Interaction Strength of Hanger and Horizontal Steel Reinforcement of Dapped End Beams

Abdul Kareem Q. Mohammad 1, Rafaa M. Abbas 1 *

1 Department of Civil engineering, University of Baghdad, Baghdad, Iraq.

Abstract

The dapped end beam members have a special end with low depth at the support area, which results in a weak area against shear stresses. Classical structural analysis doesn’t capture the precise steel reinforcement interaction at the dapped zone area. The main objectives of this study are to investigate the strength of the dapped end area and to analyze stresses in the steel reinforcement to evaluate the shear failure mechanism at the re-entrant corner. The experimental tests conducted on RC beam samples, in addition to the numerical simulation of these samples by a finite element program, have been compared with a mathematical model. The experimental program highlighted the strains in the steel reinforcement in the dapped region to calculate the magnitude of the stresses in the steel reinforcement. In the experimental program, six dapped beams were fabricated with a length of 3 m, a width of 150 mm, and a depth of 300 mm. The notched end has a 150-mm depth and 150-mm height. These beams were loaded by a concentrated load near support to investigate the shear strength capacity. From the results for steel reinforcement strain, it is found that hanger and horizontal steel reinforcements interact to provide dapped end shear strength. The study proposes a new approach to computing shear strength capacity at the re-entrant corner by adding the contributions of the horizontal and hanger steel reinforcement using an appropriate proportion strain factor. This method revealed greater carrying capacity for the dapped end beam compared with other common structural methods. The results of the numerical analysis were done by the ABAQUS finite element program, showing the same behavior as the experimental work. This study proved the common contribution of hanger and horizontal re-entrant corner steel reinforcement and proposed a new formula to determine the updated nominal shear strength.

Keywords: Prestressed; Dapped End Beam; Shear Strength; Re-entrant Corner; Reinforced Concrete.

1. Introduction

The extensive usage of dapped girders in bridges, buildings, and parking floors was not restricted by the substantial stress concentration at the re-entrant corner of a dapped end girder caused by its shape. Despite the complexity of its analysis, the design of dapped end connections is an important factor in precast concrete structures. The reduced concrete section that remains above the notch is known as the dap. Many previous studies were done to analyze the structural behavior of dapped end beams. Mattock & Teddy [1] investigated five dapped end reinforcement schemes suitable for thin-stemmed precast prestressed concrete members. The study involved subjecting full-scale specimens to a combination of shear and outward tension. Proposed design procedures were developed for each of the proposed reinforcement schemes. Barton et al. [2] conclude that the different tie reinforcement layers were seen to strain at various rates in all of the specimens. The reinforcement that makes up a single tie in the strut and tie concept is typically assumed to be similarly strained. The position of major cracks had an impact on the strains of steel reinforcement. Some of the layers of reinforcement that were closer to the external surface had less stress on the longitudinal ties.

*Corresponding author: dr.rafaa@coeng.uobaghdad.edu.iq
According to Martin & Korkosz [3], who have discussed current design practice, the most popular reinforcement scheme is one that makes use of an orthogonal arrangement of reinforcement. This reinforcement scheme’s design guidelines can be found in the PCI Design Handbook. The PCI Design Handbook suggests using an inclined bar with hooked ends as the hanger reinforcement as an alternate design. The uncertain importance of the hooks as anchorage for the hanger reinforcement is a problem in their research. Twelve high-strength concrete dapped-end beams were tested by Lu et al. [4]. They suggest a model that makes use of the Constitutive Law, equilibrium equations, and compatibility requirements to forecast the shear strength of dapped end beams. He came to the conclusion that the shear strength of dapped-terminated beams was underestimated by the PCI design approach and suggested that the methodologies be revised to include the suggested model.

Wang et al. [5] investigated the shear resistance behavior after conducting tests on 24 reinforced concrete (RC) dapped end beams. The shear capacity was found to be more influenced by inclined stirrups and longitudinally bent reinforcement than by vertical stirrups. Some design recommendations for dapped end beams’ maximum shear strength were made. Experimental studies were conducted by Peng [6] to investigate how detailing affects the behavior of dapped-end beams. The findings demonstrated that longitudinal and hanger reinforcement’s features and anchorage have a significant impact on the ductility and shear strength of dapped-end beams. Mader [7] investigated three design methods: the Prestressed Concrete Institute (PCI) design method, the Menon/Furlong design method (this method is based upon a research study conducted by the Center for Transportation Research at the University of Texas at Austin), and the strut-and-tie model (this method depends on truss analogy principles). The effects of prestressing forces on the load route in a beam were examined in each of these three ways through testing. The study’s findings were that all design methods produced beam loads that were 15 to 20% higher than expected.

