling, etc. In this section, an attempt is made to present a set of the most important and latest studies in this scope.

Ervine and Ahmed (1982) studied aeration characteristics of flow in a vertical drop shaft [1]. In 2006, Zhao undertook experimental studies on hydraulic parameters of flow at morning-glory spillways under drop flow conditions [2]. In their study, Nohani and Mousavi (2010) constructed a physical model of a morning-glory spillway and performed experimental studies to investigate the effect of the number and thickness of anti-vortex blades on the power of helical vortex and efficiency of discharge system of the spillway. Results of their research indicated that, discharge coefficient of the morning-glory spillway increased by 20% and 9% with increasing the number of the blades and the number of the blades together with blade thickness, respectively [3]. In the studies by Feng Nan et al. in 2017, experimental models of morning-glory spillways were investigated to study hydraulics of the flow and aeration at bends [4].

In his study in 2014, Nohani used an experimental physical model to determine discharge coefficient at morningglory spillway under spillway crest edge geometrical conditions. In this research, morning-glory spillways of various diameters were considered with and without anti-vortex elements at the spillway crest. It is worth noting that, the type of crest edge was further investigated in this research. On this basis, in this study, two type of spillway crest edge, namely sharp- and wide-edged crests, were examined [5].

In 2013, Savic et al. used an experimental model to present a design for throat of vertical shaft of morning-glory spillways. According to this design, flow aeration was enhanced, with the flow depressurized once passed through this section [6]. In the studies undertaken by Petaccia and Fennochi in 2015, pressure fluctuations and profile of the flow passing through siphon spillways were experimentally investigated [7]. Siphon spillways related to vertical and semi-vertical shafts have been extensively discussed in Refs. [8-10].

In 2015, Xianqi undertook studies to investigate flow characteristics at morning-glory spillways of large dams. Using a physical model, five different geometrical and hydraulic designs were applied and a relationship for discharge coefficient at morning-glory spillway was determined for each design. The following figure shows discharge relationships for each of the physical models constructed in the study by Xianqi in 2015 [11].

2. Case study

Being a permanent river flowing within Mazandaran Province, Iran, Haraz River is among the most important rivers within Caspian Sea Watershed. The river flows into Caspian Sea throw an extended delta at 24 km to the northeast of Amol Township within Sorkh-Rood Area. The river is 185 km in length, covering a watershed area of 4,100 km². Average annual discharge rate of Haraz River is about 1,100,000,000 m³.



Figure 1. Local characteristics of the dam on the map of Iranian watersheds [12]

3. Geometrical Construction Method of Morning-Glory Spillways and Their Components

In order to model hydraulic model of morning-glory spillways and its components in Flow3D Software, the entire solid body of the spillway was three-dimensionally built using geometry simulation software (e.g. AutoCAD, CATIA, SolidWorks, etc.). SolidWorks 2011 was used for this purpose in the present research. In this study, morning-glory spillways and horizontal and vertical tunnels in the spillway with the characteristics and geometry of the physical model were selected for performing three-dimensional simulation of the flow field. Geometrical characteristics of the morning-glory spillway based on the physical model of Haraz Dam in Water Research Institute are presented in the following figures and tables.

Type of spillway	Free morning-glory
Outer diameter of spillway crest entrance	35.80 m
Perimeter of spillway crest entrance	75.36 m for 360°
Diameter of spillway crest	33.76 m
Perimeter of spillway crest (length of spillway)	106 m for 360°
Spillway crest level	502

Table 1. Details and characteristics of Haraz Dam and its morning-glory spillways

The morning-glory spillway structure is located at 180 m distance to axis of dam crest on the left side of the dam body. Geographically, the spillway is centered at X = 622982.08, X = 4012945.72. Profiles of spillway crest and morning-glory are designed based on the water load of 3.6 m and *Hs/Rs* and P/RS ratios of 0.3 and 2, respectively. Height of the spillway profile is 22 m, vertical shaft level is 480 m, and the shaft diameter is 8.30 m.

Table 2. Details and specifications of vertical shaft of morning-glory spillway of Haraz Dam

	(A)			
Initial level	457.08 m			
Final level	436.91 m			
Overall length	38.17 m			
Inner diameter	8.30 m			
(B)				
Initial level	436.91 m			
Final level	426.91 m			
Vertical length	10.00 m			
Truncated cone shape	Diameter: 8.30 m to 7.5 m			

The vertical shaft is composed of two parts (diameter of 8.3 m to length of 43.09 m, and 10 m-length conversion from a diameter of 8.30 m to 7.50 m)

Table 3. Details and	specifications of	the bend along	g morning-glory	y spillway o	of Haraz Dam
			,		

Initial level of vertical arc	426.91 m			
Inner diameter	7.5 m			
Bend base level	408.6 m			
Outlet level head	5.13 m			

The vertical bend structure with a central axis of 15 m is designed along the distance from vertical shaft to vertical bend structure. It is internally shrunk along vertical arc, finally forming orifice flow control section.

Table 4. Details and characteristics of main tunnel of morning-glory spillway of Haraz Dam

Initial tunnel bed level	408.6 m
Final tunnel bed level	387.9 m
Inner section	Horseshoe-shaped
Inner diameter	7.5 m
Tunnel slope	3.89%
Length of limiting tunnel of the orifice at concrete channel	546 m



Figure 2. Characteristics of the reservoir of Haraz Dam in the physical model



Figure 3. Properties of morning-glory spillway of Haraz Dam along with the corresponding details



Figure 4. Plan of inlet span of vertical shaft of morning-glory spillway of Haraz Dam



Figure 5. Cross section across central axis of inlet span of vertical and horizontal shafts of morning-glory spillway of Haraz Dam along with the path of aeration channel and flow aeration section

In order to investigate flow field and hydraulic conditions on each of the models, one should further incorporate local topography into the spillway body. Figure 6 shows local topography across the site.



Figure 6. Morning-glory spillway of Haraz Dam and topography of surrounding area.

Three mesh blocks shall be used in this simulation. Mesh block 1 includes the entire space of the water storage of morning-glory spillway and vertical shaft and bend. In terms of cell size, this block has finer cells than other blocks. Figure 7 shows meshing configuration of the model using the adopted mesh plans.



Figure 7. Computational cells on mesh blocks of morning-glory spillway and related components in numerical model.

In order to apply boundary conditions on the parts where the wall is impermeable, Wall boundary condition was used. And for flow outlet boundary, Outflow boundary condition was adopted. For the flow inlet boundary from the reservoir, the boundary condition of equivalent pressure to the water height in the reservoir was used. Symmetry boundary condition was also applied onto areas wherein there was no boundary in real conditions.



Figure 8. Applied boundary conditions for simulating the flow on morning-glory spillway

In the next step, as an initial condition, a fluid height was defined for the model, so that at the start of simulation, following one time step of solution, initial model conditions match the boundary conditions. Figure 9 shows the modeled morning-glory spillway along with the application of initial conditions on the model. Following these steps, the numerical model was run in different conditions, with the results investigated.



Figure 9. The numerical model along with the application of initial fluid conditions

4. Results and Discussion

In all of the simulations performed in this study, three-dimensional flow field was solved using RNG turbulence model. The reason behind using this turbulence model was its features and advantages over other models such as $k - \varepsilon$. Due to the presence of the excessive term of ε in the equation of this model, it is improved for analyzing rapidly strained flows and the flows on surfaces with great geometrical changes such as morning-glory spillway. This model is of large capabilities for simulating stable flows. Furthermore, referring to previous studies, including that of Movahedi et al. in 2007, compared turbulence models using Flow3D Software, the RNG model was found to give more accurate results than other turbulence models as far as the flow on spillways was concerned, so that they used RNG turbulence model [13]. In this study, the fluid was considered to be in viscous and incompressible, with input air density of 1.2 kg/m³ and shear stress coefficient of 0.073.