Aswin et al. [8] developed a condensed STM model while taking into account the research of Wang et al. [5]. The PCI technique and STM models of the (ACI 318-08, Euro Code 2, and BS 8110) codes were used to calibrate the results. Grade of concrete, nib geometry, a/d ratio, types and locations of hanger stirrup distribution, bent form of longitudinal steel, quantity of extended end and hanger reinforcements, and bent shape of longitudinal steel were all taken into consideration as influencing parameters. It was discovered that the PCI technique and ACI-318-08 were more accurate than Euro Code 2 and BS 8110. In addition, it was shown that using inclined stirrups, bending reinforcement at an angle less than 90 degrees, deeper nubs, closer stirrup spacing, and more stirrups produced larger failure loads.

Hamoudi et al. [9] stated When a DE is under operating strain, prestressing may prevent the growth of shear cracks within the DE. Increasing prestress force and decreasing shear slenderness ratio improve the shear capacity of DEB. According to Shakir [10], the shear friction (SF) method produces findings that are less accurate than those produced by STM models because it does not sufficiently quantify the impacts of the (a/d) ratio, horizontal load effects, and hanger reinforcement. However, compared to the PCI method and ACI318-08 (STM models), the STM models of Euro Code 2 and BS 8110 were less accurate. The PCI Design Handbook defines several modes of failure and states empirical equations to calculate the required strength material. The most important equations were calculating the hanger reinforcement (A_{sh}) by using Equation 1 and the nib flexural reinforcement (A_{s}) by using Equation 2 as follows [11]:

\[
A_{sh} = \frac{\nu u}{\phi f_y} \tag{1}
\]

where: \(\nu\) is the factored vertical dap reaction, f_{y} is yield strength of the mild steel and \(\phi=0.75\) is factor of reduction.

The nib flexural reinforcement, A_{s}, using the following equation:

\[
A_{s} = A_f = \frac{\nu u a}{\phi f_y b d} \tag{2}
\]

where: a is the shear span, measured from load to center of A_{sh}, the cross-sectional dimensions are b and d, and f_{y} is the steel yield stress.

Yang et al. [12] developed a mechanism analysis based on the principles of energy and the upper-bound theorem to determine the shear strength and critical plane of failure of RC DEBs. In order to compare the outcomes to the failure load, curves from the literature were employed. Additionally, two methods—the PCI design method and the streamlined STM model of ACI 318-05—have been used to examine the DEBs. The findings showed that:

- The two techniques significantly underestimated the shear strength of DEBs. By contrast, the suggested mechanism analysis discovered adequate prediction.
- The PCI design technique did not effectively account for the impacts of the a/d ratio, the amount of HR, and the size of the horizontal load on the shear strength of DEBs. However, the mechanism analysis might take these impacts into account.
- Evidently, both the STM model and the suggested mechanism analysis establish the critical plane, and similar results were obtained with both methods.
Aswin et al. [13] showed that different layers of tie reinforcement were observed to strain at different rates. In the strut and tie model, the reinforcement making up a single tie is normally assumed to be similarly strained. The position of the major cracks had an impact on the reinforcing strains. Some of the layers of reinforcement that were closer to the external surface had less stress on the longitudinal ties. In these instances, the significant cracks appeared to reduce the available development length by an amount sufficient to impair the tie’s capacity to withstand tensile stress. In order to prevent any missing data caused by damaged strain gauges, the computation is based on strain readings at 80% of the failure load. Klein et al. [14] established practical methods for distributing essential reinforcement in dapped-end beams as well as industry-recognized standards for specifics that have undergone meticulous testing and analysis by experts. Field performance of dapped-end beams points to the necessity for better dapped-end reinforcing features, which are currently not standardized within the precast concrete sector.

Mohammed et al. [15] investigated the dapped end beam diagonal reinforcement system. In resisting diagonal cracks, they prefer diagonal reinforcement to hanger reinforcement since it is put perpendicular to the cracking path. Syed et al. [16] investigated numerically six models using one layer or two layers for the main steel reinforcement arrangement to predict the structural response of the dapped end beam at the end. Hanger and main dapped end reinforcement detailing affect sufficiently crack control. Mohammed et al. [17] stated that PCI code has not been able to adequately predict the failure load of a dapped end beam. The PCI code needs to change the upper limits and also involve the effects of the tensile strength and the high compressive strength on each dapped-end parameter in the PCI design equations. So that adequate analysis results can be achieved, Atalla et al. [18] studied the impact of steel reinforcement on the shear strength and found that the failure load was decreased by 4% when the horizontal shear reinforcement at the nib was removed, while the failure load was reduced by 7% when the vertical shear reinforcement at the nib was removed. Mohammad & Abbas [19] concluded that the drastic failure mode was the simultaneous yielding of the horizontal and vertical dapped-end steel reinforcement.

Aswin et al. [20] tested eight beams to identify the most suitable arrangement of steel reinforcement at the dapped end zone. They found that the direction of stresses, not the arrangement of a specific group in the diagonal reinforcement (DR) group, was more effective than the vertical hanger reinforcement (HR) group. According to Masenas et al. [21], other methods, such as strut and tie models, require adequate changes to accurately represent acting forces in studied zones and therefore should be used primarily for configuration-specific analysis. The PCI technique is less conservative, and Wang’s semi-empirical truss model, the EC2 6.2 section, frequently overestimates shear strength, making it unsuitable for dapped-end beam design. They proposed adding characteristics such as the height of the dap, the distance of the support from the re-entrant corner, and the concrete strength.

Abdul-Jawad [22] demonstrated experimental validation using numerical analysis. They employ the ABAQUS model to capture nonlinearity, such as post-crack tensile stiffness of concrete and stress transfer transversely through the cracked blocks of concrete. Using the current numerical model, the failure mechanism of D-E beams is proven quite well, and the maximum load expected is extremely close to the failure load of test data. Some parametric studies, such as shear span to depth ratio (a/d), concrete compressive strength, and the major D-E reinforcement on the behavior of the beams, were explained in this study. The method of analysis approach was updated by Abeyesingha et al. [23], who proposed a kinematics-based model (KBM) to account for various defects in structures specific to dapped-end beams. In comparison to the original KBM, the extended KBM produce improved predictions for the existing experimental results of degraded dapped-end beams. The forecast offset ranged from -24% to 6%. The strut and tie model (STM) provides more accurate analysis and design than the PCI method, but it also ignores strain calculation and treats ties as an axial member, which means that when reinforcement is placed in orthogonal positions, the common strength impact of the vertical and horizontal reinforcement is not taken into account.

From the previous studies and research, one can conclude that all proposed methods underestimated the capacity of the dapped end beam because these studies took the impact of reinforcement individually with respect to its position. So many researchers studied the strength of the dapped end beam using different approaches to define its actual behavior and shear strength capacity. In this study, different reinforcement scheme strains in the dapped end are investigated, and an attempt is proposed to find a new and more realistic concept to investigate the behavior of the dapped end beam by taking the common impact of vertical and horizontal reinforcement simultaneously. The objectives of this research were to attain a better understanding of the behavior of the precast prestressed concrete dapped end beams by studying the strains in the reinforcing steel in the dapped region and the associated methods of design, which combine ease of use and cost-effective manufacturing.

2. Research Methodology

This research studies the shear strength capacity of a dapped end beam by investigating reinforcement strains throughout measuring the strains of the hanger and the horizontal reinforcement in the dapped region. The experimental tests dealt with the relationship between load and strain. The response to the load is the deformation at the critical zone.
In such a case, the re-entrant corner represents the failure region. Because of the special geometry, it is found that there is a participation of the horizontal reinforcement with the vertical reinforcement from strain measurement. This part of the strain represents stress in the steel reinforcement, reflecting its strength. By adding the component of this strength to the axial strength of hanger reinforcement, one can get the total strength of the re-entrant shear strength. Basically, the most common mode of failure for dapped end beams is re-entrant shear failure. In this study, the ratio of the accompanying horizontal reinforcement strain with the hanger strain, denoted by ($\psi$), was investigated experimentally and numerically. Moreover, the strength increment factor ($\kappa$), which represents the increment in strength compared to old approaches, was calculated. This approach can be expressed as flowchart as shown in Figure 1.