The first step in any numerical model is to calibrate the model. That is, to minimize effects of external factors and get model conditions as close to those of real conditions as possible. The present numerical model has been performed on the basis of experimental studies. Because of this, calibration and verification of the numerical model was performed on this basis. In this section, calibration of the numerical model in terms of boundary conditions and simulation runs are presented. In order to extract accurate and proper results out of a numerical model, it is necessary to have the model stabilized. In the studied numerical model, following the investigation of several models, appropriate time for extracting results from the model was considered as 50 s. Figure 10 shows how the flow passes over the morning-glory spillway at different times. In this figure, it is seen that, after the 50 s, the flow on the morning-glory spillway becomes stable.





fraction of fluid contours

fraction of fluid contours







Figure 10. Flow development on morning-glory spillway at various simulation times

According to the Figure 10, at the start of calculation of the numerical model (T = 0.00 s), fluid head corresponds to the main data considered for the introduced model. Once the computations were started (T = 1 s), according to the figure, the flow goes on the bed of the morning-glory spillway. At the time T = 2.5 s, the flow is guided from reservoir of the dam toward the vertical shaft of spillway and then to the horizontal shaft. According to the figure, at time T = 20 s, the effect non-steady state of the flow on the spillway is evident, there is a turbulence in the dam reservoir waters due to the inflow of water, and input and output discharge s into and from the model are yet to be stabilized. It is worth mentioning that, in this case, the spillways are what are referred to as damped. At time T = 0.25 s, submergence and damping of the spillway is attenuated, but a steady state and stable flow is yet to be developed in the numerical model. At time T = 40 s, a rather permanent flow is established across the numerical model. At this time, full submergence of the spillway is eliminated, with a dropping flow established in the model. In order to increase accuracy of the numerical model, the simulation time was extended to T = 50 s, after when a well permanent flow was developed. In order to make sure about stability and steady state of the flow field on morning-glory spillways, the plot of passing discharge through outlet

boundary of the model versus time is presented in Figure 11. The figure indicates stability and uniformity of the flow after 50 s, confirming stability of the simulations as well.



Figure 11. Changes in inflow boundary from outlet boundary (Out Flow) versus time for pure water

In Figure 11, it is observed that, the numerical begins with removing higher flows from the model for discharging the reservoir and developing steady state flow according to the model geometry and boundary conditions. However, following the equilibrium between reservoir and input boundary condition of the numerical model, input flows into the numerical model were established in 60 s. In this case, stable flow at input boundary establishes after only 30 s, and in order to finish the calculations related to the numerical model, one should consider output boundary conditions as well.

Various tests were performed to obtain the best results out of Flow3D Software and compare possible alternatives, boundary conditions, and initial conditions, while analyzing the model sensitivity to various parameters affecting the modeling. Sensitivity of numerical models to meshing and discretization of solution domain have always been among the most important issues discussed in numerical models. In this regard, various meshing patterns were presented, each of which was associated with advantages along with weaknesses. When dealing with solid boundaries and meshing, Flow3D model exhibits a relatively different behavior than that of other computational fluid dynamics (CFD) packages. In order to delineate the meshing, some blocks were defined, within which all dimensions of the considered structure along with free space within the structure were considered. One can consider all considered details in one block. Figure 12, 13 shows the numerical model and FAVOR method in adopting various meshing configurations. From the abovementioned discussions, it is concluded that, developing appropriate solid boundaries in meshing the numerical model, particularly in models with morning-glory spillway geometry (such as the model under study), is the most significant limitation for such a model, so that appropriate selection of the meshing configuration can enhance accuracy of the computations. In this study, dual-block meshing was investigated in different tests, and the best meshing for the performed simulations was selected.