3. Experimental Work

Non-prestressed and Prestressed reinforced concrete beams having a concrete compressive strength of 40 MPa and a single strand 12.7 mm in diameter with an ultimate strength of 1860 MPa were used. 3 meters span length of the dapped end beam with depth 300 mm and 150 mm in width, the nib has 150 × 150 mm, as shown in Figures 2 and 3. A total of six prestressed RC-dapped end beams were fabricated in this study. All beams are subjected to point load until failure; the point load is applied at L/3, so the value of shear force ($Vu$) represents 2/3 of the load failure.
The testing setup consists of a steel frame to support the two dapped ends on steel supports. To prevent local concrete failure, the tested beams were supported on a rubber pad under the specimen. A hydraulic jack was used to apply the load gradually, as shown in Figure 4.
TML strain gauges were fixed on the steel reinforcement at the dapped end region, one for the first vertical stirrup as shown in Figure 3 (vertical strain dapping VD) and another one for the horizontal nib reinforcement (horizontal strain dapping HD). By using a digital data logger and the Lab View program to record the results of loads and strains. The main mode of failure was the shear at the re-entrant corner in all specimens, as shown in Figure 5.

Figure 5. shape of the failure at the reentrant corner

The results of the loading and strains of the horizontal and vertical hanger steel reinforcement for one sample are shown in Table 1.

Table 1. Digital data of the load (kN), vertical (VD) and horizontal (HD) strain in tested beam sample

<table>
<thead>
<tr>
<th>Load (kN)</th>
<th>HD, micro strain</th>
<th>VD, micro strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>142.657778</td>
<td>1090</td>
<td>2851.8</td>
</tr>
<tr>
<td>145.315556</td>
<td>1113</td>
<td>2878.4</td>
</tr>
<tr>
<td>147.617778</td>
<td>1137</td>
<td>2914.8</td>
</tr>
<tr>
<td>149.822222</td>
<td>1163</td>
<td>2945.6</td>
</tr>
<tr>
<td>151.528889</td>
<td>1186</td>
<td>3036.6</td>
</tr>
<tr>
<td>152.951111</td>
<td>1207</td>
<td>3052</td>
</tr>
<tr>
<td>153.982222</td>
<td>1227</td>
<td>3101</td>
</tr>
<tr>
<td>155.44</td>
<td>1249</td>
<td>3141.6</td>
</tr>
<tr>
<td>156.595556</td>
<td>1269</td>
<td>3171</td>
</tr>
<tr>
<td>157.733333</td>
<td>1290</td>
<td>3224.2</td>
</tr>
<tr>
<td>158.951111</td>
<td>1312</td>
<td>3243.8</td>
</tr>
<tr>
<td>157.066667</td>
<td>1321</td>
<td>3124.8</td>
</tr>
<tr>
<td>159.706667</td>
<td>1345</td>
<td>3148.6</td>
</tr>
<tr>
<td>161.173333</td>
<td>1370</td>
<td>3178</td>
</tr>
<tr>
<td>162.444444</td>
<td>1395</td>
<td>3220</td>
</tr>
<tr>
<td>164.035556</td>
<td>1419</td>
<td>3243.8</td>
</tr>
<tr>
<td>165.146667</td>
<td>1441</td>
<td>3287.2</td>
</tr>
</tbody>
</table>

4. Results and Discussion

From Table 1, it is shown that the strains increased as the load increased in both reinforcement rebars as they resisted the loading stress together, and that the vertical steel reinforcement has yielded while the horizontal reinforcement is still below yielding. It is also noted that when the strain for vertical reinforcement reaches its maximum value, the strain for horizontal reinforcement still has a lower value. The ratio of the horizontal reinforcement strain is about 0.28–0.58 with respect to the maximum strain of vertical steel reinforcement in the tested model reinforcement. On the other hand, the horizontal steel reinforcement flexure force is balanced with the top concrete in the region above the dap, as shown in Figure 5. The curves in Figure 6 show the relationships between load and strain of vertical and horizontal reinforcement for the tested dapped end beams. The ratio of vertical strain to horizontal strain ($\varepsilon_{\text{HD}} / \varepsilon_{\text{VD}}$) is shown in Table 2, in which the horizontal strain is expressed as a percent of the vertical ultimate strain. The strain gauge data for steel reinforcement show the strain value at the pure shear zone at the dapped end zone in the re-entrant corner. In this table, considering a reference strain value of 3000 micro strains in the vertical hanger reinforcement, the accompanying strain in the horizontal steel reinforcement is estimated as compiled in this table. For each sample, the ratio of two perpendicular strains was determined as shown in Table 2.
Table 2. Results strains and failure load for the dapped end beams

<table>
<thead>
<tr>
<th>Sample</th>
<th>Load failure (kN)</th>
<th>Vu (kN)</th>
<th>Vertical reinf. strain (VD), $\mu \in VD$</th>
<th>Accompanying horizontal reinf., (HD), $\mu \in HD$</th>
<th>VD: HD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>165</td>
<td>110</td>
<td>3000</td>
<td>1110</td>
<td>1 : 0.37</td>
</tr>
<tr>
<td>2</td>
<td>128.2</td>
<td>85.46</td>
<td>3000</td>
<td>944</td>
<td>1 : 0.31</td>
</tr>
<tr>
<td>3</td>
<td>112.6</td>
<td>75.06</td>
<td>3000</td>
<td>1452</td>
<td>1 : 0.48</td>
</tr>
<tr>
<td>4</td>
<td>148.1</td>
<td>98.73</td>
<td>3000</td>
<td>1760</td>
<td>1 : 0.58</td>
</tr>
<tr>
<td>5</td>
<td>141.3</td>
<td>94.2</td>
<td>3000</td>
<td>1428</td>
<td>1 : 0.47</td>
</tr>
<tr>
<td>6</td>
<td>145.8</td>
<td>97.2</td>
<td>3000</td>
<td>856</td>
<td>1 : 0.28</td>
</tr>
<tr>
<td>average</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1 : 0.42</td>
</tr>
</tbody>
</table>

Figure 6. Relationships between the load and the strains of the vertical and horizontal steel reinforcement
5. Finite Element Analysis Verification

The nonlinear behavior of the dapped end members was modelled and analyzed using the commercial version of the ABAQUS computer program (ABAQUS 2019). Simulation of the dimension and geometry of concrete, longitudinal and transverse reinforcing steel, endplates, and support and loading plates All parameters are obtained from the experimental work including: compressive strength $f'_c$, young modulus $E_c$, and yield stress of reinforcing steel $f_y$ were assigned to the material properties. The procedure of loading was incrementally increasing loading until model failure, as shown in Figures 7 and 8.

![Figure 7. Finite element model by ABAQUS program](image1)

![Figure 8. Finite element model of the dapped end region](image2)

The results and output of the analysis for the strain and stresses at the dapped end of a beam sample presented in Figure 9 show rational agreement with the experimental tests. The strain results of the horizontal steel reinforcement are shown in comparison with the vertical reinforcement. Also, the result of the maximum in plane principal strains shows the contribution of horizontal reinforcement to resisting the applied load forces. From the behavior of the two perpendicular reinforcements, one can note that vertical hanger reinforcement strain exceeded yielding strain, whereas horizontal reinforcement is still away from yielding for all models. The finite element analysis shows the ratio of hangers to horizontal strains is about (1:2).

![Figure 9. Strain and stress of the vertical and horizontal steel reinforcement](image3)

The numerical analysis clearly shows that when the vertical strain reaches the maximum capacity, the horizontal strain represents the ratio of the ultimate strain in the hanger steel, as shown in Figure 10.
6. Results Trend Line

The experimental and numerical tests show the common strength contribution of the hanger and horizontal reinforcement at the dapped end re-entrant corner. This result confirms the combined effect of the two orthogonal steel reinforcements at the dapped end, which differs from common design formulas and procedures that take into consideration the orthogonal direction steel reinforcement strength individually.