In order to determine the size of meshing across computational field of the flow on morning-glory spillways, firstly, computational cells of 1 m in length, width and height ($X \times Y \times Z$) were considered (these values are presented as the smallest mesh size to cover the body of the morning-glory spillway, because he smallest cells across the meshing were in Block 1). In this case, once finished with verifying dimensions of the meshing cells using FAVOR module of the Flow3D numerical modeling software, it was found that, dimensions of the selected cells are not suitable for the constructed morning-glory spillway geometry. In other words, the morning-glory spillway and, in particular, the spillway crest, horizontal shaft, and cup launcher of the model are not adequately considered. For this reason, higher numbers of computational cells were considered for the numerical modeling. Accordingly, computational cells of 0.75 m in length, width and height ($X \times Y \times Z$) were considered. Following another round of mesh dimensional investigation and verification, it was again observed that, some of the cells comprising the geometrical model (thickness of the morning-glory spillway edge thickness) defined by the numerical model were inappropriate. For this reason, by varying the dimensions of mesh cells in the numerical model, the most optimal and suitable dimension for meshing cells was found to be 0.39 *cm* in length, 0.375 *cm* in width, and 0.52 *cm* in depth. Difference cases of computation cell dimensions are demonstrated in the Figure 12.



Figure 12. Performance of FAVOR method with solid boundaries and different meshing configurations, and calibrating the meshing configuration of the morning-glory spillway using 2,500,000 cells



Figure 13. Performance of FAVOR method with solid boundaries and different meshing configurations, and calibrating the meshing configuration of the morning-glory spillway using 4,030,000 cells

Table 5 presents cell size conditions for the mesh block used in the numerical model. It should be noted that, a total of 4,030,000 computational cells were used to simulate flow in this model.

X direction		
Total number of real cells = 130		
Minimum cell size = 0.399994	ati = 19	
Maximum cell size = 10.275	ati= 1	
Maximum adjacent cell size ratio = 2.64583	ati = 17	
Y direction		
Total number of real cells = 155		
Minimum cell size = 0.375	atj = 26	
Maximum cell size = 4.5	atj= 1	
Maximum adjacent cell size ratio = 1.44	atj = 26	
Z direction		
Total number of real cells = 200		
Minimum cell size = 0.526314	atk = 171	
Maximum c <mark>ell</mark> size = 0.556297	at k = 1	
Maximum adjacent cell size ratio = 1.00234	atk = 178	
	Maximum aspect ratios	
Total number of real cells	X_Y direction: 27.4	
4020000	Y_Z direction: 8.55003	
4030000	Z_X direction: 19.5226	

Civil Engineering Journal

Calibration and verification of the present numerical model against the most important hydraulic parameters were performed for four discharges on morning-glory spillway of Haraz Dam. In this regard, the parameters of average water depth on spillway crest, output discharge of the spillway, average flow velocity on the spillway crest, and average pressure on the spillway crest were extracted from the numerical model and compared against available experimental data for evaluating relative error of the numerical model. The values of relative error were obtained from the following relationship:

$$Error \% = 100 * \frac{X_{Exp} - X_{Num}}{X_{Exp}}$$
(1)

Where X_P refers to actual values of a parameter (experimental values) and X_M refers to simulated values of the same parameter.

Table 6. A comparison between associated errors with numerical modeling of hydraulic parameters of the flow, as
compared to experimental data

Run	Q (m ³ /s)			H (m)		v (m/s)			p (pa)			
No.	Exp	Num	Er %	Exp	Num	Er %	Exp	Num	Er %	Exp	Num	Er%
1	92	85.9	6.7	2.2	2.025	8	2.35	2.17	7.7	5494	4981	9.3
2	386	358	7.3	5.3	5.05	4.7	3.9	3.51	10	4316	3992	7.5
3	642	605	5.8	7.2	6.55	9.1	5.23	5.02	4.2	1024	923	9.9
4	1240	1165	5.9	18	17.08	5.1	5.34	5.01	6.2	14520	13550	7.2
Ave Error %		6.4			6.7			7			8.4	

As can be observed in the above table, the numerical modeling under the mentioned boundary conditions, meshing configuration and initial conditions is of some acceptable error percentage. According to the results of the numerical model, relative error of modeling in determining output discharge of spillway was calculated as 6.4%. The corresponding relative error to flow depth at spillway crest was evaluated as 6.7%, and tat to average flow velocity on spillway crest was 7%, when compared against the experimental data. Furthermore, associated relative error with hydrostatic pressure of the flow on the spillway crest was found to be 8.4%. In total, considering the presence of different factors affecting hydraulic parameters of the flow, the associated error with numerical modeling, as compared to experimental data, was evaluated as acceptable, so that the presented numerical modeling could be introduced as a calibrated and verified model for the rest of this study. Using the results of the numerical model, variations of discharge versus fluid depth head measured from the spillway crest, what is referred to as spillway rating curve, is presented in Figure 14.



Figure 14. Variations in output discharge from the morning-glory spillway based on free head of the flow (rating curve)

As can be seen in Figure 14, based on the fluid head at morning-glory spillways, the passing discharge is in either of three modes, as follows:

Control mode 1: crest control, zone (1)

Control mode 2: tube or orifice control, zone (2)

Control mode 3: pipe control, zone (3)

Figure 14 has delineated different zones in terms of flow control mode. Table 7 present discharge properties for control mode in each zone.

Table 7. Classification of the rating curve of morning-glory spillway of Haraz dam using the results of the numerical model.

No.	Description	Water load on the spillway (m)	Comments
1	Minimum discharge up to 460 m ³ /s	0-6.1	Spillway crest control
2	$840\ m^{3}/s > discharge > 460\ m^{3}/s$	6.1 - 8.00	Spillway crest control
3	1120 m ³ /s > discharge > 840 m ³ /s	8.00 - 12.15	Change of flow control at discharge of 800 $m^{3/s}$
4	1120 m ³ /s > discharge > 1240 m ³ /s	12.15 – 18	Flow control at shaft and end of bend
5	discharge > 1240 m ³ /s	> 18	Full flow control in the tunnel

As the vertical shaft is filled completely, the spillway is completely damped at the end of the bend for the discharge of 1240 m³/s. Figures 15-17 show each of the flow control modes on the numerically modeled morning-glory spillway.



Figure 15. Cross sectional profile of the flow passing through the morning-glory spillway with crest flow control at the spillway crest at a discharge of 460 m³/s



Figure 16. Cross sectional profile of the flow passing through the morning-glory spillway with crest flow control at the spillway crest at a discharge of 840 m³/s



Figure 17. Cross sectional profile of the flow passing through the morning-glory spillway with crest flow control at the spillway crest at a discharge of 1250 m³/s

An analysis on Figures 15-17 shows that, as long as the morning-glory spillway crest serves as the control section the flow on the crest profile follows an increasing trend, with the flow attached to the spillway body down to the increasing slope of the crest, following where it detaches from the spillway body and hits the end of the bend through free falling. In the numerical model, with increasing the discharge, the water table overflowing the crest becomes thicker, so that the water table turns into a vertical water jet eventually. Figure 18 shows the interception between water and vertical jet (crotch point) created by vortex motion of flow at the spillway throat.



Figure 18. The intercept crotch point of falling jets under the effect of vortex motion of flow at the spillway throat

In the numerical model, with increasing the water head on the spillway, the crotch point and uplifted part of the flow rise. For very high heads, the uplifted and conversion points can be submerged fully, leaving a small uplift and vortex on water surface. In this case, flow control can be performed by a compressed jet developed by the input span of the crest.

5. Conclusion

In the present research, an attempt was made to use Flow3D numerical model to calibrate and verify three-dimensional numerical modeling of flow on morning-glory spillways of one of the largest and most important Iranian dams using the information of physical model, determining the modeling accuracy and associated relative error with the simulations. Afterwards, by applying boundary conditions, initial conditions, and conditions for meshing the solution domain for the flow in the presented numerical model for morning-glory spillways, the discharge passing through the spillway was investigated. Based on the results of the numerical model, relative error of the numerical modeling in determining output discharge of the spillway was 6.4%. The corresponding relative error to flow depth at spillway crest was evaluated as 6.7%, and tat to average flow velocity on spillway crest was 7%, when compared against the experimental data. Furthermore, associated relative error with hydrostatic pressure of the flow on the spillway crest was found to be 8.4%. In total, considering the presence of different factors affecting hydraulic parameters of the flow, the associated error with numerical modeling, as compared to experimental data, was evaluated as acceptable, so that the presented numerical model results, with increasing the water head on the spillway, the crotch point and uplifted part of the flow rise. For very high heads, the uplifted and conversion points can be well submerged, leaving a small uplift and vortex on water surface.