In this study, the suggested formula takes into consideration the contribution of the two components of the steel reinforcement by adding a ratio of the accompanying horizontal steel reinforcement strength to the strength of the vertical steel reinforcement. If \( \psi \) denotes the experimental strain ratio as in Equation 4, in which (VD) the vertical strain and (HD) the accompanying horizontal reinforcement strain. Taking into account the structural reinforced concrete concepts as follows:

- Balance between concrete compressive strength and horizontal steel reinforcement in the dapped zone.
- Minimum shear steel reinforcement.

\( \varepsilon_{VD} \) is Maximum strain in the vertical reinforcement; \( \varepsilon_{HD} \) is Accompanying strain in the horizontal reinforcement.

Let, \( A_h \) is Steel area of horizontal reinforcement, \( A_v \) is Steel area of hanger reinforcement (stirrup), by vectors summation rule, the resultant of strains is:

Total strain \( \varepsilon_T \) = \( \sqrt{(\varepsilon_{VD})^2 + (\varepsilon_{HD})^2} \)  

Let, \( \frac{\varepsilon_{HD}}{\varepsilon_{VD}} = \psi \)  

\( \psi \) is experimental strains ratio:

\( \varepsilon_{HD} = \psi \times \varepsilon_{VD} \)

Total strain \( \varepsilon_T \) = \( \sqrt{(\varepsilon_{VD})^2 + (\psi \varepsilon_{VD})^2} \)  

\( \varepsilon_T = \varepsilon_{VD} \times \sqrt{1 + \psi^2} \)

for elastic-perfectly plastic steel model;

\( f_s = \{(\varepsilon_{VD} \times \sqrt{1 + \psi^2} \times E) \leq f_y \)  

Nominal shear strength \( V_n \) = \( f_s \times A_v \)

\( V_n_{modified} = [\sqrt{1 + \psi^2} \times \varepsilon_{VD} \times E] \times A_v \)

At yielding stage, \( \varepsilon_{VD} \times E = f_y \)

So, \( V_n \ (Updated) = \sqrt{1 + \psi^2} \times f_y \times A_v \)
Let, $\kappa = \sqrt{1 + \psi^2}$, ($\kappa > 1$)  

\[ V_{n\text{ (Updated)}} = \kappa (f_y \times A_v) \]  

From above analysis one can conclude that the updated nominal shear strength is greater than the common PCI and STM method nominal shear strength by about $(1.04 - 1.16)$, where $\kappa$ factor $= (1.04 - 1.16)$ since $(\psi)=0.42$. 

This formula complies with the test result of load failure for the six dappled end beam in the experimental study, whereas the previous calculation of strength capacity was more conservative.

7. Case Study

The dapped end beam with above test specimen dimensions has $3$ meter length, depth $= 300$ mm, width $= 150$ mm and extended depth $= 150$ mm, dap length $= 150$ mm, concrete cover $= 20$ mm, re-entrant span length $= 125$ mm with $d_1=117$ mm, $f_y=500$ MPa, proportion of strains $(\psi)=0.42$ (Table 1), $A_s=200$ mm$^2$, $A_v=237$ m$^2$, $V_u$ and $V_n$: ultimate and nominal shear capacities.

7.1. Structural Analysis

The experimental test for the dapped beam was conducted by applying concentrated load of $P=165$ kN, so shearing force $V_u = 2P/3 = 2 \times 165/3 = 110$ kN and reinforcement as shown in the Figure 2. From equation below we can calculate the shear strength capacity of the dapped end beam at the re-entrant corner zone:

Hanger reinforcement have been calculated by using Equation 9 for each PCI & STM method:

\[ V_n = \frac{V_u}{\psi f_y} = \frac{V_{n\text{ (old formula)}}}{f_y} \]

For this study take: $\psi = \text{about (0.31 – 0.58)}$, average $\psi = 0.42$ (Table 2)

$\psi = \varepsilon_HD/\varepsilon_VD$,

$\varepsilon_HD = \psi \times \varepsilon_VD = 0.42 \varepsilon_VD$

Total strain $= \sqrt{(\varepsilon_VD)^2 + (0.42\varepsilon_VD)^2}$

Strength of re-entrant dap $= \sqrt{(1.1746 \varepsilon_VD^2 \times E)} \leq f_y$

\[ f_s = 1.0846 \varepsilon_VD \times E \]

\[ V_n = 1.0846 \varepsilon_VD \times E \times A_v \quad \text{(where } V_n = f_s \times A_v) \]

\[ V_n = 1.0846 \times 200 \times 500 = 10846 \text{ N This value is nearer to the experimental actual value of } V_n; 11000 \text{ N indicating increment in nominal strength of hanger reinforcement by 8.46%} \]

Therefore, the common strength of the orthogonal reinforcement is the best interpretation for the dapped end beam failure at the re-entrant area. Where the bigger value of nominal strength comes from the participation of the orthogonal reinforcement at the dapped end zone, from tests and FE analysis, the strength capacity at failure equals the nominal strength of one set of the hanger reinforcement plus the ratio from the orthogonal one, depending on the strain case. In comparison of this study’s results with other research, it was found that the increment in strength was 4–16% greater than the strength extracted by the PCI and STM approaches.

8. Conclusions

More research is needed to accurately evaluate the strength of the re-entrant dapped zone. However, the following conclusions can be drawn from this study:

- The strut and tie model (STM) provides more accurate analysis and design than the PCI method, but it doesn’t represent the actual behavior of the dapped end beam at the re-entrant zone.
- The mode of failure of the tested dapped end beams was re-entrant corner cracking, while failure by direct shear or flexural at the extended end didn’t take place.
- The current study proposes a new technique to compute the shear strength capacity of the dapped end beam by adding the contribution percentage of accompanying horizontal reinforcement strength to the strength of vertical hanger steel reinforcement using an appropriate proportion strain factor.
- The proposed range of the proportion strain factor is about $0.30-0.60$. 
The updated nominal shear strength is greater than the common PCI and STM method nominal shear strengths by about 1.04–1.16.

The finite element model has rational agreement with the experimental work in the case of elastic and plastic strains and stresses in horizontal and vertical re-entrant corner reinforcement.

The study revealed that enhancement of the shear strength of the dapped end beam might be attained by increasing the horizontal reinforcement at the re-entrant corner.

This approach needs more investigation to define the gap between the theoretical and actual re-entrant strengths.

Numerical analysis by a finite element program with more arrangements of steel reinforcement is needed to compare with common approaches.

9. Nomenclatures

<table>
<thead>
<tr>
<th>STM</th>
<th>Strut and Tie Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_u$</td>
<td>Ultimate shear force</td>
</tr>
<tr>
<td>$V_n$</td>
<td>Nominal shear force</td>
</tr>
<tr>
<td>RC</td>
<td>Reinforced Concrete</td>
</tr>
<tr>
<td>$E$</td>
<td>Elasticity modulus</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Steel reinforcement stress</td>
</tr>
<tr>
<td>$f_y\text{DEP}$</td>
<td>Yield stress steel reinforcement</td>
</tr>
<tr>
<td>$\varepsilon_{\text{VD}}$</td>
<td>Vertical steel reinforcement strain</td>
</tr>
<tr>
<td>$\varepsilon_{\text{HD}}$</td>
<td>The accompanying horizontal steel reinforcement strain</td>
</tr>
<tr>
<td>$\varepsilon_T$</td>
<td>Total strain</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Updated strength factor</td>
</tr>
</tbody>
</table>

10. Declarations

10.1. Author Contributions

Conceptualization, A.Q.M. and R.M.A.; methodology, A.Q.M.; software, A.Q.M.; validation, R.M.A.; formal analysis, A.Q.M.; investigation, A.Q.M.; resources, A.Q.M. and R.M.A.; data curation, A.Q.M.; writing—original draft preparation, A.Q.M.; writing—review and editing, R.M.A.; visualization, A.Q.M. and R.M.A.; supervision, R.M.A.; project administration, R.M.A. All authors have read and agreed to the published version of the manuscript.

10.2. Data Availability Statement

The data presented in this study are available on request from the corresponding author.

10.3. Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

10.4. Acknowledgments

Acknowledgment is dedicated to the Civil Engineering Department at the University of Baghdad for their useful support during investigation time.

10.5. Conflicts of Interest

The authors declare no conflict of interest.

11. References