Following this stage, the effect of submergence on spillway begins to exist, ending up with full submergence of the crest. In this case, flow control can be performed by a compressed jet developed by the input span of the crest.

6. References

[1] Ervine, D. A., and A. A. Ahmed. "A Scaling relationship for a two-dimensional vertical dropshaft." In Proc. Intl. Conf. on Hydraulic Modelling of Civil Engineering Structures, pp. 195-214. 1982.

[2] Zhao, Can-Hua, David Z. Zhu, Shuang-Ke Sun, and Zhi-Ping Liu. "Experimental Study of Flow in a Vortex Drop Shaft." Journal of Hydraulic Engineering 132, no. 1 (January 2006): 61–68. doi:10.1061/(asce)0733-9429(2006)132:1(61).

[3] Nohani, E and H. mousavi, 2010. The Effect of number and thickness Vortex Breakers on Discharge Coefficient for the Shaft Spillways. Proceedings of 1th National Conference of water, soil, plant and agricultural mechanization, Islamic Azad University.

[4] FENG H, CHENG G, LIANG J, GAO F. Inspiration of Rainwater Utilization Plan in Denmark for the Construction of Sponge City in China. DEStech Transactions on Environment, Energy and Earth Science [Internet]. DEStech Publications; 2017 Jul 6;(icepe). Available from: http://dx.doi.org/10.12783/dteees/icepe2017/11827

[5] Nohani, Ebrahim. "Numerical Simulation of the Flow Pattern on Morning Glory Spillways." International Journal of Life Sciences 9, no. 4 (June 5, 2015). doi:10.3126/ijls.v9i4.12671.

[6] Savic, Ljubodrag, Radomir Kapor, Vladan Kuzmanovic, and Bojan Milovanovic. "Shaft Spillway with Deflector Downstream of Vertical Bend." Proceedings of the Institution of Civil Engineers - Water Management 167, no. 5 (May 2014): 269–278. doi:10.1680/wama.12.00111.

[7] Petaccia, G., and A. Fenocchi. "Experimental Assessment of the Stage–discharge Relationship of the Heyn Siphons of Bric Zerbino Dam." Flow Measurement and Instrumentation 41 (March 2015): 36–40. doi:10.1016/j.flowmeasinst.2014.10.012.

[8] Houichi L, Ibrahim G, Achour B. Experiments for the discharge capacity of the siphon spillway having the Creager Ofitserov profile. Int J Fluid Mech Res 2006; 33(5):395–406. http://dx.doi.org/10.1615/InterJFluidMechRes.v33.i5.10.

[9] Houichi L, Ibrahim G, Achour B. Experimental comparative study of siphon spillway and overflow spillway. Cour Savoir 2009; 9:95–100.

[10] Gramatky G. Siphon spillway [Dissertation]. University of California; 1928 .

[11] Narany, Tahoora, Mohammad Ramli, Ahmad Aris, Wan Sulaiman, and Kazem Fakharian. "Spatial Assessment of Groundwater Quality Monitoring Wells Using Indicator Kriging and Risk Mapping, Amol-Babol Plain, Iran." Water 6, no. 1 (December 31, 2013): 68–85. doi:10.3390/w6010068.

[12] Zhang, Xianqi. "Hydraulic Characteristics Of Rotational Flow Shaft Spillway For High Dams." International Journal of Heat and Technology 33, no. 1 (March 30, 2015): 167–174. doi:10.18280/ijht.330123.

[13] Altomare, C., J.M. Domínguez, A.J.C. Crespo, J. González-Cao, T. Suzuki, M. Gómez-Gesteira, and P. Troch. "Long-Crested Wave Generation and Absorption for SPH-Based DualSPHysics Model." Coastal Engineering 127 (September 2017): 37–54. doi:10.1016/j.coastaleng.2017.06.004.